MATHEMATICAL MODELLING OF POLLUTANT CONCENTRATION DISTRIBUTION IN RIVER

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This is my present to my beloved husband, parents, family and for those who are always believes and appreciates in the richness of learning.

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ABSTRACT

In recent years, the interest in preserving the quality of water for distribution processes in order to maximize the fulfillment of various sectors has considerably increased. Be it domestic sources, industrial or agricultural effluents, the massive increase of industrial productions accompanied by high growth of large urban populations has led to severe water pollution problems. Therefore, to identify water pollution, water quality models becomes an important tool to recognize the behaviors of pollutants in water environment. In this dissertation, the most important objective is to understand and formulate a mathematical model involving the study of pollutant transport in water environment via an advection-diffusion equation in river. The analytical solution of the model is found using Laplace transform method. Once the equation is solved, the solution is plotted using Maple for an easier analysis of the result. Graph of concentration of the pollutant against distance will be interpret and discuss. The result suggests that the concentration of pollutant is decrease against distance.

ABSTRAK

Dalam tahun-tahun kebelakangan ini, kepentingan mengekalkan kualiti air bagi proses pengagihan untuk memaksimumkan memenuhi pelbagai sektor telah meningkat dengan banyaknya. Sama ada sumber dalam domestik, pengaruh industri atau pertanian, peningkatan secara besar-besaran pengeluaran industri yang disertai dengan pertumbuhan penduduk bandar besar telah membawa kepada masalah pencemaran air yang teruk. Oleh itu, untuk mengenalpasti masalah pencemaran air, model kualiti air menjadi alat penting untuk mengenalpasti ciri-ciri bahan pencemar dalam sumber persekitaran air. Dalam disertasi ini, objektif paling penting adalah untuk memahami dan merangka model ringkas matematik yang melibatkan kajian dalam aliran bahan pencemar dalam persekitaran melalui persamaan olahan-resapan dalam sungai. Penyelesaian analisis model didapati dengan menggunakan kaedah Laplace. Setelah persamaan tersebut diselesaikan, penyelesaiannya akan di plot dengan menggunakan perisian Maple untuk menganalisis keputusan yang diperoleh. Graf kepekatan bahan pencemar terhadap jarak akan ditafsir dan dibincangkan. Hasilnya menunjukkan bahawa kepekatan bahan pencemar berkurangan terhadap jarak.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

L	-	Laplace operator
L^{-1}	-	Laplace inverse operator
t	-	time(days)
x	-	distance(meter)
c	-	concentration pollutant
σ	-	coefficient of diffusion
v_0	-	velocity of the river
α	-	substance discharge rate
γ	-	coefficient source of pollutant

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter will briefly discuss on the dissertation flow from study of background, problem statement and objective of the study. Hence, follow by scope of the study, significance of the study and report structure. Each subtopic related to each other in order to make the readers understand the process of this dissertation conducted.

1.2 Pollution

Based on Environment Quality Act 1974, pollutants means any natural or artificial substances, whether in a solid, semi-solid or liquid form, or in the form of gas or vapour, or in mixture of at least two of these substances, or any objectionable odour or noise or heat emitted, discharge or deposited or is likely to be emitted, discharged or deposited from any source which can directly or indirectly cause pollution and includes any environmentally hazardous substances.

Pollutant contributes pollution. Environment Quality Act 1974 also stated that pollution is any direct or indirect alteration of the physical, thermal, chemical, or biological properties of any part of the environment by discharging, emitting or depositing environmentally hazardous substances, pollutants or wastes so as to affect any beneficial use adversely, to cause a condition which is hazardous or potentially hazardous to public health, safety, or welfare, or to animals, birds, wildlife, fish or aquatic life, or to plants or to cause a contravention of any condition, limitation or restriction to which a license under this Act.

1.2.1 Land Pollution

Land pollution is pollution of the earth's natural land surface by industrial, commercial, domestic and agricultural activities. The deposition of solid or liquid waste materials on land or underground in a manner that can contaminate the soil and groundwater, threaten public health, and cause unsightly conditions and nuisances. Chen and Guidotti (2011) stated that the major unsolved issues in managing hazardous waste is finding methods of disposal that safe and cheaper. The effect over hazardous waste is potential health effect when exposure to toxic chemicals and particularly the risk of cancer.

The waste materials that cause land pollution are broadly classified as municipal solid waste (MSW), construction and demolition (C&D) waste or debris and hazardous waste. MSW includes nonhazardous garbage, rubbish, and trash from homes, institutions, commercial establishments, and industrial facilities. Garbage contains moist and biodegradable food wastes such as meat and vegetable scraps. The rubbish comprises mostly dry materials such as paper, glass, textiles, and plastic objects and trash includes bulky waste materials and objects such as discarded mattresses, appliances, pieces of furniture.

C&D waste includes wood and metal objects, wallboard, concrete rubble, asphalt, and other inert materials produced when structures are built, renovated or demolished. Hazardous wastes include harmful and dangerous substances generated primarily as liquids but also as solids, sludge or gases by various chemical manufacturing companies, petroleum refineries, paper mills, smelters, machine shops, dry cleaners, automobile repair shops, and many other industries or commercial facilities. In addition to improper disposal of MSW, C&D waste, and hazardous waste, contaminated effluent from subsurface sewage disposal can also be a cause of land pollution.

