MODELING OF BLOOD FLOW IN STENOSED ARTERY WITH A CLOT

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To my beloved mother.

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ABSTRACT

The presence of blood clot in the blood stream can lead to partial or even complete blockage of the blood vessel. The condition can get worse in a stenosed artery. The clot will alter the flow field and modify the pressure distribution. To understand the changes in the blood flow due to presence of the clot, the fluid mechanics of blood flow in a stenosed artery with a clot is studied through a mathematical analysis. Blood is modelled as an incompressible Newtonian fluid and the flow is treated to be axi-symmetric. The artery is modelled as a rigid tube with a stenosis and an elongated shape representing a clot inside it. Both the stenosis and the clot are axi-symmetric over the length of the artery. The governing equations of motion are solved by using regular perturbation analysis. The results show that in the presence of the clot, the important physiologically flow characteristics such as pressure gradient and wall shear stress vary markedly across the stenotic lesion. The height of the peak of the clot increases the wall shear stress. Results are sensitive to not only the geometry of the stenotic wall but also to the position of the clot.

ABSTRAK

Kehadiran darah beku di dalam aliran darah boleh menyebabkan salur darah tersumbat separa atau tersumbat sepenuhnya. Keadaan boleh menjadi lebih buruk di dalam arteri dengan stenosis. Gumpalan akan mengubah medan aliran dan mengubah taburan tekanan. Untuk memahami perubahan dalam aliran darah disebabkan kehadiran gumpalan, mekanik bendalir aliran darah dalam arteri yang mengandungi gumpalan dikaji melalui analisis matematik. Darah dimodelkan sebagai cecair Newtonian yang tidak boleh mampat dan aliran dianggap simetri terhadap paksi. Arteri dimodelkan sebagai tiub yang rigid dengan stenosis dan sebuah bentuk memanjang yang mewakili gumpalan di dalamnya. Stenosis dan gumpalan adalah simetri terhadap paksi di sepanjang arteri. Persamaan-persamaan kawalan gerakan diselesaikan dengan menggunakan analisa perturbasi biasa. Hasil kajian menunjukkan bahawa dengan kehadiran gumpalan, ciri-ciri fisiologi aliran yang penting seperti kecerunan tekanan dan tegasan ricih dinding berubah agak ketara di seluruh kawasan stenosis. Ketinggian puncak gumpalan meningkatkan tegasan ricih dinding. Keputusaan tidak hanya sensitif terhadap geometri dinding stenosis, tetapi juga terhadap posisi gumpalan.

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LIST OF SYMBOLS/NOTATIONS

Roman Letters

d	-	Diameter
$f(z)$	-	function describing wall variation
k	-	Catheter radius (ratio of catheter size to vessel size, $k\,R_o/R_o$
l	-	Length
$\tilde{p}$	-	Dimensional pressure
p	-	Non-dimensional pressure
Q	-	Dimensional flow rate
q	-	Non-dimensional flow rate
$\tilde{r}$	-	Dimensional radial distance
r	-	Non-dimensional radial distance
R_o	-	Undisturbed/ mean radius of the stenosed artery
R_{st}	-	Steady streaming Reynolds number $({U_o}^2/\omega\mu)$
R_{en}	-	Reynolds number $(U_o R_o/\mu)$
R_s	-	Dimensional radius of stenosed artery
r_s	-	Non-dimensional radius of stenosed artery
R_c	-	Dimensional radius of clot artery
r_c	-	Non-dimensional radius of clot artery
$\tilde{t}$	-	Dimensional time
U_o	-	Characteristic axial velocity
(U, W)	-	Dimensional velocity in $(\tilde{r}, \tilde{z})$ direction
(u,w)	-	Non-dimensional velocity in (r, z) direction
$\tilde{z}$	-	Dimensional axial distance
z	-	Non-dimensional axial distance

I_o, K_o	-	Modified Bessel's function of order 0
I_1, K_1	-	Modified Bessel's function of order 1

