A GENERALIZED POWER-LAW MODEL OF BLOOD FLOW THROUGH TAPERED ARTERIES WITH AN OVERLAPPING STENOSIS

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To my beloved parents,

my awesome sisters,

and all family members–

Thank you for your support and unconditional love.

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ABSTRACT

A mathematical model of a generalized Power-law blood flow through a tapered artery with an overlapping stenosis is considered. The flow is assumed to be two-dimensional, unsteady, laminar, incompressible and axisymmetric. The artery is considered to be elastic and time-variant due to the pulsatile flow contributed by the pumping of heart. The continuity equation and momentum equation are derived in the cylindrical coordinate system. Then the radial coordinate transformation is used to transform the equations and boundary conditions in terms of radius of lumen before they are solved numerically using a finite difference scheme. Numerical results obtained show that the blood flow characteristics such as the velocity profiles, flow rate, resistance and wall shear stress are significantly affected by the taper angle of artery, severity of stenosis and time-variant nature of artery. As the taper angle increases, both axial velocity and flow rate increase, while resistive impedance and wall shear stress decrease. However, the radial velocity may increase or decrease with taper angle, depending on radial distance and time. In constrast, increasing the level of stenosis causes the axial velocity and flow rate to decrease, and resistance and wall shear stress to increase. As time progresses, the values of axial velocity, flow rate and resistance decrease during the first phase of cardiac cycle and increase during the second phase. Radial velocity and wall shear stress exhibit different behavior from other flow characteristics. The value of wall shear stress increases during the first phase and decreases during the second phase of cardiac cycle. The value of radial velocity decreases for all time.

ABSTRAK

Model matematik bagi aliran darah bercirikan model 'Power-law' umum melalui arteri yang mengembang dan menirus dengan stenosis bertindih dipertimbangkan. Aliran ini dianggap sebagai dua dimensi, tidak mantap, lamina, tidak boleh mampat dan simetri pada paksi. Arteri dianggap sebagai elastik dan berubah mengikut masa disebabkan oleh aliran darah yang bergantung kepada denyutan jantung. Persamaan keselanjaran dan persamaan momentum diperolehi dalam sistem koordinat silinder. Kemudian transformasi koordinat jejarian digunakan untuk mengubah persamaan dan keadaan sempadan dalam bentuk radius lumen sebelum diselesaikan secara berangka menggunakan skema beza terhingga. Keputusan berangka yang diperolehi menunjukkan bahawa ciri-ciri aliran darah seperti profil halaju, kadar aliran, rintangan dan tegasan ricih dinding secara ketara dipengaruhi oleh sudut tirus arteri, tahap stenosis dan sifat arteri yang berubah mengikut masa. Seiring dengan peningkatan sudut tirus, kedua-dua halaju paksi dan kadar aliran meningkat, manakala rintangan dan tegasan ricih dinding menurun. Walau bagaimanapun, halaju jejarian meningkat atau menurun dengan sudut tirus, bergantung kepada jarak jejarian dan masa. Sebaliknya, peningkatan tahap stenosis menyebabkan halaju paksi dan kadar aliran berkurangan, dan rintangan dan dinding tegasan ricih meningkat. Seiring dengan pertambahan masa, nilai halaju paksi, kadar aliran dan rintangan menurun semasa fasa pertama kitar kardiak dan meningkat semasa fasa kedua. Halaju jejarian dan tegasan ricih dinding mempamerkan tingkah laku yang berbeza dari ciri-ciri aliran yang lain. Nilai tegasan ricih dinding meningkat semasa fasa pertama dan berkurangan semasa fasa kedua kitar kardiak. Nilai halaju jejarian berkurangan sepanjang tempoh tersebut.

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LIST OF SYMBOLS

Roman

A	–	surface area
A_o	–	constant amplitude of pressure gradient
A_1	–	amplitude of pulsatile component of pressure gradient
$a(t)$	–	time-variant parameter
b	–	constant parameter in time variant parameter
d	–	location of stenosis
dr	–	small change in r-direction
$d\theta$	–	small change in θ-direction
dz	–	small change in z-direction
f_p	–	pulse frequency
h_m	–	height of stenosis at the middle of stenosis
$\vec{g}$	–	gravitational acceleration
g_r	–	gravitational acceleration in r-direction
g_θ	–	gravitational acceleration in θ-direction
g_z	–	gravitational acceleration in z-direction
l	–	half-length of stenosis
L	–	finite length of arterial segment
m	–	constant characterizing non-Newtonian fluid
m_s	–	slope of tapered artery
$\dot{m}$	–	mass flow rate

