PERISTALTIC MOTION OF A MAGNETOHYDRODYNAMICS OLDROYD-B FLUID IN PLANAR CHANNEL WITH HEAT TRANSFER

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Specially dedicated to my beloved family for their love and support throughout my journey in my studies. Thank you for everything.

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ABSTRACT

Peristalsis is defined as wave of area relaxion contraction and expansion along the length of channel containing the fluid. Peristalsis is also a well known mechanism of fluid transport in biological system. It occurs in gastrointestinal, urinary, reproductive tracts and many other glandular ducts in a living body. Peristaltic motion occurs widely when stenosis is formed in the fluid transport trough the esophagus, urine transport from kidney to bladder through the urethra, chyme movement in gastrointestinal tract, transport of lymph in the lymphatics vessels and in the vasomotion of small blood vessels such as arteries, venules and capillaries. In this dissertation, heat transfer for peristaltic flow of a magnetohydrodynamic Oldroyd-B fluid in a planar channel is studied under the long wavelength and low Reynolds number considerations. This analysis can model the transport of food bolus through esophagus. The analytical expressions for the stream function, the axial velocity, the pressure gradient, the temperature and coefficients of heat transfer are obtained. The effects of various parameters such as the Hartman number, the amplitude ratio, Eckert number and Prandtl number on the flow characteristics are discussed. The results indicate that increasing the Hartmann number decreases the axial velocity, increases the pressure rise in the entire pumping region, and decreases the temperature in the downstream region.

ABSTRAK

Peristalsis ditakrifkan sebagai gelombang otot di kawasan pengecutan dan pengembangan di sepanjang saluran yang mengandungi bendalir. Peristals juga suatu mekanisma yang penting untuk pengangkutan bendalir dalam sistem biologi. Mekanisma ini berlaku di gastrousus, urinari, saluran reproduktif dan di kebanyakkan saluran kelenjar pada badan manusia. Pergerakan peristalsis berlaku secara meluas apabila stenosis ditubuhkan dalam pengangkutan bendalir melalui esofagus, pengangkutan air kencing dari buah pinggang ke pundi kencing melalui uretra, pergerakan chyme dalam saluran gastrousus, pengangkutan limfa di kapal-kapal lymphatics dan vasomotion pada saluran darah kecil seperti arteri, venules dan kapilari. Dalam disertasi ini, pemindahan haba bagi aliran peristalsis magnetohidrodinamik bendalir Oldroyd-B melalui saluran mengufuk dikaji dengan mempertimbangkan penghampiran panjang gelombang yang panjang dan nombor Reynolds rendah. Analisis ini memberi model pengangkutan bolus makanan melalui esofagus. Ungkapan-ungkapan analitikal diperolehi merangkumi fungsi arus, halaju paksi, kecerunan tekanan, and closed form for temperature distributions and pekali bagi pemindahan haba. Kesan pelbagai parameter seperti nombor Hartman, nisbah amplitude, nombor Eckert dan nombor Prandtl terhadap cirri-ciri aliran dibincangkan. Hasil kajian menunjukkan bahawa apabila nombor Hartmann meningkat, halaju paksi menyusut, peningkatan tekanan bertambah di seluruh kawasan pengepaman, dan suhu menyusut dalam kawasan hiliran.

TABLE OF CONTENTS

CHAPTER	TITLE		PAGE
	DECLARATION		**ii**
	DEDICATION		**iii**
	ACKNOWLEDGEMENTS		**iv**
	ABSTRACT		**v**
	ABSTRAK		**vi**
	TABLE OF CONTENTS		**vii**
	LIST OF FIGURES		**x**
	LIST OF TABLES		**xi**
	LIST OF SYMBOLS/NOTATIONS		**xii**
1	**INTRODUCTION**		**1**
	1.1	Research Background	2
	1.2	Problem Statement	5
	1.3	Objectives of the Study	5
	1.4	Scope of the Study	5
	1.5	Significance of the Study	6
	1.6	Dissertation Organization	6

