EFFECTS OF PRESSURE GRADIENT ON STEADY FORCED CONVECTION USING HOMOTOPY ANALYSIS METHOD (HAM)

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This dissertation is dedicated to my family, friends, all the people that matter most in my life for their love endless support and encouragement. I am supremely indebted to you all.

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ABSTRACT

The study of convective heat transfer has generated many interests and become more important recently because of their wide applications in engineering and in several industrial processes. In this dissertation, the effects of pressure gradient and Prandtl number on velocity and temperature profiles on convective heat transfer in boundary layer over a flat plate are discussed. The governing boundary layer equations are transformed into a system of non-dimensional equations and then solved using Homotopy Analysis Method. This method provides the freedom to choose the initial guess function. This is used to solve the boundary layer problem. The approximate analytical results are then compared with published results obtained by Homotopy Perturbation Method and Finite Difference Method. This study is focused on comparing the accuracy of results and applicability of the three methods which are HAM, HPM and NM. Homotopy analysis method which is an approximate analytical approach compares between the HPM and NM reveals that the HAM technique is a powerful tool for the boundary layer. Results of HAM in the absence of pressure gradient are better than the HPM and Numerical Method (Finite Difference Method).

ABSTRAK

Kajian pemindahan haba perolakan telah menimbulkan pelbagai minat. Pada masa ini, aplikasi tersebut telah menjadi sangat penting kerana aplikasi tersebut sangat luas terutamanya dalam bidang kejuruteraan dan beberapa proses dalam industri. Disertasi ini membincangkan kesan Kecerunan Tekanan dan nombor Prandtl pada halaju dan suhu dalam proses pemindahan haba perolakan di lapisan sempadan di atas plat rata. Persamaan lapisan sempadan ditukar ke dalam sistem persamaan bukan dimensi dan kemudian diselesaikan dengan Kaedah Analisis Homotopi. Kaedah ini memberi kebebasan untuk memilih fungsi tekaan awal. Kaedah ini digunakan untuk menyelesaikan masalah lapisan sempadan. Keputusan analisis anggaran kemudiannya dibandingkan dengan keputusan yang diperolehi oleh Kaedah Homotopi Pengusikan dan Kaedah Berangka dengan menggunakan Kaedah Perbezaan Terhingga. Matlamat utama adalah memberi tumpuan kepada perbandingan ketepatan keputusan dan kebolehgunaan tiga kaedah iaitu HAM, HPM dan NM. Perbandingan pendekatan analisis antara NM dan HPM mendedahkan bahawa teknik HAM adalah kaedah yang berkesan untuk lapisan sempadan. Keputusan HAM tanpa kecerunan tekanan lebih baik berbanding HPM dan kaedah berangka (Kaedah Perbezaan Terhingga).

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LIST OF SYMBOLS

u, v	-	Fluctuating velocity components in x and y directions
T	-	Temperature
C_f	-	Skin friction coefficient
T_w	-	Wall temperature
T_∞	-	Local ambient temperature
U_∞	-	Characteristic velocity
H	-	Convective heat transfer coefficient
P	-	Pressure
x, y	-	Coordinates along and perpendicular to the plane
τ_w	-	Wall shear stress
q_w	-	Heat flux at wall
K	-	heat transfer coefficient
a, b, c, d	-	constants of velocity profile
e, f, g, i	-	constants of temperature profile
h	-	auxiliary convergence-control parameter
L	-	linear operator
N	-	Non-linear operator
NM	-	Numerical Method
PM	-	Perturbation Method
HPM	-	Homotopy Analysis Method
HAM	-	Homotopy Perturbation Method
f	-	Dimensionless velocity function
H	-	Dynamic similarity variable
m	-	Pressure gradient parameter
Pr	-	Prandtl number
Re	-	Reynold number

p	-	Parameter of Homotopy
q	-	embedding parameter
k	-	Thermal conductivity
ν	-	Kinematic viscosity
Nu	-	Nusselt number

GREEK SYMBOLS

ρ	-	Density
η	-	Dimensionless variable
δ	-	Boundary layer thickness
ν	-	Kinematics viscosity
θ	-	Dimensionless temperature
α	-	Thermal diffusivity
ψ	-	Streamline function

SUPERSCRIPTS

f	-	Fluid
‘	-	Differentiation with respect to η

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CHAPTER 1

INTRODUCTION

1.1 Introduction

In this chapter, Section 1.2 discusses the background of the study. Problem statements of the study are presented in Section 1.3. Section1.4 deals with an Objective of the study. Scopes of the study is presented in Section 1.5. Significance of the study is explained in Section 1.6. Section 1.7 deals with the dissertation outline.