Greek Letters

α	-	Womersley number ($R_o\sqrt{\omega/\mu}$)
ρ	-	Density of the fluid
λ	-	Length of stenosis
δ	-	Geometric parameter (R_o/λ)
δ_2	-	Radius of peak of clot
θ	-	Dimensional azimuthal distance
ω	-	Angular frequency
μ	-	Kinematic viscosity
τ	-	Non-dimensional time
τ_o	-	Phase angle
ψ	-	Stream function

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CHAPTER 1

INTRODUCTION

1.1 Research Background

The medical term for a blood clot is a thrombus (pl. thrombi). When the thrombus becomes larger, the shear forces that act on it also increase due to the acceleration of blood around the thrombus. If the shear forces exceed the strength of the inter-cellular/inter-molecular bonds, a portion of the thrombus can break off and enter the circulation. A portion of the thrombus that breaks off of the main thrombus is called a thromboembolus or embolus (pl. emboli). Emboli that enter the circulation can have many devastating effects within the body. As they pass through the circulatory system, it is possible that emboli with different sizes become lodged in different sections of the cardiovascular system. This will reduce or completely cut off the oxygen supply to the cells in that area. As a result, the part of the heart muscle that is deprived of oxygen dies, and a heart attack can occur. If the emboli become stuck in the cerebral circulation, a stroke can occur.

There are many situations in which a blood clot is more likely to form including being on long-term bed rest, crossing legs for long periods of time when sitting, or sitting for long periods of time, such as in a plane or car, during and after pregnancy, not having enough water in body (dehydration), taking birth control pills or estrogen hormones (especially in women who smoke) and using an intravenous catheter long-term. Blood clots are also more likely in people with cancer, recent surgery or injury, obesity, and liver or kidney disease. People who have high cholesterol level are also at risk for blood clots. A build up of cholesterol that narrows an artery may change or slow the flow of blood, making it easier for a blood clot or thrombus to form. Conditions that are passed down through families (inherited) may make you more likely to form abnormal blood clots. Inherited conditions such as Factor V Leiden thrombophilia, Prothrombin G20210A mutation and other rare conditions such as protein C, protein S, and antithrombin III deficiencies which affect clotting can also make a person more likely to form abnormal blood clots.

Blood clot can also enter the blood stream during the insertion of catheter into the blood vessel. The catheter may cause a thrombus on the wall of the blood vessel to break free and enter the blood circulation. There is also a risk of bringing pathological constituents into the blood stream during the catheterization process. The constituents that go inside the blood vessel will be attached to the vessel wall but later some constituents can get detached from the wall, then join the blood stream again, and form a clot that can lead to partial or even complete blockage of the blood vessel. One example of the constituents is bacteria such as Coagulase negative staphylococci. Coagulase negative staphylococci can produce an enzyme called coagulase which can cause blood clot formation.

1.2 Problem Statement

In the previous research, most of the studies on blood flow in the arteries involved the insertion of catheter and the existence of stenosis. Only a few of them included clot formation in the blood vessel. The existence of blood clot in the blood vessel especially in the stenosed artery will affect the blood flow. An annular region will be formed in the blood vessel between the blood clot and the wall of the stenosed blood vessel. Thus this research will investigate whether there are any significant changes on the flow field due to the existence of clot inside the stenosed artery.

1.3 Objectives of Study

The purpose of this study is to determine the blood flow characteristics such as the pressure gradient and the wall shear stress in a stenosed artery when there is a presence of a clot inside it. The objectives of the study are

a) to formulate a mathematical model to represent the flow of blood in the annular region between a clot and a stenosed artery.
b) to solve the governing equations of the blood flow by using perturbation method.
c) to investigate the effect of the clot on the blood flow in terms of pressure gradient, wall shear stress and streamline pattern.

