

**THE EFFECT OF ROAD PROFILE ON ONE QUARTER CAR SUSPENSION  
MODEL**

**ABDUL RAZAK BIN AZIT**

**UNIVERSITI TEKNOLOGI MALAYSIA**

**THE EFFECT OF ROAD PROFILE ON ONE QUARTER CAR SUSPENSION  
MODEL**

**ABDUL RAZAK BIN AZIT**

A dissertation submitted in partial fulfillment of the  
requirements for the award of the degree of  
Master of Science (Engineering Mathematics)

Faculty of Science  
Universiti Teknologi Malaysia

JUNE, 2014

*Specially..*

*To my beloved parents*

*To my beloved wife*

*And not forgetting to all friends*

*For their*

*Encouragements, Love, Sacrifice and Best Wishes*

## **ACKNOWLEDGEMENT**

Praise be to Allah S.W.T to Whom we seek help and guidance and under His benevolence we exist and without His help this project could not have been accomplished.

I would like to express my sincere thanks and appreciation to Associated Professor Dr. Shamsuddin Bin Ahmad, my project supervisor, for all the help, generous and guidance time given throughout the course of completing this dissertation.

In addition, I would also thanks to all my friends and course mates for their assistance, guidance, cooperation and friendship throughout the completion of dissertation.

Last but not least, my gratitude also goes to all my family members for their continuous encouragement and support. Thanks you all.

## **ABSTRACT**

A vehicle suspension system consists a system of springs, shock absorbers and linkages that connects a vehicle to its wheels. The above system serves as comfort and security for the driver and passengers in the vehicle. The main objective of this study is to one quarter car suspension systems subject to a Yellow Transverse Bar road profile. In this study, the suspension of travel limit and magnitude of car's vertical acceleration are validated using the standard of Ford Scorpio car. The vibration of the car is compared with the effect of vibration the car driving on the sinusoidal road profile. The results of the study is obtained and analyzed by using mathematical model in term of Ordinary Differential Equation via Maple 12.

## **ABSTRAK**

Sistem gantungan kenderaan terdiri daripada sistem spring, penyerap hentak dan sambungan mekanikal yang menghubungkan kenderaan dengan rodanya. Sistem tersebut memberi keselesaan dan keselamatan kepada pemandu dan penumpang akibat daripada hentakan roda kereta dengan permukaan jalan. Objektif utama kajian ini adalah mengkaji satu per empat sistem gantungan kereta yang melalui permukaan jalan berbentuk Garisan Rentas Kuning. Had gantungan perjalanan dan magnitud cepatan mencancang kereta disahkan dengan menggunakan piawaian kereta Ford Scorpio. Getaran kenderaan bagi sistem gantungan kereta di atas dibandingkan dengan kesan getaran kereta yang melalui profil jalan berbentuk sinus. Keputusan kajian diperolehi dan dianalisis menggunakan model matematik dalam sebutan Persamaan Pembezaan Biasa dengan pengaturcaraan Maple 12.

## TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	<b>DECLARATION</b>	ii
	<b>DEDICATION</b>	iii
	<b>ACKNOWLEDGEMENT</b>	iv
	<b>ABSTRACT</b>	v
	<b>ABSTRAK</b>	vi
	<b>TABLE OF CONTENTS</b>	vii
	<b>LIST OF TABLES</b>	x
	<b>LIST OF FIGURES</b>	xi
	<b>LIST OF APPENDICES</b>	xiv
<b>1</b>	<b>INTRODUCTION</b>	<b>1</b>
	1.1 Introduction	1
	1.2 Background of the Problem	2
	1.3 Statement of Problem	2
	1.4 Objectives of the Research	3
	1.5 Scope of the Research	3
	1.6 Significance of the Research	4
<b>2</b>	<b>LITERATURE REVIEW</b>	<b>5</b>
	2.1 Introduction on Car Suspension	5
	2.2 The Definition of Suspension	6
	2.3 The Suspension Systems	6
	2.3.1 The Active Suspension System	7
	2.3.2 The Semi-Active Suspension System	8