1.2 Background of the Study

Heat transfer occurs when there is a temperature difference in a region or between a region and its surroundings. Heat is transferred during the process by convection and conduction. In motion of a gas or liquid individual particles which are at different temperatures come in contact with each other [Kakac and Yener, 1980]. Convection which means the transfer of heat through a fluid is caused by molecular motion.

Forced convection arises when heat transfer by convection is assisted by external means such as fan, wind, mixer, and many others. Forced convection is also driven by other external forces such as flow speed and pressure gradient. Heat transfer by convection is a complex process. When the fluid flow changes significantly with temperature, we cannot predict the velocity without knowing the temperature presented [Cebeci and Bradshaw, 1984].

In 1904, Ludwig Prandtl a German Scientist introduced the concept of boundary layer theory as the platform of modern fluid dynamics today. Boundary layer is a thin layer that exists close to the surface of a solid body when a real fluid flows past the body. He assumed a no-slip condition at the surface and that frictional effects were experienced only in the boundary layer, a thin region near the surface. A Boundary layer in a flowing fluid can either be laminar (layered) or turbulent (disordered) depending on the value of the Reynolds number. Boundary layered is said to be laminar when the Reynolds number is smaller and the stream wise velocity changes uniformly as it moves away from the wall. In case of turbulent the value of Reynolds number is higher and unsteady swirling flows of the stream wise velocity inside the boundary layer [Head and Bandyopadhyay, 1981].

A Newtonian fluid is one that exhibits a constant viscosity despite application of any external force or stress. A Newtonian fluid is fluid which obeys the Newton's law of viscosity. The viscosity of such a fluid may however change if exposed to different temperatures or pressures in contrast to force. Many different fluids are Newtonian including water, air and certain oils. A non-Newtonian fluids which is a broad class of fluids, is a relation connecting shear rate and shear stress which is a non-linear, higher order and complex in form. Fluids which do not obey the Newton's law of viscosity are called non-Newtonian fluids. Pressure gradient over a flat plate subject to a constant temperature is of great importance in boundary layer convection heat transfer. Pressure gradient as result of force convection affects boundary layer growth and results in reducing boundary layer thickness by [Rebay and Padet, 2005]. The fluid flow over a flat plate starts as a boundary layer, in which motion is highly ordered and fluid flow can be identified in stream lines. As mentioned by Miyake et al. (1995), it was shown that thin layer approximation is valid; the effect of pressure gradient in the flow is quite different from that in a boundary layer flow.

Our study is on convective heat transfer equations of boundary layer with pressure gradient over a flat plate using homotopy analysis method (HAM). We will look into the effects of pressure gradient and Prandtl number upon velocity and temperature profiles and then compared our result with Fathizadeh and Rashidi (2009). The momentum and energy equations are solve by Homotopy Analysis Method. An analytic method is proposed for determining the valid region of convergence of control parameter of the homotopy series, as an alternative to the classical way of adjusting the region through graphical analysis of Turkyilmazoglu (2010).

1.3 Problem Statements

This research is an extension work of Fathizadeh and Rashidi (2009), in which they studied boundary layer convective heat transfer with pressure gradient using Homotopy Perturbation Method (HPM) over a flat plate.

We solve the problem using Homotopy Analysis Method (HAM) and extended to the problem by studying the effects of Prandtl number alongside with pressure gradient. In order to study this problem, some questions need to be answered: How to solve the problem using Homotopy Analysis Method (HAM) and extend the problem by studying the effects of Prandtl number alongside with pressure gradient? How does Prandtl number and pressure gradient affect the velocity and temperature profiles in boundary layer convective heat transfer over a flat plate? How to compare our HAM results obtained in the absence of pressure gradient with methods used such as Homotopy Perturbation Method (HPM) and Numerical Method (NM)? How to transform the governing boundary layer equations into a system of non-dimensional equations?