$\dot{M}_c$	–	convective momentum rate
$\dot{M}_m$	–	molecular momentum rate
n	–	a constant characterizing non-Newtonian fluid
p	–	pressure
Q	–	flow rate
R	–	radius of lumen in constricted arterial segment
R_o	–	radius of lumen in unconstricted arterial segment
V	–	volume
$\vec{v}$	–	velocity vector
v_r	–	velocity component in r-direction (radial velocity)
v_θ	–	velocity component in θ-direction
v_z	–	velocity component in z-direction (axial velocity)

Greek

α	–	taper angle of artery
$\dot{\gamma}$	–	shear strain rate
$\dot{\boldsymbol{\gamma}}$	–	shear strain rate tensor
η	–	non-Newtonian fluid viscosity
Λ	–	resistive impedance
λ	–	wavelength of pressure wave
μ	–	Newtonian fluid viscosity
ρ	–	fluid density
$\boldsymbol{\sigma}$	–	molecular momentum flux tensor
$\boldsymbol{\tau}$	–	shear (viscous) stress tensor
τ	–	shear (viscous) stress
ω	–	angular frequency of pressure wave

CHAPTER 1

INTRODUCTION

1.1 Introduction

The circulatory system, also known as cardiovascular system, consists of the heart, blood vessels and blood. It is a very crucial human system that any abnormality can leads to malfunction of organs or other body system which in turns cause more serious physiological problems and death. In order to understand the reason of vascular diseases, many experimental and numerical analyses have been carried out to investigate the blood flow through different parts of circulatory system. However, a special attention has been given on study of blood flow in arteries.

The arteries are one of the three major types of blood vessels, other than capillaries and veins. Arteries are the high-pressure blood vessels that transport blood from the heart, through increasingly small blood vessels; smaller arteries, arterioles, and capillaries. The role of systemic arteries to carry oxygenated blood is well known that many people mistakenly think of arteries only as vessels that carry oxygenated blood. This is not true since the blood flowing through the pulmonary artery is deoxygenated and the blood flowing through the pulmonary vein is oxygenated. A more appropriate distinction between arteries and veins is that arteries carry blood at a relatively higher pressure than the pressure within veins [1].

Because of that, the structure of an artery is different from the vein. An artery is composed of three layers. Each layer has its own functions in blood vessel mechanics and transport phenomena. The innermost layer, tunica intima consists of a thin monolayer of endothelial cells that line the inner surface of the vessel. Their anatomical location causes these cells to be subjected to large variations in stress and strain. The middle layer, tunica media is comprised of alternating layers of interconnected smooth muscle cells and elastic lamellae. The outermost layer, tunica adventitia, consists of loose, more organized fiberous connective tissue and may have less influence on mechanics [2]. The inside of the vessel where the blood flows is called lumen of an artery. Figure 1.1 below illustrates the normal layers of an artery.

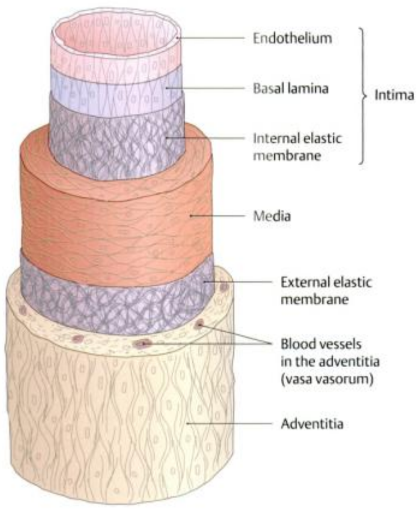

Figure 1.1: Normal layers of artery. This figure is from [3].

The normal condition of arteries and blood flow can be affected by vascular diseases. One of the most common vascular diseases that cause serious morbidity and death is atherosclerosis. Atherosclerosis is a disorder characterized by progressive abnormal narrowing and occlusion of the lumen of artery. The narrowing is caused by obstruction or stenosis, which is formed by deposition of fatty substances, cholesterol, cellular waste products, calcium, and fibrin in the inner layer of an artery. As the deposition continues to accumulate, it will build up into plaque. If a piece of plaque breaks away, it can cause bleeding into the plaque. The formation of thrombus (blood clots) around the plaque may cause the condition to get worse.

