

DOUBLE POPULATION MRT LBM IN AN ENCLOSURE USING NANOFUID

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*To my beloved “husband, mother and father”
for their endless support and encouragement*

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ABSTRACT

In this study, lattice Boltzmann method is applied to investigate the natural convection flows utilizing nanofluids in a square enclosure. Al_2O_3 and CuO water based nanofluids with 5, 6, 7, 8 and 9% nanoparticle volume fraction is used as the fluid. This study has been carried out for the pertinent parameters in the following ranges: the Rayleigh number of nanofluid, $\text{Ra}=10^3, 10^4, 10^5$ and 10^6 , the volumetric fraction of nanoparticles 5, 6, 7, 8 and 9% and the aspect ratio (Ar) of the enclosure is 1.0. The effects of solid volume fraction of nanofluids on hydrodynamic and thermal characteristics are investigated and discussed. The average and local Nusselt numbers, streamlines, temperature contours and vertical component of velocity for different values of solid volume fraction and Rayleigh number are illustrated. Results show that by increasing Rayleigh number and nanoparticle volume fraction, average Nusselt number increases in whole range of Rayleigh numbers that lead to decreasing thermal boundary layer and enhancement of heat transfer of fluid in the cavity. As expected, Al_2O_3 with higher heat conductivity has higher Nusselt number with respect to CuO with lower heat conductivity.

ABSTRAK

Dalam kajian ini, kaedah kekisi Boltzmann telah digunakan untuk menyiasat aliran olakan semulajadi dengan menggunakan cecair nano dalam sebuah rongga segiempat. Cecair nano yang berasaskan Al_2O_3 dan CuO dengan 5, 6, 7 dan 9% pecahan isipadu zarah nano telah digunakan. Kajian ini telah dijalankan untuk parameter yang berkenaan: number Rayleigh untuk cecair nano, Ra , adalah 10^3 , 10^4 , 10^5 dan 10^6 ; pecahan isipadu untuk zarah-zarah nano adalah 5, dan 9%; dan nisbah aspek (Ar) untuk kekandang adalah 1.0. Kesan-kesan pecahan isipadu pepejal cecair nano ini terhadap ciri-ciri hydro-dinamik dan terma telah disiasat dan akan dibincangkan. Nombor-nombor Nusselt purata dan tempatan, garisan-garisan arus, suhu kontur, dan komponen menegak halaju untuk nilai-nilai pecahan isipadu pepejal serta number Rayleigh yang berbeza juga akan ditunjukkan. Keputusan kajian menunjukkan bahawa peningkatan dalam nombor Rayleigh dan pecahan isipadu zarah nano turut akan meningkatkan nilai nombor purata Nusselt untuk keseluruhan lingkungan nombor Rayleigh. Keadaan sedemikian akan menyebabkan penyusutan sempadan lapisan terma dan peningkatan kadar pemindahan haba untuk cecair yang berada dalam rongga tersebut. Seperti yang diramalkan, Al_2O_3 dengan kekonduksian haba yang lebih tinggi turut memiliki nombor Nusselt yang lebih tinggi daripada CuO yang mempunya kekonduksian haba yang lebih rendah.

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LIST OF SYMBOLS

Cp	-	Specific heat capacity (J/kg K)
C	-	Lattice streaming speed (m/s)
Ar	-	Aspect ratio
Fi	-	Density distribution function
$f_{i_{eq}}$	-	Equilibrium distribution function
Gi	-	Energy distribution function
$g_{i_{eq}}$	-	Equilibrium distribution function
Lx	-	Length of channel
Ly	-	Height of channel
K	-	Thermal conductivity (W/m K)
Nu	-	Nusselt number= $-(k_{nf}/k_f) \times (\partial\theta/\partial Y)$
Nu_{ave}	-	Average Nusselt number= $\int Nu \cdot dx$
Pr	-	Prandtl number= ν/α
Ra	-	Rayleigh number
T	-	Temperature (K)
U	-	Velocity in x direction
V	-	Velocity in y direction
H	-	Enclosure height
W	-	Enclosure weight
<i>Abbreviations</i>		
BGK	-	Bhatnagar-Gross-Krook
CFD	-	Computational Fluid Dynamics
D2Q9	-	Three Dimensions nineteen velocities lattice nann method
FEM	-	Finite Element Method
FDM	-	Finite Difference Method
FVM	-	Finite Volume Method
LB	-	Lattice Boltzmann
LBE	-	Lattice Boltzmann Equation
LBM	-	Lattice Boltzmann Method
LGA	-	Lattice Gas Approach
PDE	-	Partial Differential Equations
MRT	-	Multi-Relaxation Time
SRT	-	Single Relaxation Time
<i>Greek symbols</i>		

α	-	Thermal diffusivity (m ² /s)
ε	-	Internal energy (J/kg)
ϕ	-	Particle volume fraction
μ	-	Dynamic viscosity (kg/m s)
ρ	-	Density (kg/m ³)
τ	-	Relaxation time for f_i
τ_c	-	Relaxation time for g_i
ν	-	Kinematic viscosity (m ² /s)
<i>Subscripts</i>		
avg	-	average
C	-	Cold
F	-	Fluid
H	-	Hot
nf	-	Nanofluid

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study

In the last century, Computational Fluid Dynamic (CFD) had been widely used due to advancement in computation technology. Basically, CFD has been used and compared the solution between the experiment results and analytical results [1]. In addition, CFD also helps to interpret as well as to study the behavior of fluids. Although CFD is a powerful tool to demonstrate the fluid flow behavior, however the error gain in the simulations is still an issue that needs great attention from researcher. Besides that, Latt.J [2] discovered that the conventional CFD is difficult in solving multi-phase flow due to complexity of the partial differential equation. In most of the cases, Navier-Stokes (NS) equation becomes the fundamental basic for CFD in simulating fluid flow. Rather than NS equation, CFD also has been used to solve the continuity equation, the energy equation and other equation which are derived from equation mention before. There are many type of numerical approaches can be choose to solve all kind of these equation in order to solve the fluid problems.