2.3.3	The Passive Suspension System	9
2.4	Comparison of Active Suspension and Passive Suspension	10
2.5	Modeling of Quarter Car Suspension	12
2.6	Road Profile	13
2.6.1	Sinusoid Road Profile	14
2.6.2	Yellow Transverse Bar Road Profile	15
2.7	The Newton's Law of Motion	19
2.8	The Hooke's Law of Motion	20
2.9	Second Order Differential Equation	21
2.9.1	Solution of Homogeneous Equations	21
2.9.2	Solution of Non-homogeneous equation	22
2.9.2.1	Method of Undetermined Coefficients	23
2.9.3	Fourier Series	24
<b>3</b>	<b>METHODOLOGY</b>	<b>28</b>
3.1	Square-wave Function	28
3.2	Sinusoidal Car Suspension System Model	31
3.2.1	Method of Undetermined Coefficients	33
3.2.2	Fourier Series	36
3.3	Maple 12 Software	43
<b>4</b>	<b>FORMULATION OF A QUARTER OF CAR SUSPENSION MODEL</b>	<b>44</b>
4.1	Factors to Set Up A Quarter of Car Suspension Model	44
4.2	Assumptions	45
4.3	Formulate a Quarter Car Suspension Model	46
4.4	Solution of a Quarter Car Suspension Model	52



4.4.1 Analytical Solution of a Quarter Car Suspension Model	52
4.4.2 Interpretation and Validation of Graphs	56
<b>5 YELLOW TRANSVERSE BAR ROAD PROFILE</b>	<b>62</b>
5.1 Yellow Transverse Bar (YTB) Road Profile	62
5.2 Formulate a Model on YTB Road Profile	64
5.2.1 Interpretation and Validation of Graph	73
<b>6 DISCUSSION</b>	<b>92</b>
6.1 Discussion on Car Suspension System	92
6.2 Discussion of quarter car suspension system model's via sinusoidal and YTB road profile	93
<b>7 CONCLUSION AND FUTURE RESEARCH</b>	<b>97</b>
7.1 Conclusion	97
7.2 Recommendation for Future Research	98
<b>REFERENCES</b>	<b>100</b>
<b>APPENDIX A - C</b>	<b>103 - 107</b>

**LIST OF TABLE**

<b>TABLE NO.</b>	<b>TITLE</b>	<b>PAGE</b>
2.1	Yellow Transverse Bar Specification	17
2.2	Roots of the Characteristic Equation	22
2.3	General Solution	22
2.4	Expected trial functions $y_p(x)$	24
6.1	Effect of thickness and width of alert bar on quarter car suspension via YTB road profile	94

**LIST OF FIGURES**

<b>FIGURES NO.</b>	<b>TITLE</b>	<b>PAGE</b>
2.1	A Soft Active Suspension System	7
2.2	A Stiff Active Suspension System	8
2.3	A Semi-Active Car Suspension System	9
2.4	A Passive Suspension System	10
2.5	Front Vertical Wheel Displacement	11
2.6	Rear Vertical Wheel Displacement	12
2.7	Quarter Car Suspension Model	12
2.8	Sinusoid Road Profile	15
2.9	Yellow Transverse Bar (YTB)	16
2.10	(a) Curve                      (b) Junction (c) Pedestrian Crossing	16
2.11	Standard Measurement of Alert Bar	18

2.12	YTB Construction	18
2.13	Alert Bar	18
2.14	Square waveform road profile	19
2.15	Sine, Square, Triangle and Sawtooth Waveforms	25
3.1	Square waveform	28
3.2	Partial sums of the Fourier series for the square-wave function	31
3.3	Road Profile	32
3.4	Model of a Car	32
4.1	Sinusoid Road Surface Profile	46
4.2	Simple Car Body Model	47
4.3	Car Body Force Diagram	49
4.4	Displacement $x$ against time $t$	57
4.5	Displacement $x$ against time $t$	58
4.6	Displacement $x$ against time $t$	59
4.7	Displacement $x$ against time $t$	60
4.8	Displacement $x$ against time $t$	61

