

EVALUATION OF REED-SOLOMON (R-S) CODING
PERFORMANCE FOR M-ARY MODULATION IN AWGN AND
MULTIPATH FADING CHANNELS

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To my beloved mother, father and family.

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ABSTRACT

In digital communication, Reed-Solomon (RS) codes refer to as a part of channel coding that had becoming very significant to better withstand the effects of various channel impairments such as noise, interference and fading. This signal processing technique is designed to improve communication performance and can be deliberate as medium for accomplishing desirable system trade-offs. As such, it is crucial that thorough evaluation on its performance is being carried out for justification. The purpose of this study is to evaluate the performance of RS coding system using M-ary modulation over AWGN and multipath fading channels. Rayleigh fading channel is used to represent the multipath fading channel and Multiple Frequency Shift Keying (MFSK) will be used as the modulation technique throughout this study. To achieve this, extensive simulation were performed by varying different type of parameters pertaining RS codes based on its characteristic. Computer simulation tool, MATLAB will be used to create and run extensively the entire simulation model for performance evaluation. It is discovered that the performance of RS codes can be assess through the function of its block size, redundancy and code rate and it is observed using Bir-Error Rate (BER) performance curve. Because RS codes work at byte level, thus it is also apparent that RS codes can perform well against burst noise. For more comprehensive study on RS Codes performance, different type of modulation techniques may be applied in the model and many other RS Codes with larger block sizes may be use.

ABSTRAK

Didalam komunikasi digital, kaedah kod Reed-Solomon (RS) adalah merujuk kepada penulisan kod untuk saluran (channel). Konsep kod untuk saluran adalah amat penting untuk memastikan keupayaan sesuatu system penghantaran dalam menghadapi pelbagai campurtangan hingar dan kehilangan signal semasa proses penghantaran melalui saluran transmisi. Dengan mempunyai teknik kod yang betul, prestasi sesuatu penghantaran signal dapat diperbaiki. Projek ini akan membuat penilaian terhadap prestasi kod RS dengan menggunakan kaedah modulasi (M-ary) dengan melalui saluran Pertambahan Bunyi Hingar Putih (AWGN) dan kelenturan pelbagai laluan (multipath fading). Model saluran kelenturan pelbagai laluan Rayleigh digunakan untuk menggambarkan kelenturan pelbagai laluan dan teknik modulasi MFSK digunakan sebagai kaedah transmisi. Untuk menjayakan projek ini, kaedah simulasi model digunakan dengan menggunakan program MATLAB untuk mereka model-model simulasi yang dikehendaki. Daripada keputusan simulasi yang diperolehi, prestasi kod RS boleh dinilai melalui fungsi saiz, kod berulang (redundancy) dan kadar kod. Semua penilaian tersebut dibuat berdasarkan lenturan graf prestasi kod RS dengan menilai graf kadar kesilapan bit. Disamping itu, kod RS juga adalah amat agam dalam mengatasi masalah letusan (burst) kesilapan dalam sistem. Penilaian seterusnya boleh dibuat dengan menggunakan pelbagai kaedah modulasi ataupun dengan menggunakan kod RS yang lebih besar untuk dalam memberikan lenturan graf prestasi yang terbaik.

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LIST OF SYMBOLS

m	-	No of bits per symbol
N,n	-	No of codeword length
k	-	No of data symbol (message)
t	-	No of error correcting capability
R	-	Code rate
ϕ	-	Phase
ω_o	-	Frequency
t	-	Time
f_d	-	Doppler shift
v	-	Velocity
λ	-	Wavelength

LIST OF ABBREVIATIONS

RS	-	Reed-Solomon
BER	-	Bit-Error Rate
AWGN	-	Additive White Gaussian Noise
MFSK	-	Multiple Frequency Shift Keying
BPSK	-	Binary Phase Shift Keying
GF	-	Galois Field
SNR	-	Signal-to-Noise Ratio
GUI	-	Graphical User Interface

CHAPTER 1

INTRODUCTION

Introduction will go about the background or layout of the project that will define the area of work that being performed throughout the task. It gave a clear definition and sense of direction so it will not be misled during the entire project.