2	**LITERATURE REVIEW**	8
	2.1 Introduction	8
	2.2 The Physiology of Peristalsis	8
	2.3 Heat Transfer	10
	2.4 Studies On Previous Research On Peristaltic Motion In A Plannar Channel	10
	2.5 Basic Equation of Fluid Mechanic	12
	2.5.1 Continuity Equation	12
	2.5.2 Momentum Equation	14
	2.5.3 Energy Equation	17
3	**THE MATHEMATICAL MODEL**	20
	3.1 Introduction	20
	3.2 Basic Equation	21
	3.3 Formulation of The Problem	23
	3.4 Dimensionless Formulation	26
4	**SOLUTION PROCEDURE**	30
	4.1 Introduction	30
	4.2 Rate of Volume Flow and Boundary Conditions	30
	4.3 Analytical Solution	32
	4.4 The Solution for M=0	35
	4.5 Graph for Analytical Solution	37

5	**RESULTS AND DISCUSSION**		41
	5.1	Introduction	41
6	**CONCLUSION**		57
	6.1	Summary of Research	57
	6.2	Conclusion	58
	REFERENCES		59

LIST OF FIGURES

FIGURE NO	TITLE	PAGE
1.1	The gastrointestinal system	3
1.2	The digestive system	4
1.3	The heart-lung machine	4
2.1	Peristalsis	9
2.2	Finite control volume fixed in space	13
2.3	An arbitrary volume V enclosed by material surface	15
3.1	The diagram of a planar channel	23
5.1	Variation of the pressure gradient dp/dx	45
5.2	Pressure rise per wavelength Δp_{λ}	46
5.3	Frictional force F_{λ}	47
5.4	M, Pr, ϕ , E vs the temperature θ	48
5.5	Variation of the heat transfer coefficient at the wall	49
5.6	Effects of different parameters on the axial velocity	50
5.7	Streamlines for F = 0.5, ϕ = 0.2	51
5.8	Streamlines for F = 0.5, M = 2	52
5.9	Streamlines ϕ = 0.3, M = 1	53
5.10	Streamlines ϕ = 0.3, M = 0	54

LIST OF TABLES

TABLE NO	TITLE	PAGE
5.1	The pressure rise per wavelength for Hartmann M and the flow rate F when $\phi = 0.2$	55
5.2	The pressure rise per wavelength amplitude ratio ϕ and the flow rate F when M = 2.	55
5.3	The frictional force per wavelength for Hartmann number M and the flow rate F when $\phi = 0.2$.	56
5.4	The frictional force per wavelength scheduled for amplitude ratio ϕ and the flow rate F when M = 2.	56

LIST OF SYMBOLS

τ	-	Cauchy stress tensor
$\mathbf{S}$	-	extra stress tensor
p	-	The pressure
ρ	-	Fluid density
$\mathbf{I}$	-	Unit tensor
∇	-	Gradient operator
$\mathbf{A}_1$	-	The first Rivlin-Erickson tensor
η_1	-	Relaxion time
η_2	-	Retardation time
$\mathbf{L} = \mathrm{r}\mathbf{V}$	-	Velocity gradient
λ	-	Wavelength
$\mathbf{B}$	-	Magnetic field
$\mathbf{J}$	-	Current density
σ	-	Electrical conductivity of the fluid
$\mathbf{E}$	-	Total electric field
δ	-	Wave number
Re	-	Reynolds number
M	-	Hartman number
We	-	Weissenberg number
ψ	-	Stream function
ϕ	-	Amplitude ratio
θ	-	The mean flow rate in the fixed frame
F	-	The flow rate in the wave frame
u	-	Axial velocity

Δp_{λ}	-	Pressure rise per wavelength
F_{λ}	-	Frictional force per wavelength
$\mathbf{f}$	-	Body forces
q	-	Heat flux
C_P	-	Capacity of the fluid
T	-	Temperature
K	-	Conductivity of the fluid
$(\overline{X},\overline{Y})$	-	Dimensional coordinates in the laboratory frame
$(\overline{U},\overline{V})$	-	Dimensional velocity components in the laboratory frame
$(\overline{u},\overline{v})$	-	Dimensional velocity components in the wave frame
(u,v)	-	Non-dimensional velocity components in the wave frame
$(\overline{x},\overline{y})$	-	Dimensional coordinates in the wave frame
(x,y)	-	Non-dimensional coordinates in the wave frame
$\overline{t}$	-	Dimensional time parameter