Figure 1.1 Land Pollution

1.2.2 Air Pollution

World Health Organization (WHO) stated that air pollution is contamination of the indoor or outdoor environment by any chemical, physical or biological agent that modifies the natural characteristic of the atmosphere. Its consists of gaseous, liquid, or solid substances that, when present in sufficient concentration, for a sufficient time, and under certain conditions, tend to interfere with human comfort, health or welfare, and cause environmental damage.

The smoke from the burning of coal, peat and other fuel contribute of air pollution. Around large power plants and industrial, the wind can carries the smoke over long distance and air can be contaminated with a radius of 1 to 5 kilometres. Other source of air pollution is from industrial enterprise significant to dust generation. The amount of dust, its characteristic and influence on the human body vary and depend on the location and source of dust and on its composition. (Ekaterina, 2011).

Air pollution causes acid rain, ozone depletion, photochemical smog, and other such phenomena. Some of the main contributors to air pollution are automobile emissions, tobacco smoke, combustion of coal, acid rain, noise pollution from cars and construction, power plants, manufacturing buildings, large ships, aerosol sprays, wildfires and nuclear weapons.

Figure 1.2 Air pollution

1.2.3 Water Pollution

Water pollution is the introduction of chemical, biological and physical matter into large bodies of water that degrade the quality of life that lives in it and consumes it. Water pollution happens when toxic substances enter water bodies such as lakes, rivers, oceans and so on, getting dissolved in them, lying suspended in the water or depositing on the bed as defined by as the U.S Public Health Service. This degrades the quality of water.

Not only does this spell disaster for aquatic ecosystems, the pollutants also seep through and reach the groundwater, which might end up in our households as contaminated water we use in our daily activities, including drinking. Water pollution can be caused in a number of ways, one of the most polluting being city sewage and industrial waste discharge.

Some of the main contributors to water pollution are waste treatment facilities, mining, human sewage, oil spills, failing septic systems, soap from washing your car, household chemicals and animal waste. Soils and groundwater contain the residue of human agricultural practices and also improperly disposed of industrial wastes. Indirect sources of water pollution include contaminants that enter the water supply from soils or groundwater systems and from the atmosphere via rain.

Figure 1.3 Water Pollution

1.3 Water Quality Standard

Water Quality Standards are the foundation of the water quality based control program mandated by the Clean Water Act. The standards are also the technical basis for reducing runoff from rural and urban areas. A standard can consist of either numeric or narrative limits for a specific physical or chemical parameter. Ultimately, a water quality standard is developed to help protect and maintain water quality necessary to meet and maintain designated or assigned uses, such as swimming, recreation, public water supply, and aquatic life.

The water quality status of rivers in Malaysia has been a cause for concern for various local authorities, government agencies as well as the public at large. Rives in Malaysia are generally considered polluted with coherent examples such as Sungai Klang in Selangor, Sungai Juru in Penang and Sungai Sengget in Johor. In Malaysia, the existing methodology for river water quality classification and monitoring is quite extensive. In fact, the country's current water quality monitoring network is at par, if not better, than many developed countries. At the moment, Malaysia has over 100 manual and automatic river quality monitoring stations in 146 basin maintained by the Department of Environment(DOE) alone (Malaysia Environmental Quality Report,2006).

Environmental acts and regulations were established in 1970's. "Environmental Quality Act 1974" is an act related to the prevention, abatement, control of pollution and enhancement of the environment. Under this act, the Minister after consultation with the Environmental Quality Council elaborates regulations for prescribing ambient water quality standards which are applied to surface waters and marine waters. "The Environmental Quality (Sewage and Industrial Effluents) Regulations 1979" was also established under the act to prescribe discharge standards.

According to Malaysia Environmental Quality Report (2006), in 1985 the government undertook a national study known as the “Development of Water Quality Criteria and Standard for Malaysia”, whose researcher consisted of a multidisplinary team of experts from universities throughout the country. The study was carried out in four phases with the intention of developing a national benchmark of water quality conditions on a water quality conditions on a per parameter basis. Over 120 psyhco-chemical and biological parametes were reviewed in the study. In the end, The Interim National Water Quality Standard (INWQS) was drafted.