1.1 Background

In digital communication today, one of the most significant function lies on the channel coding scheme for error control. Channel coding refers a class of signal transformations that is design to improve communications performances by enabling the transmitted signal to better withstand the effects of channel impairments such as noise, interference and fading occurred during transmission. Channel coding can be either waveform coding or structured sequences. Waveform coding deals with transforming the waveforms to make detection process less subject to errors where structured sequences deals with transforming data for better sequences by having structured redundancy. Reed-Solomon (RS) Codes is considered a structured sequence that most widely used in error control codes. It was invented by Larry S. Reed and Gustave Solomon in 1960, who wrote “Polynomial Codes over Certain Finite Fields”. In following years, rapid advancements were made for RS Codes for error correction and detection techniques. Now, it had been used vigorously in storage and digital communication applications including countless wired and wireless systems.

1.2 Problem Statement

Digital communications had becoming very significant the world of telecommunication systems. It had become very essential and the demand for improving the communication system had become very crucial thus, transmission rate of a system had increase gradually to accommodate larger data payload to be transferred.

As the payload of communication system increased, the probability for an error to occur is also increased. Thus besides good modulation techniques, a good channel coding is very critical to ensure that data transferred can endure any channel impairments such as noise, interference and channel fading. In general, an excellent code must:

1. have the ability to correct and detect the errors found in the channel,
2. be suited to the noise environment of the channel,
3. be efficient in the use of redundancy, and
4. have a coding and decoding algorithm which can be economically implemented using available technology.

Besides that, basically, there are 2 important issues arise when choosing a code for any given application for digital communications;

1. What is the noise environment of the channel and what types of errors are expected in the system?
2. Of the available codes, which code or codes are well suited to the noise environment and which ones can be implemented into the system with the best cost/performance tradeoffs?

With that considerations, RS Codes is being chosen because it is a powerful coding that operate at multi-bit symbols which have long block length that tend to average out the random errors and make it suitable for random error correction. Therefore suitable to combat against error occurred in burst that usually happen in wireless digital communications.

1.3 Project Objectives

The main objective of this project is to evaluate Bit Error Rate (BER) performances of Reed-Solomon (RS) code in error correction control. It is assumed that the signal is transmitted in M-ary modulation technique with the presence of Additive White Gaussian Noise (AWGN) and multipath fading channel. These impairments are the essence in digital communication system that needs to overcome during transmission. The project will evaluate BER performance for RS codes when the number of bits per symbol, number of information symbols per block and also number of correctable symbol errors are varies by using a certain type of modulation technique.

This is also a good foundation to identify and acknowledge the limitation and weakness of R-S codes in dealing with AWGN and multipath fading channel in digital transmission. It will certainly assist in better designing the systems used by knowing the restriction that should be taken into account. On a general ground, it may verify coding-modulation system trade-offs, which are the fundamentals to all digital communication design. The designing goals for digital communications are:

1. To maximize the transmission bit rate, R .
2. To minimize the probability of bit error, P_b .
3. To minimize required power, or equivalently, to minimize required bit energy to noise power spectral density E_b/N_0 .
4. To minimize required system bandwidth, W .
5. To maximize system utilization, that is, to provide reliable service for a maximum number of users with minimum delay and with maximum resistance to interference.
6. To minimize system complexity, computational load and system cost

However, the goals in digital communications design are clearly in conflict when there are several constraints and theoretical limitations that seem to be very hard to avert.

1.4 Scope of Work

The project scope will define the area of work that that being carried out to ensure successful outcome. First and foremost, some literature review of RS codes implementations in today's environment to give a good idea on how RS codes being implemented. Since this project paper is a comparison study of the BER performance of RS codes, thus literature review will only be an assist tool to see whether it is on the right track or not.

Since this project does not specify any m-ary modulation techniques, thus it is assumed that the modulation technique used in the simulation is M-Frequency Shift Keying (MFSK). MFSK is used because it allows the simulation to be done without any other digital communications elements such as interleaver or deinterleaver. This will ensure the BER performance is definitely for RS codes. As for channel impairments, AWGN and Rayleigh Multipath Fading Channels is used to depict wireless communication system. The research covers on the performances of the error correction capabilities when the variables in RS codes are being change, whether it is the number of bits per symbol (m), the number of information symbols per block (n,k) or the number of correctable symbols error (t). All of these variables are being applied under the influence for MFSK modulation within AWGN and Rayleigh Multipath fading channel.