As a result, it increasingly disrupts the blood flow or completely blocks the flow of blood to organs, body tissues and structures. It is particularly dangerous in the coronary and carotid arteries due to the critical oxygen requirement of the heart and brain. The carotid arteries provide blood to the brain, while coronary arteries provide blood to the heart. When the blood supply is limited, patients can suffer stroke and heart attack, respectively. If renal arteries that supply blood to the kidneys are severely obstructed, there is a serious risk of developing chronic kidney disease [1, 4]. Stenosis also can cause an increase in the wall stiffness or a decrease in compliance of blood vessels [1], impairing the function of the vessel in transporting blood effectively.

Many studies suggest that diseases related to abnormality occurring in blood vessel appear to be strongly influenced by hemodynamics. It is widely accepted that the hemodynamic concept of atherosclerosis considers the laws of fluid dynamics as the primary factor in the mechanisms development of atherosclerosis. Regions experiencing relatively lower levels of wall shear stress and regions experiencing oscillatory flow reversal are believed to have higher tendency to form stenosis. It is deduced that regions near branching, bifurcation junctions and curvature are common sites for the formation and development of atherosclerosis [5, 6, 7, 8]. In addition, investigations have shown that the flow behavior in the stenosed artery is quite different than one in the normal arteries.

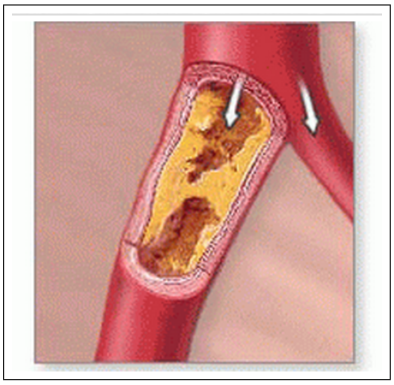

Figure 1.2: Example of presence of stenosis in artery. This figure is from [9].

1.2 Problem Statement

Realizing that the initiation and progression of vascular diseases are strongly influenced by the characteristics of the blood, the flow as well as the vessels, extensive studies have been done by researchers to acquire more knowledge on flow parameters such as velocity, flow rate, pressure drop and shear stress. By understanding the fluid mechanics aspects of arterial stenoses, medical practitioners and bio-medical engineers can design better bio-medical instruments and use more effective approaches in treatment and diagnosis of diseases. The significance of studying the problem which affects the worldwide community motivates the current study of blood flow in artery. In this research, we seek to understand the properties of moving blood, the characteristics of the flow and the effect of stenoses on the flow.

1.3 Objectives of Study

The main aim of the study is to develop a mathematical model for non-Newtonian blood flow in a stenosed artery. In particular, the objectives are:

(i) to derive the governing equations of blood flow in terms of pressure and viscous shear stress;

(ii) to construct the geometry of an overlapping stenosis mathematically;

(iii) to solve the governing equations numerically using finite difference method;

(iv) to analyse the velocity profile, flow rate, wall shear stress, and resistance of the blood flow for different parameters.

1.4 Scope of Study

The study focuses on small tapered artery with overlapping stenoses. The flow of blood is considered to be two-dimensional,incompressible, unsteady, laminar and axisymmetric. The blood is treated as non-Newtonian fluid following generalized Power-law model. The artery is modelled as a distensible cylindrical tube where the considered wall motion is only due to the systolic and diastolic phase of pumping heart.

1.5 Significance of Study

This research helps reseachers to understand the basic concept of blood flow in constricted artery, where a mathematical model is presented to describe the blood flow through tapered artery with an overlapping stenosis. From here, people interested in this field can move to more challenging problems which may involved more hemodynamic factors and flow parameters, and solve the mathematical model with more sophisticated methods.

1.6 Outline of Study

This dissertation is divided into six chapters. This chapter presents an introduction to the research background, objectives, scope and significance of research. The upcoming Chapter 2 will touch on some basic rheology of blood, and review some fluid models, types of stenoses, and methods of solution that other researchers had work on. Chapter 3 concerns with the formulation of the problem and comprised of six sections including the introduction. In Section 3.2, the geometry of the arterial segment is constructed mathematically. Then, the continuity equation and momentum equation are derived directly in cylindrical form in Section 3.3 and 3.4. In Section 3.5, the expressions for stress tensor components are given where the derivation of the formulae can be referred in Appendix A. The mathematical model of considered problem is stated in Section 3.6.

Next, the solution procedure is presented in Chapter 4, where Section 4.1 introduces the content of the chapter, and in Section 4.2, the equations and boundary conditions are transformed using the radial coordinate transformation. Then, in Section 4.3, the radial velocity component is derived from the transformed continuity equation. The finite difference method is applied in Section 4.4 to discretize the velocity components, stress components and boundary conditions. Discretizations of other blood flow characteristics are also shown in this section. For numerical computation, the solution procedure is written as a MATLAB programming code, and is supplemented in Appendix B.