5.1	Alert Bar Measurement	62
5.2	YTB Road Profile	63
5.3	YTB Road Profile with $h = 0.1\text{ m}$ , $d = 2\text{ m}$	74
5.4	Graph of $x$ Various Time $t$ within $0 < t < 15$	75
5.5	Graph of $x$ Various Time $t$ within $0 < t < 15$	77
5.6	YTB Road Profile with $h = 1\text{ m}$ and $d = 2\text{ m}$	78
5.7	Graph of $x$ Various Time $t$ within $0 < t < 15$	79
5.8	YTB Road Profile with $h = 0.01\text{ m}$ and $d = 2\text{ m}$	80
5.9	Graph of $x$ Various Time $t$ within $0 < t < 15$	81
5.10	YTB Road Profile with $h = 0.001\text{ m}$ and $d = 2\text{ m}$	82
5.11	Graph of $x$ Various Time $t$ within $0 < t < 15$	83
5.12	YTB Road Profile with $h = 0.0001\text{ m}$ and $d = 2\text{ m}$	84

5.13	Graph of $x$ Various Time $t$ within $0 < t < 15$	85
5.14	YTB Road Profile with $h = 0.003$ m and $d = 0.3$ m	86
5.15	Graph of $x$ Various Time $t$ within $0 < t < 15$	90
7.1	“Heaviside Waveform” road profile	99
7.2	“Sawtooth Waveform” road profile	99

**LIST OF APPENDIX**

<b>APPENDIX NO.</b>	<b>TITLE</b>	<b>PAGE</b>
A	Maple Code of Figure 4.4, Figure 4.5, Figure 4.6, Figure 4.7 and Figure 4.8	103
B	Maple Code of Figure 5.4	104
C	Maple Code of Figure 5.5, Figure 5.7, Figure 5.9, Figure 5.11, Figure 5.13 and Figure 5.15	106

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Introduction**

A suspension system connects the wheel of the car to the body. In such a way that the body is cautioned from jolts resulting from driving on uneven road surfaces. The suspension affects a car's comfort and performance. The suspension system must also keep tyres firmly in contact with the ground and to provide a comfortable ride for the passengers. The suspension also prevents the car from shaking itself and without the suspension the ride will be so harsh and inconvenient.

The shock absorber is modeled as a simple spring-dashpot system with spring stiffness and dashpot constant. Most car suspension system use springs in the form of a coil or a series of leaves made of steel. The dashpot which is commonly known as damper is usually a hydraulic device which is effectively a piston moving inside a housing containing fluid.



## **1.2 Background of the Problem**

The suspension system has two basic functions to keep the car's wheel in firm contact with the road and provides a comfortable ride for the passengers.

As the plunger in the shock absorber causes the fluid to move, a certain resistance to this motion is exhibited; a sort of internal molecular friction. This drag is caused by short-range molecular cohesive forces and it opposed the upward or downward motion of the wheel. Without coil and leaf springs to absorb the vertical energy would jump off uneven road. The suspension system suspends the car's body a short distance above the ground and maintains the body at relatively constant height to prevent it from pitching and swaying. The car suspension system also keeps all four tyres firmly in contact with the ground.

The car's suspension system is also needs to be designed to compromise between the comforts as the road handling can be improved by further investigation and research.

## **1.3 Statement Problem**

A one quarter car suspension system consists a shock absorber and spring. The different of road profiles will give to different vibration on a car suspension system. The suspension system must also keep tyres firmly in contact with the road and to provide a comfortable ride for the passengers.

To study the effect of road profiles on the car suspension system, a specific model of the car suspension system need to be formulated.

#### **1.4 Objectives of the Research**

The objectives of this research are:

- a) to formulate a quarter car suspension models.
- b) to find the solution of the car suspension models.
- c) to study the effect of sinusoidal and Yellow Transverse Bar on the car suspension models.