CHAPTER 1

INTRODUCTION

Fluid dynamics deals with fluid flow. There are two categories of fluid; ideal and real according to Singh (2007). Incompressible and no viscosity are qualities of an ideal fluid. Real fluids are known as Newtonian fluids and non-Newtonian fluids. Newtonian fluids are a fluid obeys Newton's law of viscosity which states that shear stress is proportional to the rate of shear strain. Water, benzene, ethyl alcohol, hexane and most solutions of low molecular weight are examples of Newtonian fluid. Non-newtonian fluid such as shampoo, ketchup, paint, blood, starch suspensions, toothpaste, custard whose shear stress and shear rate is non linear.

Non-Newtonian fluids can be organized according to their constitutive equations. These models are arranged into fluids of differential, rate and integral types stated by Dunn and Rajagopal, (1995). One of the simplest subclass of rate type fluids model is an Oldroyd-B model. The Oldroyd-B model is used to describe the flow of viscoelastic fluids. The model is named after its Oldroyd, (1950). He was the first who constructed a model of three dimensional rate type models. The Oldroyd-B fluid has acquired a special status among the many fluids of the rate type, as it includes as special cases of classical Newtonian and Maxwell fluids as stated by Shah *et al*., (2009). According to Fetecau *et al*.,(2009), the Oldroyd-B model has had some success in describing the response of some polymeric liquids.

1.1 Research Background

The behavior of most of the physiological fluids, oil, hydrocarbons and polymer is known to be non-Newtonian. One of the major chemical mechanisms for fluid transport in many biological systems is well known to physiologists to be peristalsis. Peristalsis is an important mechanism for mixing and transporting fluids, which is generated by a progressive wave of contraction or expansion moving along a tube. This travelling-wave phenomenon is referred to as peristalsis.

Several theoretical and experimental attempts have been done in Latham, (1966) to find out peristaltic action in different situations. Pandey and Chaube, (2010), state that peristaltic motion happens in blood flow through blood vessels, urine flow through ureter, chime flow through the gastrointestinal, the bile flow through bile duct and spermatic fluid flow through vas deferens, etc as shown in Figure 1.1. The transport of a bolus of food through esophagus to stomach is achieved by the chemical mechanism of peristalsis as shown in Figure 1.2. In biomedical, peristaltic pumping is used as devices like heart lung machine to pump the blood as shown in Figure 1.3.

Magnetohydrodynamics (MHD) (*magneto fluid dynamics* or *hydromagnetics*) is the study of the dynamics of electrically conducting fluids. Examples of such fluids include plasmas, liquid metals, and salt water or electrolytes. The word *magnetohydrodynamics* (MHD) is derived from *magneto*- meaning magnetic field, *hydro*- meaning liquid, and -*dynamics* meaning movement. The field of MHD was initiated by Alfvén, (1942). MHD peristaltic has applications in bioengineering and medical sciences such as the development of magnetic devices for cell separation, targeted transport of drugs applying magnetic partices as drug carriers, magnetic wound by Nadeem, and Akbar, (2011). The MHD peristaltic studies are useful for having a proper understanding of the functioning of different machines used by clinicians for pumping blood and magnetic resonance imaging (MRI).

The focus of this study is on the analytical solutions of the system of non-linear partial differential equations resulting from the mathematical modeling of an MHD peristaltic motion of an Oldroyd-B fluid. There are no exact methods for these problem, so approximation methods are being used.