Here homotopy analysis method, which gives us good freedom to choose the initial guess, is used to solve boundary layer problem and the analytical results are compared with those of Homotopy Perturbation Method (HPM) and the Numerical Method (NM) using Finite Difference Method obtained by Ghiaasiaan (2011). We focused on the results accuracy and applicability of different methods. In particular we look at the accuracy of the HAM with HPM to NM.

1.4 Objectives of the Study

The objectives of this study are:

(i) To determine the mathematical formulation of boundary layer convective heat transfer over a flat plate.

(ii) To find the corresponding approximate analytic solutions using Homotopy Analysis Method (HAM).

(iii) To investigate boundary layer convective heat transfer over a flat plate in the presence of pressure gradient.

(iv) To compare the results of the three methods: HAM, HPM and NM, in particular their accuracy and applicability on the problem.

(v) To show and discuss the effects of Prandtl number and pressure gradient on velocity and temperature profiles.

1.5 Scope of the Study

The study of boundary layer convective heat transfer with pressure gradient using HAM over a flat plate is considered in this dissertation. The problem will be narrow down to force convective heat transfer over a flat plate with pressure gradient. The analysis is only based on Prandtl number and pressure gradient. The convective heat transfer equations of boundary layer with pressure gradient over a flat plate are solved using Homotopy Analysis Method (HAM). The results are discussed based on the velocity profile and temperature profile.

1.6 Significance of the Study

In this dissertation we study the boundary layer convective heat transfer with pressure gradient using Homotopy Analysis Method (HAM) over a flat plate. The basic governing equations describing the boundary layer convective heat transfer phenomenon are non-linear in nature. The Homotopy Perturbation Method and numerical solution which are widely used by: physicists, and engineers to handle these types of non-linear physical phenomena.

In numerical method; stability and convergence should be considered, so as to avoid divergent or inappropriate results. Most of the time the Homotopy Perturbation method is reliable, we obtain many interesting and important results, due to reduction in the size of computational domain which give this method a wider applicability. This technique does not require a small parameter in the system unlike the perturbation methods. The approximation obtained in this method is valid not only for small parameters, but also for very large parameters. For special choices of the initial guesses it is shown that the convergence-control parameter does not cover the HPM. Clearly, this limitation of HPM yields a non-convergence series of solution. So the significant part is to tackle the governing nonlinear equations which govern the flow with such a method which does not require this limitation. We will use Homotopy Analysis Method (HAM). By the convergence-control parameter this method (HAM) can however prevented to occur. The effect of small parameter increases on the accuracy of the analytical results of two methods that is Finite Difference Method (FDM) and Homotopy Perturbation Method (HPM) also have been studied. HAM can give much better approximations for nonlinear differential equations than the previous solutions HPM. The solutions of HAM can be expressed with different functions and therefore they can be originated from the nature of the problems. The related results in HAM converge more rapidly. In HAM, the auxiliary parameter provides us with a convenient way to adjust and control the convergence and its rate for the solutions series.

1.7 Dissertation Outline

This dissertation consists of 6 chapters; Chapter 1 begins with introduction, background of study which outlines the general introduction followed by problem statement, objectives of the study, and scopes of research and significance of the present work. The literature review regarding the problems outlined in the objectives is presented and discussed in detail, in Chapter 2.

In Chapter 3, we presented the mathematical formulation of boundary layer convective heat transfer over a flat plate with constant temperature with pressure gradient. The transformations of the governing equations are also showed from the dimensional to non-dimensional equations in this Chapter. In Chapter 4, the methodology used to solve the problem using Homotopy Analysis Method is discussed. Next, in Chapter 5 we presented results and discussions on the method employed. Graphs and tables are also presented.

Finally, we give a detailed conclusion in Chapter 6 with the summary of research alongside with some future recommendations. Further, all the references are listed after Chapter 6. After that is the appendices includes the MATHEMATICA codes for the graphs used in this dissertation.

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