This project is carried out entirely using scientific simulation software. The error performance will be measured and analyze in terms of Bit Error Rate (BER) that will be simulated using MATLAB 7.0. Because MATLAB is an interpreted language for numerical computation, it allows one to perform numerical calculations, and visualize the results without the need for complicated and time consuming programming and also allows its users to accurately solve problems, and produce graphics easily. For this particular project, Simulink (applications inside MATLAB) is used to create the model for verifying RS Codes in digital communications.

1.5 Project Methodology

Research methodology is an organized set of procedures and guidelines for the research project to be carry out in a systematic way. It will also help to govern the works and finding the materials on the right way and the right sources. For this project, several methods are being applied:

1. *Literature review on research done in this area.*

The first step is to familiarize with as much information about the topic of the project. It is also important to gain some value added knowledge by knowing the present status on the development or research that been done in this area. This will give a very good picture on the development of the project.

2. *Study the theory of channel coding of RS Codes.*

Since it is a research on R-S code performances, understanding the theory thoroughly is very important for further analysis. That is why every theory that been disclose, how it is implemented and the mathematical equation involved in RS code must be well versed to ensure the project can be execute correctly.

3. *Learn how to used MATLAB 7.0 and simulation.*

MATLAB was chosen to be the analysis tool for this project. That is why it is very important to learn to use MATLAB and its features to help producing the desired results. There are 2 distinctive features in MATLAB that used to create the simulation model; SIMULINK and BERTool. Thus, after familiarize with its tools and functions, the simulation system demo is then to be tested. The results will be discussed in later chapters.

By having these methodologies as guidelines, good results will be obtained and it discussion will be more meaningful and constructive.

REFERENCES

- [1] B. Sklar, "Digital Communications: Fundamentals and Applications", Prentice-Hall, 2nd Edition, 2002.
- [2] B. Sklar, "Rayleigh Fading Channel in Mobile Digital Communication System Part 1: Characterization", IEEE Communication Magazine, pp. 90-100, July 1997.
- [3] H. Labiod, "Performance of Reed Solomon Error-Correcting Codes On Fading Channels", IEEE, 1999.
- [4] B. Vucetic, V. Ponampalam and J. Vuckovic, "Low Complexity Soft Decision Decoding Algorithms for Reed-Solomon Codes", IEICE Trans. Commun, Vol E84-B. March 2001.
- [5] L.L. Joiner and J.J. Komo, "Improved Performance of M-ary Orthogonal Signalling Using Reed Solomon Codes", IEEE, 1999.
- [6] S.W. Kim and W. Stark, "Performance Limits of Reed-Solomon Coded CDMA with Orthogonal Signaling in a Rayleigh-Fading Channel", IEEE Transaction On Communications, Vol. 46, No. 9, September 1998.
- [7] G. Noubir, "Collision Free One-Way Communications Using Reed-Solomon Codes", 1998.
- [8] W.J. Ebel and W.H. Tranter, "The Performance of Reed-Solomon Codes on a Bursty-Noise Channel", IEEE Transaction On Communications, Vol. 43, No. 2/3/4, 1995.
- [9] E. R. Berlekamp, "Bit-Serial Reed-Solomon Encoders" IEEE Transaction On Information Theory, Vol. IT-28, No. 6, November 1982.
- [10] G. Breed, "Bit Error Rate: Fundamental Concepts and Issues", Summit TechnicalMedia, LLC. 2003.
- [11] W. Stallings, "Wireless Communications and Networks", Prentice Hall, International Edition. 2002.
- [12] V. Ponnampalam and A. Grant, "An Efficient SISO Algorithm for Reed-Solomon Codes", Proceedings 4th Australian Communication Theory

Workshop, 2003.

- [13] X.M. Zhang and K. K. Parhi, “Fast Factorization Architecture in Soft-Decision Reed-Solomon Decoding”, IEEE Transactions On VLSI Systems, March 2005.
- [14] L.L Yang and L. Hanzo, “Performance of Errors and Erasures Decoded Reed Solomon Codes Over Frequency Selective Fading Channels Using M-ary Orthogonal Signalling”, IEEE, 2000.
- [15] John G. Proakis, “Digital Communications”, McGraw-Hill, 3rd Edition, 1995.