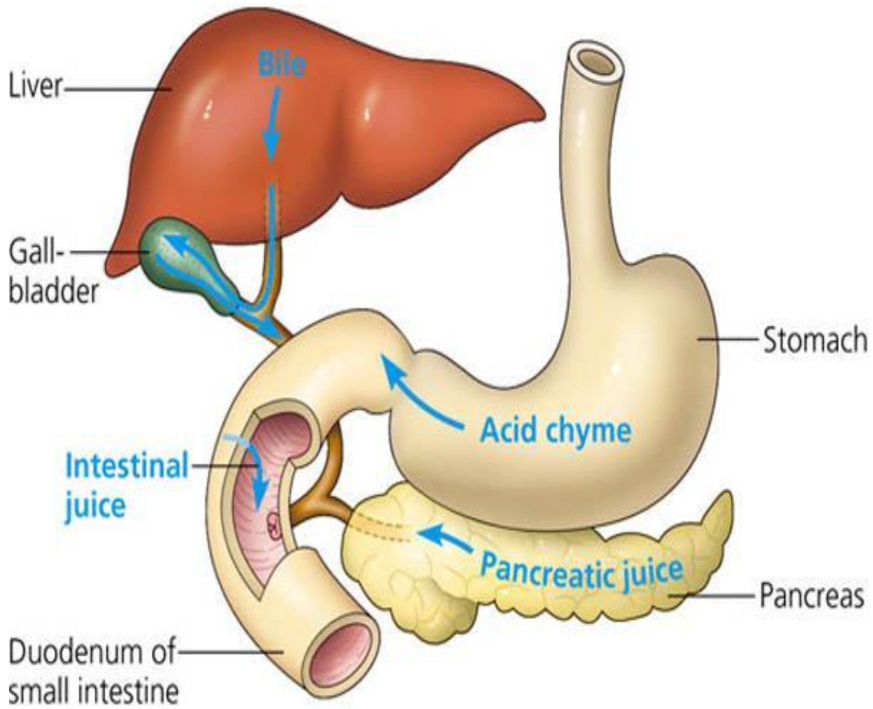

Figure 1.1: The gastrointestinal system

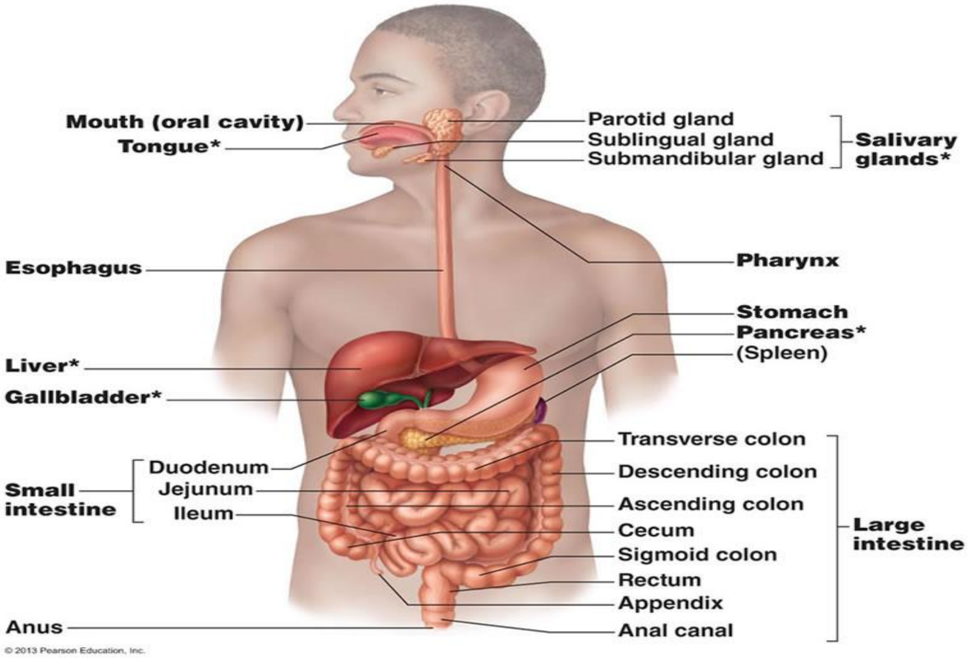

Figure 1.2: The digestive system

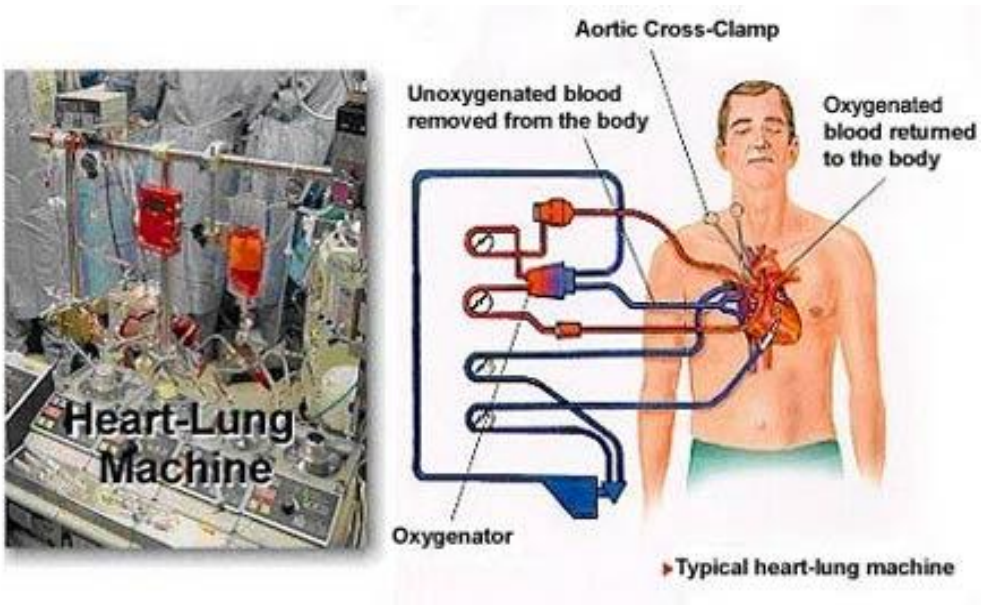

Figure 1.3: The heart-lung machine

1.2 Problem Statement

Questions that have motivated this study is how the peristalsis flow of an Oldroyd-B fluid characteristics are altered under the effect of magnetic field?

1.3 Objectives of The Study

The aim of the study are:

1. To formulate peristaltic motion of a magnetohydrodynamic Oldroyd-B fluid in a planar channel model.
2. To obtain analytical solutions of the system of non-linear partial differential equations resulting from the mathematical modeling of an MHD peristaltic motion of an Oldroyd-B fluid.with heat equation.
3. To determine the effects of various parameters such as the Hartman number, the amplitude ratio, Eckert number and Prandtl number on the flow characteristics.

1.4 Scope of The Study

The study focuses on obtaining approximate solutions for the 2D peristaltic motion of an Oldroyd-B fluid in a planar channel under the effect of magnetic field. This work base on the work done by K. A. Zakaria and N.S Amin (2013).

1.5 Significance of The Study

Peristaltic flow of a fluid is very important for medical diagnosis and it has many clinical applications. These applications are swallowing of food bolus through esophagus, the urine transport from kidney to bladder, the movement of chime in the tract, the transport of lymph in the lymphatic vessels and the vasomotion of small blood vessel. The flow of non-Newtonian fluids also has benefit to the field environmental engineering, chemical and biomedical. Furthermore, peristaltic pump are found in many applications of medicine, engineering and water waste.. That is dialysis machine, heart lung machine, infusion pump, concrete pump, sewage sludge and etc.

1.6 Dissertation Organization

This thesis is organized into six chapters including this introductory chapter. Chapter 1 presents some general introduction, research background, objectives, scope and significance of the study. Chapter 2 discusses the general literature review of non-Newtonian fluid related to the study.