The numerical results are discussed in Chapter 5. This chapter consists of six sections. The first section introduces the content of the chapter. In Section 5.2, we discuss the results for the axial velocity at different taper angles, at different axial positions, and different times. Similar analyses are done on radial velocity in Section 5.3. Next, Section 5.4 shows the results for flow rate, Section 5.5 on resistance and Section 5.6 on wall shear stress. The final chapter, Chapter 6 will wrap up the discussed problem and give some recommendations for future research.

REFERENCES

1. Waite, L. and Fine, J. *Applied Biofluid Mechanics.* USA: McGraw Hill Companies, Inc. 2007.

2. Canfield, T. R. and Dobrin, P. B. Mechanics of Blood Vessels. In Peterson, D. R. and Bronzino, J. D. (Eds.). *Biomechanics: Principles & Applications.* Boca Raton, FL: CRC/Taylor &Francis. 11.1–11.13. 2008.

3. Schuenke, M., Schulte, E. and Schumacher, U. *General Anatomy and Musculoskeletal System: THIEME Atlas of Anatomy.* Stuttgart, Germany: Georg Thieme Verlag. 2010.

4. Levick, J. R. *An Introduction to Cardiovascular Physiology.* 5th Edition. London: Hodder Arnold. 2010.

5. Ku, D. N. Blood Flow in Arteries. *Annual Review of Fluid Mechanics*, 1997. 29(1): 399–434.

6. Westerhof, N., Stergiopulas, N. and Noble, M. I. M. *Snapshots of Hemodynamics: an Aid for Clinical Research and Graduate Education.* USA: Springer. 2005.

7. Caro, C. G. Discovery of the Role of Wall Shear in Atherosclerosis. *Arteriosclerosis, Thrombosis, and Vascular Biology*, 2009. 29(2): 158–161.

8. Campbell, I. and Taylor, W. R. Flow and Atherosclerosis. In Hsiai, T. K., Blackman, B. and Jo, H. (Eds.). *Hemodynamics and Mechanobiology of Endothelium.* Singapore: World Scientific. 1–38. 2010.

9. Gu, X. C., Yu, Y. and Wang, C. Surgical Treatment for Diffuse Coronary. In Aronow, W. S. (Ed.). *Artery Bypass.* InTech: DOI:10.5772/54416. 277–289. 2013.

10. Shukla, J. B., Parihar, R. S. and Rao, B. R. P. Effects of Stenosis on Non-Newtonian Flow of the Blood in an Artery. *Bulletin of Mathematical Biology*, 1980. 42: 283–294.

11. Pralhad, R. N. and Schultz, D. H. Modeling of Arterial Stenosis and Its Applications to Blood Diseases. *Mathematical Biosciences*, 2004. 190: 203–220.

12. Haldar, K. Effects of the Shape of Stenosis on the Resistance to Blood Flow Through an Artery. *Bulletin of Mathematical Biology*, 1985. 47(4): 545–550.

13. Chakravarty, S., Datta, A. and Mandal, P. K. Analysis of Nonlinear Blood Flow in a Stenosed Flexible Artery. *International Journal of Engineering Science*, 1995. 33(12): 1821–1837.

14. Tu, C. and Deville, M. Pulsatile Flow of Non-Newtonian Fluids Through Arterial Stenoses. *Journal of Biomechanics*, 1996. 29(7): 899–908.

15. Chakravarty, S. and Mandal, P. K. Mathematical Modelling of Blood Flow Through an Overlapping Arterial Stenosis. *Mathematical and Computer Modelling*, 1994. 19(1): 59–70.

16. Whitmore, R. *Rheology of Circulation.* Pergamon Press. 1968.

17. Jeffords, J. V. and H., K. M. Concerning the Geometric Shape of Arteries and Arteriols. *Angiology*, 1956. 7: 105–136.

18. Bloch, E. H. A Quantitative Study of the Hemodynamics in the Living Microvascular System. *American Journal of Anatomy*, 1962. 110: 125–153.

19. Mandal, P. K. An Unsteady Analysis of Non-Newtonian Blood Flow Through Tapered Arteries with a Stenosis. *International Journal of Non-Linear Mechanics*, 2005. 40: 151–164.