1.4 Scope of Study

This work is based on the journal paper by Sarkar and Jayaraman (2005) titled '*Nonlinear analysis of arterial blood flow: steady streaming effect*'. The stenosis and the clot are assumed to be axi-symmetic over the length of the artery. The flow is considered axi-symmetric and oscillatory in nature. The blood is considered Newtonian and incompressible.

1.5 Significance of Study

The blood clot in the blood circulation can lead to partial or even complete blockage of the blood vessels. The seriousness can be very acute in the case of a stenotic artery. This study is significant to be carried out in order to understand the seriousness of the situation by investigating the blood flow characteristic in the artery. In reality, clot formation in blood vessel after the catheter insertion procedure can lead to serious problems such as heart attack and stroke due to the blockage in the blood vessel. Usually, after an insertion of catheter procedure into the blood vessel, the doctor will likely recommend the patient to take aspirin and another anticlotting medicine to prevent blood clot formation. So the investigation of blood flow in stenosed artery together with blood clot is significance to be carried out.

REFERENCES

Anderson, J.R. (1995). *Computational Fluid Dynamics; The Basics with Applications.* New York: McGraw-Hill.

Bell, W.W.(1968). *Special Functions for Scientists and Engineers*. New Jersey: Princeton.

Chapra, S.C. and Canale R.P.(2010). *Numerical Methods for Engineers.* (6^{th} ed.). New York: McGraw-Hill.

Dash, R.K., Jayaraman, G., Metha, K.N. (1996). Estimation of Increased Flow Resistance In A Narrow Catheterized Artery; A Theoretical Model. *J. Biomech*. 29(7): 917-930.

Dash, R. K., Jayaraman, G., Metha, K. N. (1999). Flow in a Catheterized Curved Artery with Stenosis. *J. Biomech* 32: 49-61.

Doffin, J. and Chagneau, F. (1981). J. Oscillating flow between a clot model and a stenosis. *Biomech*. 14 (3): 143-148.

Jayaraman, G. and Sarkar, A. (2005). Nonlinear Analysis of Arterial Blood Flow; Steady Streaming Effect. *Nonlinear Anal*. 63: 880–890. 50

Kanai, H., Iizuka, M., Sakamoto, K. (1970). One of the problems in the measurement of blood pressure by catheter-insertion; wave reflection at the tip of the catheter. *Medical and Biological Engineering*: 483-496.

Ku, David N. (1997). Blood Flow In Arteries, *Annu. Rev. Fluid Mech.* 29:399–434.

Mader, Sylvia S.(2005). *Understanding Human Anatomy & Physiology*, McGraw-Hill.

Martini, F.H., and Ober, W.C., Garrison, C.W., Welch, K., and Hutchings, R.T. (2001). *Fundamental of anatomy & Physiology*. (5th Ed.). Upper Saddle River, N.J.: Prentice Hall.

McDonald, D.A. (1974). *Blood Flow in Arteries*. Baltimore: Williams & Wilkins.

Norzieha binti Mustapha (2009). *Numerical Modelling and Simulation of Blood Flow Through a Multi-Irregular Stenosed Artery.* Universiti Teknologi Malaysia: Dissertation Doctor of Philosophy.

Rubenstein David A., Frame Mary D., Wei Yin, (2012). *Biofluid Mechanics: An Introduction to Fluid Mechanics, Macrocirculation , and Microcirculation*, Elsevier.

Sarkar, A. and Jayaraman, G. (1998). Correction to Flow Rate Pressure Drop Relation in Coronary Angioplasty: Steady Streaming Effect. *J. Biomech* 31: 781-791.

Srivastava, V.P., Vishnoi, R., Mishra. S., and Sinha, P. (2010). Blood Flow through a composite stenosis in catheterized arteries. *E-Journal of Science & Technology.* 4(5) 55 – 64.

White, F.M. (2003). *Fluid Mechanics*. McGraw Hill: New York.

Womersley JR. (1955). Method for the calculation of velocity, rate of flow and viscous drag in arteries when their pressure gradient is known. *J. Physiol.* 127:553–63.