In Chapter 3, the governing non-linear differential equations for an Oldroyd-B fluid are modeled. The peristaltic motion of a magnetohydrodynamic Oldroyd-B fluid in a planar channel with heat transfer is studied under the long wavelength and low Renolds number assumption. The purpose is to study the peristaltic transport of food bolus through the oesophagus modeled by an Oldroyd-B fluid.

The analytical solutions of non-linear equation have been obtained for the axial stream velocity, the axial velocity, the axial pressure gradient, the pressure rise, the frictional force per wavelength, the temperature distribution and the coefficient of heat transfer using the long wavelength and the low Renolds number approximation and exact methods are discussed in Chapter 4.

In Chapter 5, the effects of Hartman number, the flow rate and the amplitude ratio on the pressure gradient, the pressure rise and the frictional force per wavelength have been discussed through numerical computations.

Finally, the conclusions and suggestions for future work are presented in Chapter 6

REFERENCES

Ali, N. and Hayat, T. (2007). Peristaltic motion of a Carreau fluid in an asymmetric channel. *Applied Mathematics and Computation*. 193(2): 535 -552.

Alven, H (1942). Existence of Electromagnetic-Hydrodynamic Waves. *Nature*, Vol. 150, p.p 405.

Anderson, J.R. (1995). *Computational Fluid Dynamics; The Basics with Applications.* New York: McGraw-Hill.

Dunn, J. and Rajagopal, K. (1995). Fluids of differential type: Critical review and thermodynamic analysis. *International Journal of Engineering Science*. 33(5): 689 – 729.

Elshahed, M. and Haroun, M. H. (2005). Peristaltic transport of Johnson-Segalman fluid under effect of a magnetic field. *Mathematical Problems in Engineering*. 2005(6): 663–677.

Fetecau, C., Kamran, M. and Vieru, D.(2009). Exact solutions for the flow of a generalized Oldroyd-B fluid induced by a constantly accelerating plate between two side walls perpendicular to the plate. *Journal of Non-Newtonian Fluid Mechanics*, . 156(3): 189 – 201.

Hayat, T., Alvi, N. and Ali, N. (2008). Peristaltic mechanism of a Maxwell fluid in an asymmetric channel. Nonlinear Analysis: *Real World Applications*. 9(4): 1474 – 1490.

Hayat, T. and Hina, S. (2010). The influence of wall properties on the MHD peristaltic flow of a Maxwell fluid with heat and mass transfer. *Nonlinear Analysis: Real World Applications*. 11(4): 3155 – 3169.

Hayat, T. Wang, Y. Hutter, K. Asghar, S. and Siddiqui, A. M. (2004). Peristaltic transport of an Oldroyd-B fluid in a planar channel. *Mathematical Problems in Engineering*. 2004(4): 347-376.

Hayat, T. Wang, Y., Siddiqui, A.M., and Hutter, K. (2003). Peristaltic motion of Johnson-Segalman fluid in a planar channel. *Mathematical Problems in Engineering*, 1, 1-23.

Kothandapani, M. and Srinivas, S. (2008). Peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. *International Journal of Non-Linear Mechanics.* 43(9): 915 – 924.

Latham, T.W. (1966). *Fluid motion in peristaltic pumping*. Master's Thesis. MIT. Cambridge, MA.

Mishra, M. and Ramachandra Rao, A. (2003). Peristaltic transport of a Newtonian fluid in an asymmetric channel. *Zeitschrift fr angewandte Mathematik und Physik ZAMP*. 54: 532–550.

Mekheimer, K. S., El Shehawey, E. F. and Elaw, A. M. (1998). Peristaltic Motion of a Particle-Fluid Suspension in a Planar Channel. *International Journal of Theoretical Physics.* 37: 2895–2920

Mekheimer, K. S.and Abd Elmaboud.Y. (2008). Peristaltic transport of a particle-fluid through a uniform and non-uniform annulus. *Applied Journal Applied Bionics and Biomechanics.* Vol 5 Issue 2, 47-57.

Mohamed, H. and Haroun. (2007). Effect of Deborah number and phase difference on peristaltic transport of a third-order fluid in an asymmetric channel. *Communications in Nonlinear Science and Numerical Simulation*. 12(8): 1464 – 1480.