20. Chakravarty, S. and Mandal, P. K. Two-dimensional Blood Flow Through Tapered Arteries Under Stenotic Conditions. *International Journal of Non-Linear Mechanics*, 2000. 35: 779–793.

21. Ismail, Z., Abdullah, I., Mustapha, N. and Amin, N. A Power-law Model of Blood Flow Through a Tapered Overlapping Stenosed Artery. *Applied Mathematics and Computation*, 2008. 195: 669–680.

22. Mandal, P. K., Chakravarty, S. and Mandal, A. Numerical Study of the Unsteady Flow of Non-Newtonian Fluid Through Differently Shaped Arterial Stenoses. *International Journal of Computer Mathematics*, 2007. 84(7): 1059–1077.

23. Lieber, B. B. Arterial Macrocirculatory Hemodynamics. In Bronzino, J. D. (Ed.). *The Biomedical Engineering Handbook*. Boca Raton: CRC Press LLC. 2nd Edition. 10.1–10.10. 2000.

24. Lee, K. W. and Xu, X. Y. Modelling of Flow and Wall Behaviour in a Mildly Stenosed Tube. *Medical Engineering & Physics*, 2002. 24: 575–586.

25. Moayeri, M. S. and Zendehbudi, G. R. Effects of Elastic Property of the Wall on Flow Characteristics Through Arterial Stenoses. *Journal of Biomechanics*, 2003. 36: 525–535.

26. Merrill, W. Rheology of Blood. *Physiological Reviews*, 1969. 49(4): 863–888.

27. Caro, C. G., Pedley, T. J., Schroter, R. C. and Seed, W. A. *The Mechanics of Circulation*. Oxford: Oxford University Press. 1978.

28. Venkatesan, J., Sankar, D. S., Hemalatha, K. and Yatim, Y. Mathematical Analysis of Casson Fluid Model for Blood Rheology in Stenosed Narrow Arteries. *Journal of Applied Mathematics*, 2013. 2013. doi:10.1155/2013/583809.

29. Riahi, D. N., Roy, R. and Cavazos, S. On Arterial Blood Flow in the Presence of an Overlapping Stenosis. *Mathematical and Computer Modelling*, 2011. 54(11-12): 2999–3006.

30. Chakravarty, S. and Datta, A. Effects of Stenosis on Arterial Rheology Through a Mathematical Model. *Mathematical and Computer Modelling*, 1989. 12(12): 1601–1612.

31. Ishikawa, T., Guimaraes, L. F. R., Oshima, S. and Yamane, R. Effect of Non-Newtonian Property of Blood on Flow Through a Stenosed Tube. *Fluid Dynamics Research*, 1998. 22: 251–264.

32. Mazumdar, J. *Biofluid Mechanics*. Singapore: World Scientific Publishing Co. Pte. Ltd. 1992.

33. Bird, R. B., Stewart, W. E. and Lightfoot, E. N. *Transport Phenomena*. 2nd Edition. USA: John Wiley & Sons, Inc. 2002.

34. Lee, B. Y., Assadi, C., Madden, J. L., Kavner, D., Trainor, F. S. and McCann, W. J. Hemodynamics of Arterial Stenosis. *World Journal of Surgery*, 1978. 2(5): 621–629.

35. Ikbal, M. A. Viscoelastic Blood Flow Through Arterial Stenosis Effect of Variable Viscosity. *International Journal of Non-Linear Mechanics*, 2012. 47(8): 888–894.

36. Cengel, Y. A. and Cimbala, J. M. *Fluid Mechanics: Fundamentals and Applications*. McGraw Hill. 2006.

37. Li, W.-H. and Lam, S.-H. *Principles of Fluid Mechanics*. Addison-Wesley Publishing Co., Inc. 1964.

38. Pedley, T. J. *The Fluid Mechanics of of Large Blood Vessels*. Cambridge University Press. 1980.

39. Burton, A. C. *Physiology and Biophysics of the Circulation: An Introductory Text*. Year Book Medical Publisher. 1966.

40. McGee, S. and Seshaiyer, P. Finite Difference Methods for Coupled Flow Interaction Transport Models. *Seventh Mississippi State - UAB Conference on Dierential Equations and Computational Simulations*, Nov 1-3, 2007. San Marcos: Texas State University. 2009: 171–184.

41. McDonald, D. A. *Blood Flow in Arteries*. London: Edward Arnold. 1974.

42. Fry, D. L. Acute Vascular Endothelial Changes Associated with Increased Blood Velocity Gradient. *Circulation Research*, 1968. 22: 165–197.