In 1990's, a new CFD method introduced to solve complex system tools which historically it's originated from lattice gas automata (LGA). This method is based on mesoscopic numerical approach which is something between macroscopic (FDM, FVM, FEM...) and microscopic method and is suitable for solving each fluid dynamic and either system related to partial differential equations [3].

In this method, fluids can be simulated by modeling its individual molecules that is consisted. So it will behave as a fluid if all the interactions between molecules be calculated correctly. But simulating such a fluid with this much numbers of molecules need a huge amount of data that should be calculated by computers. It's the biggest disadvantage of such a method that computer resources are not in hand. In fact, LB method is a bridge between molecular description that defines as kinetic of fluid motion and the real macroscopic world [4]. The kinetic theory tries to understand the macroscopic properties of fluids from the properties of their molecules: molecular mass, electrical properties shape parameters, the mean free path and so on [5].

Recently, the lattice Boltzmann equation (LBE) method has gained much attention for its ability to simulate fluid flows, and for its potential advantages over conventional numerical solution of the Navier–Stokes (NS) equations. A few standard, benchmark problems have been simulated by LBE and the results are shown to agree quite well with the corresponding NS solutions. Currently, a number of other complex flow problems are being simulated using the LBE approach.

LBM has several advantages compare to traditional CFD method especially when solving the complex boundaries problems. Most of the CFD methods are time consuming, but LBM can save a lot of time due to its flexibility on boundary treatment. This is because LBM only calculates due to its number of mesh points and the lattice model rather than calculate random motion of every particle. After LBM has been introduced for many years, it already shows its high capability in simulating the behavior of flow in macroscopic channel. Most of the results obtained from LBM are in good agreement with analytical results and other numerical results. The flow pattern and its behavior can be studied through analyzing the outcome of results [6].

The LBM uses ensemble averaged distribution function to describe the kinetic system and consider that the collective behavior of the imagined particles which characterize the system, is in agreement by the principle of macroscopic physics. Nowadays the lattice Boltzmann method (LBM) has established itself as a powerful tool for the simulation of a wide range of physical phenomena. One of its main applications is the field of computational fluid dynamics where it has proven

successful to solve the weakly compressible Navier–Stokes equations and models associated with more complex flows involving several phases or components. It has also been successfully applied to the simulation of flows of pseudo plastic and viscoelastic fluids. This method does not solve directly the macroscopic conservation equations, but somewhat models the statistics of collision of particles and may offer more modeling freedom than the classical methods based on finite difference, finite volume or finite element to which it is a competitive alternative.

Considering the rapid pace with which the subject is developing, in the foreseeable future the LBE method is likely to play a significant role in the numerical prediction of flows. A particularly simple linearized version of the collision operator makes use of a relaxation time towards an equilibrium value using a single relaxation time parameter. The relaxation term is known as the Bhatnagar–Gross–Krook (BGK) collision operator [3]. This model is called the lattice Boltzmann BGK model. Use of this collision operator makes the computations much faster. Due to the extreme simplicity, the lattice BGK (LBGK) equation [4] has become the most popular lattice Boltzmann model.

To solve Lattice Boltzmann equation partial differential must be considered. In this regard partial differential equation presents fluid flow through the space and time. As a matter of fact, certain solutions only exist for a few specific cases with simple geometries and suitable boundary conditions. It is certainly true that to obtain simplified equation; the complex phenomena must be ignored. However, nowadays digital computers have rapidly developed and many researchers prefer to use high performance computers in their field of study.

1.2 Statement of Problem

The properties of water based nanofluids have been presented in the form of tables or equations and heat transfer effects is investigated because the demand on usage of nanofluids is rapidly increasing but research on the nanofluids is at the early stage and still big gap between numerical and experimental results exist. Also behavior

of nanofluids is not well understood so in this study tried to investigate different factors which effect on heat transfer conditions of different nanofluids to find the best aspect-ratio and volume fraction of each nanofluid to have the best heat transfer in a square cavity.

1.3 Objectives of the study

The specific objectives of this study are as follows:

- Investigation of effect of nanoparticle volume fraction on heat transfer enhancement in a square cavity.
- Investigation of effect of increasing Rayleigh number on heat transfer enhancement of nanofluids and compare the Nusselt number in each case.

1.4 Scope of the Study

In this study, two dimensional Lattice Boltzmann scheme is used to simulate heat transfer in a square cavity. D2Q9 lattice model is chosen for the simulation. Nanoparticles that are used in this study are Al_2O_3 and CuO in water base fluid and the range of Rayleigh number is 10^3 - 10^4 and 10^5 . For the cavity aspect ratio of 1.0 is chosen. The effect of particle interactions is neglected.

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