Naby, A. E. H. A. E., Misery, A. E. and Kareem, M. A. E. (2004). Separation in the flow through peristaltic motion of a carreau fluid in uniform tube. *Physica A: Statistical Mechanics and its Applications*. 343(0): 1–14.

Nadeem, S., Akbar, N., Hayat, T. and Hendi, A. (2011). Peristaltic flow of Walter's B fluid in endoscope. *Applied Mathematics and Mechanics*. 32: 689–700.

Oldroyd, J. G. (1950). *On the Formulation of Rheological Equations of State*. Proceedings of the Royal Society of London. Series: A Mathematical and Physical Sciences. 200(1063).

Pandey, S. and Chaube, M. (2010). Peristaltic transport of a visco-elastic fluid in a tube of non-uniform cross section. *Mathematical and Computer Modelling*. 52(3-4): 501 – 514.

Radhakrishnamacharya, G. (1982). Long wavelength approximation to peristaltic motion of a power law fluid. *Rheologica Acta*. 21: 30–35.

Radhakrishnamacharya, G. and Radhakrishna Murty, V. (1993). Heat Transfer to Peristaltic Transport in a Non-Uniform Channel. *Defence Science Journal*. Vol 43. No 3, pp 275-280.

Reddy, M. S., Rao, A. R. and Sreenadh, S. (2007). Peristaltic motion of a powerlaw fluid in an asymmetric channel. *International Journal of Non-Linear Mechanics.* 42(10): 1153 – 1161.

Shah, S. H. A. M., Khan, M. and Qi, H. (2009). Exact solutions for a viscoelastic fluid with the generalized Oldroyd-B model. *Nonlinear Analysis: Real World Applications.* 10(4): 2590 – 2599.

Shapiro, A. H., Jaffrin, M. Y. and Weinberg, S. L. (1969). Peristaltic pumping with long wavelengths at low Reynolds number. *Journal of Fluid Mechanics*. 37(04): 799–825.

Siddiqui, A. M. and Schwarz, W. H. (1993). *Peristaltic motion of a third-order fluid in a planar channel*. Rheologica Acta. 32: 47–56.

Siddiqui, A. M., Provost, A. and Schwarz, W. H. Peristaltic pumping of a second order fluid in a planar channel. *Rheologica Acta*, 1991. 30: 249–262.

Singh, R. P. *Introduction to food engineering*. Amsterdam [u.a.]: Elsevier. 2007.

Sobh, A. and Oda, H. (2008). *Blood flow in uniform planar channel*. Asian Journal of Applied Sciences. 1: 46–58.

Soundalgekar, V. (1967). On the flow of an electrically conducting, incompressible fluid near an accelerated plate in the presence of a parallel plate, under transverse magnetic field. *Proceedings Mathematical Sciences*. 65: 179–187.

Srinivas, S. and Kothandapani, M. (2008). Peristaltic transport in an asymmetric channel with heat transfer A note. *International Communications in Heat and Mass Transfer*. 35(4): 514 – 522.

Romig, M. F. (1964). The Influence of Electric and Magnetic Fields on Heat Transfer to Electrically Conducting Fluids. In: Irvine, T. F. and Hartnett, J. P., eds. *Advances in Heat Transfer*. Elsevier, *Advances in Heat Transfer*, vol. 1. 267– 354.

Tripathi, D., Pandey, S. and Das, S. (2010). Peristaltic flow of viscoelastic fluid with fractional Maxwell model through a channel. *Applied Mathematics and Computation*. 215(10): 3645 – 3654.

Wang, Y., Hayat, T. and Hutter, K. (2007). Peristaltic flow of a Johnson-Segalman fluid through a deformable tube. *Theoretical and Computational Fluid Dynamics*. 21: 369–380

Zakaria, K. A. and Amin, N. S. (2013). Analytical Solutions of Peristaltic Motion of Magnetohydrodynamics Non-Newtonian Fluids. Universiti Teknologi Malaysia.

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