COMPARING CHEBYSHEV POLYNOMIALS AND ADOMIAN DECOMPOSITION METHOD IN SOLVING NONLINEAR VOLTERRA INTEGRAL EQUATIONS OF SECOND KIND

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Specially dedicated to my beloved parents,

Mohamad Sapawi bin Ramli and Zanariah bt Mahmud

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ABSTRACT

The nonlinear integral equations are usually difficult to solve analytically and in many cases, it is required to obtain the approximate solutions. The nonlinear Volterra integral equation of second kind is one of them. This dissertation compares two methods that are used in order to solve nonlinear Volterra integral equation of second kind. Those are Chebyshev polynomials and Adomian decomposition method. The Chebyshev polynomials are developed to approximate the solution of linear and nonlinear Volterra integral equations. While, Adomian decomposition method, is a method that can be applied directly for all type of linear and nonlinear integral equations and maintain high accuracy of numerical solution. Hence, the best method is picked based on the absolute error that will be compared with the exact solution.

ABSTRAK

Persamaan kamiran tidak linear kebiasaannya sukar diselesaikan secara analitik dan untuk menyelesaikannya memerlukan penyelesaian anggaran. Persamaan kamiran Volterra tidak linear merupakan salah satu darinya. Penyelidikan ini membandingkan dua kaedah untuk menyelesaikan Persamaan kamiran Volterra tidak linear iaitu kaedah polynomial Chebyshev dan kaedah penguraian Adomian. Kaedah polynomial Chebyshev dibentuk untuk menganggarkan penyelesaian persamaan kamiran Volterra linear dan tidak linear. Manakala, kaedah penguraian Adomian merupakan kaedah yang boleh digunakan secara langsung samaada pada persamaan kamiran tidak linear atau linear dengan mengekalkan ketepatan daripada penyelesaian berangka. Kaedah yang paling baik dipilih dari penyelidikan ini berdasarkan ralat mutlak apabila dibandingkan dengan penyelesaian tepat.

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CHAPTER 1

INTRODUCTION

1.1 Research Background

In mathematics, the integral equation can be classified into two classes, which are Volterra integral equations and Fredholm integral equations. The Volterra integral equations are special type of integral equations. Integral equations are used as mathematical models for many and varied physical situations and integral equations also occur as reformulations of other mathematical problems. Besides, it play an important role in many branches of linear and nonlinear functional analysis and their applications in the theory of elasticity, engineering mathematical physics, potential theory, electrostatics and radiative heat transfer problems (Awadeh *et al*, 2009). In general, an integral equation is any equation which involves integrals of an unknown function $x(t)$. For Volterra integral equations, the unknown function $x(t)$ will be real or complex valued function of a single real variable t (Miller). A nonlinear Volterra integral equation of second kind has the form

$$y(x) = f(x) + \int_a^x K\big(x, t, f(t)\big)dt \qquad , x \geq a \qquad (1.1)$$

where the functions $f(x)$ and $K(x,t,f(t))$ are known and $y(x)$ is unknown function. In this research nonlinear Volterra integral equations will be solved using Chebyshev polynomials and Adomian decomposition method.

Several numerical methods for solving the nonlinear Volterra integral equations have been presented. According to Maleknejad *et.al* (2007) several authors present numerical solutions for nonlinear Volterra integral equations by using Galerkin, Taylor polynomials and other methods. Chebyshev polynomial is a special group of polynomials whose properties and applications were discovered by the Russian mathematician, Pafnuty Lvonich Chebyshev. It also play significant role in nearly every area of numerical analysis, including polynomial approximation, numerical integration and integral equations. Besides that, Chebyshev polynomial is one of the orthogonal polynomials that are developed to approximate the solutions of linear and nonlinear Volterra integral equations.

Another method that will be used is Adomian decomposition method. Adomian decomposition method is a semi-analytical method for solving ordinary and partial nonlinear differential equations. The method was developed from the 1970s to the 1990s by George Adomian, chair of the Center for Applied Mathematics at the University of Georgia. This method has proven rather successful in dealing with linear as well as nonlinear problems. It also continues to develop and gain ground in applied mathematics and integral methods. Besides, Adomian decomposition method is mathematical tools providing analytical and rapidly convergent solutions to a variety of problems in nonlinear science (Azreq, 2009). Recently, Adomian decomposition method has been applied to solve the systems of linear and nonlinear Volterra integral equations of first and second kind.

Computers have brought a fundamental change in the nature research and in education in science and engineering. Experimentalists and theoreticians use computer to collect and analyse data and manipulate equations numerically and symbolically. In this research we are going to use MATHEMATICA software in order to solve both methods. MATHEMATICA is a computational software program used in many scientific, engineering, mathematical and computing fields, based on symbolic mathematics. It was conceived by Stephen Wolfram and is developed by Wolfram Research of Champaign, Illinois.

Besides, this software have many features such as, it can automatic translate English sentences into MATHEMATICA code, has special mathematical function library, it also can support complex number, arbitrary precision, interval arithmetic and symbolic computation. MATHEMATICA is a very large and seemingly complex system. In this research, MATHEMATICA will be used as a tool to solve both methods numerically.

1.2 Problem Statement

There are several numerical method used to solved nonlinear Volterra integral equations. This research is conducted to find the best method between Chebyshev polynomials and Adomian decomposition method in solving the nonlinear Volterra integral equations of second kind. The problem statements that will be discussed in this research are as follows. The nonlinear Volterra integral equation is solved using Chebyshev polynomials and Adomian decomposition method. Both method are compared using MATHEMATICA software in order to get the absolute error. Hence, we can determined the method that is better in solving nonlinear Volterra integral equations. The calculations of both method can be seen in Chapter 3 and Chapter 4.

1.3 Objectives of the Study

The objectives of the study are:

a) To find the solution of nonlinear Volterra integral equations of second kind using Chebyshev polynomials.
b) To find the solution of nonlinear Volterra integral equations of second kind using Adomian decomposition method.
c) To compare the method of Chebyshev polynomials and Adomian decomposition method using MATHEMATICA software.

1.4 Scope of the Study

This research will focus on finding the best method between Chebyshev polynomials and Adomian decomposition method in solving nonlinear Volterra integral equations of second kind. The computations involved in this study are performed using mathematical software MATHEMATICA.

1.5 Significance of the Study

This research can gives us the best method in solving nonlinear Volterra integral equations. Furthermore, we can sharpen our knowledge about the mathematical software MATHEMATICA and learn new numerical methods which are Chebyshev polynomials and Adomian decomposition method in order to solve this kind of problem.

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