Table 1.1 The Interim National Water Quality Standard for Malaysia by Malaysia Environmental Quality Report (2006)

PARAMETER	UNIT	CLASS					
		I	IIA	IIB	III	IV	V
Ammoniacal Nitrogen	mg/l	0.1	0.3	0.3	0.9	2.7	> 2.7
Biochemical Oxygen Demand	mg/l	1	3	3	6	12	> 12
Chemical Oxygen Demand	mg/l	10	25	25	50	100	> 100
Dissolved Oxygen	mg/l	7	5 – 7	5 - 7	3 – 5	< 3	< 1
pH	-	6.5 - 8.5	6 – 9	6 - 9	5 – 9	5 - 9	-
Colour	TCU	15	150	150	-	-	-

Electrical Conductivity*	µS/cm	1000	1000	-	-	6000	-
Floatables	-	N	N	N	-	-	-
Odour	-	N	N	N	-	-	-
Salinity	%	0.5	1	-	-	2	-
Taste	-	N	N	N	-	-	-
Total Dissolved Solid	mg/l	500	1000	-	-	4000	-
Total Suspended Solid	mg/l	25	50	50	150	300	300
Temperature	°C	-	Normal + 2 °C	-	Normal + 2 °C	-	-
Turbidity	NTU	5	50	50	-	-	-
Faecal Coliform**	count/100 ml	10	100	400	5000 (20000)a	5000 (20000)a	-
Total Coliform	count/100 ml	100	5000	5000	50000	50000	> 50000

1.4 Effect of Water Pollution

The effects of water pollution can be catastrophic, depending on the kind of chemicals, concentrations of the pollutants and where there are polluted. These effects varied and depend on what chemicals are dumped and in which locations. Many water bodies near urban areas such as cities and towns are highly polluted. This is the result of both garbage dumped by individuals and dangerous chemicals legally or illegally dumped by manufacturing industries, health centres, schools and market places.

The main problem caused by water pollution is that it kills life that depends on these water bodies. Dead fish, crabs, birds and sea gulls, dolphins, and many other animals often wind up on beaches, killed by pollutants in their habitat. Pollution disrupts the natural food chain as well. Pollutants such as lead and cadmium are eaten by tiny animals. Later, these animals are consumed by fish and shellfish, and the food chain continues to be disrupted at all higher levels. Eventually, peoples can get diseases such as hepatitis by eating seafood that has been poisoned.

Ecosystems can be severely changed or destroyed by water pollution. Many areas are now being affected by careless human pollution, and this pollution is coming back to hurt humans in many ways. We all drink water that comes from a lake or local river. In countries that have poor screening and purification practices, people often get water-borne disease outbreaks such as cholera and tuberculosis. Every year, there are an estimated three to five millions cholera cases and 100 000 to 120 000 deaths due to cholera.

In developed countries, even where there are better purification methods, people still suffer from the health effects of water pollution. Take toxins emitted by algae growth for instance and this can cause stomach aches and rashes. Excess nitrogen in drinking water also pose serious risks to infants. There is some real financial implications that will result from water pollution. It can cost a lot more to purify drinking water that takes its source from nutrient polluted water bodies. Fishing stock is affected negatively when there is a depletion of oxygen. Consumers are also weary of fish from these sources and tend to stay away from them, costing fisheries to lose revenue.

1.5 Statement of Problem

To help better understand our world, we often describe a better particular phenomenon mathematically. Such mathematical model is idealization of the real world phenomenon and never a completely accurate representation. In modelling world, we are interested in predicting the value of a variable at some time in the future.

In this research, in order to study the river flow with the effect of diffusion and advection in medium, we need to study a suitable mathematical model that represents the physical phenomenon. In this purpose, we are determining one-dimensional mathematical model of river pollution with the effect of without and with additional of pollutant. The model will then be solved via analytical. Once we have obtained the solution to the model, the graph of the solution will be computed using Maple. Then, we will be able to identify the concentration level of pollutant at any point of the river.

1.6 Objectives of the Research

The objectives of this research are:

1. to formulate advection-diffusion model without and with source.

2. to get the solutions of advection-diffusion model without and with source.

3. to compare the solution of advection-diffusion model between without and with source.

1.5 Scope of the Research

This study emphasize on the formulation of the mathematical model to determine the concentration of pollutant based on advection-diffusion equation. This study focus on two problem which advection-diffusion equation without and with source. Source is a chemical substance that pour into the river. The mathematical model of river pollution is in term of partial differential equation. The analytical solution of the model is found using a transformation and Laplace transform. Once the result have been obtained, we will interpret the result based on the graphical output.

1.6 Significance of the Research

From this research, we can predict the concentration of pollutant in the river at any distance. The solution can helps the authority in making quick and accurate decision related to cleaning the river and making policy about level pollution of river in certain area

1.7 Report Structure

The report structure for this dissertation are listed as below:

i. Chapter 1
Introduction. Briefly introduction about pollution and Water Quality Standard. The statement of problem, objective, scope and significance of the research are explained in this chapter.

ii. Chapter 2

Literature Review. All previous studies or research that has been done and related with the current project is assembled in this chapter, including the theories, models, methods and figures that may support this project.

iii. Chapter 3

Methodology. The dissertation's methodology is describe as method or approached that used to solve the problem given.

iv. Chapter 4

Mathematical Modelling. The techniques that used to analyze and to get solutions for the method proposed in the previous chapter will be shown and use to obtain result in the next chapter using appropriate mathematical package.

v. Chapter 5

Results and Data Analysis. The solutions from previous chapter will be used to analyze the problem proposed. All results and graph will be shown and attached in this chapter.

vi. Chapter 6

Conclusion and Recommendation. This dissertation will be concluded with a summary after the whole studied is completed.

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