#### **1.5 Scope of the research**

There are many models to study the effect of road profile on car suspension systems. In this study we use an ordinary differential equation model.

## 1.6 Significance of the Research

In recent years, the automobile industry has tried to fulfill customer demand. In order to make it happen is with, put a great efforts in producing highly develop vehicle's suspension system in determining the quality of ride. This research will give benefits to Mathematicians, Physicists, Engineers and others field indirectly to improve the vehicle's suspension system and also improve vehicle's performance.

The advantage that we gain is we will able to investigate into the relationship between alert bar at the road surface and the tyre that rides through the alert bar with a view to identify some peculiar characteristics of this interaction. In particular, the model evaluates the effective distance that should be installed between two road bars. This would help in controlling the speed of vehicles, reduce noise pollution due to vehicle movement and sudden break application, and maintain minimum impact on the vehicles. Otherwise, the application of mathematics is specific to a car suspension system by different road profile.

## REFERENCES

1. Cheah Tat Wee. *Study and Analysis of a Passenger Car Suspension System*. Undergraduate Research Report. University Technology Malaysia. 2001.
2. Ezzati Farhani Bt Muhammad Nur. *Active Suspension System for Quarter Car Model*. Undergraduate Research Report. University Technology Malaysia. 2006.
3. Sheldon L. Abbott and Ivan D. Hinerman. *Automotive Suspension and Steering*. Glencoe Publishing Co. 1982
4. Glossary of Automotive Terms. Society of Automotive Engineers. Feb, 1998.
5. Yahaya, Md. S. *Robust Control of Active Suspension System for a Quarter Car Model*. Journal of Control and Instrumentation Engineering Department, Faculty of Electrical Engineering, University Malaysia. 2006.
6. Yahaya, Md. S. *Modelling and Control of Active Suspension System Using Proportional Intergral Sliding Mode Control*. Ph. D. University Technology Malaysia. 2004.

7. Vigneswaran, M. R. *Computer Simulation of Passive and Active Suspension System Using Simulink*. Journal of School of Electrical System Engineering Department, University Malaysia Perlis, April 2008.
8. Yahaya, Md. S. and Johari, H. S.O. *Sliding Mode Control of A Hydraulically Actuated Active Suspension*. Journal Technology, 44(D): 37-48, University Technology Malaysia. 2006.
9. Pang A.Y. *Mathematical Modelling In Car Suspension System*. Master Research Report. University Technology Malaysia. 2010.
10. Dirman, H. and Mohd. Fua'ad R. *System Identification of Nonlinear Model of A Quarter Car Passive Suspension with Backpropagation Neural Network*. Journal of Control and Instrumentation Engineering Department, Faculty of Electrical Engineering, University Technology Malaysia. 2005.
11. Abu Bakar Adham Bin Hell Mee. *Modeling and Controller Design for an Active Car Suspension System Using Half Car Model*. Journal of Control and Instrumentation Engineering Department, Faculty of Electrical Engineering, University Technology Malaysia. 2008/2009.
12. Emanuele Guglielmino, Tudor Sideteanu, Charles W. Stammers, Gheorghe Ghita and Marius Giuclea. *Semi-active Suspension Control: Improved Vehicle Ride and Road Friendliness*. Springer. 2008.
13. Dennis G.zill. *Differential Equation With Compute Lab Experiments*. Second Edition. Casebound. 1998.
14. Normah. M, Zaiton M.I, Halijah O., Sharidan S. and Khairil A.A. *Differential Equation Module*. Department of Mathematics, Faculty of Science, University Technology Malaysia. Esktop Publisher. 2008.

15. James, W. B. and Ruel, V. C. *Fourier Series and Boundary Value Problems, Seventh Edition*. Mc Graw Hill, Higher Education. America, New York. 2008. Chapter 1, pg1-23.
16. Edwards, C.H. and David, E.P. *Differential Equation and Boundary Value Problems: Computing and Modelling. 2<sup>nd</sup> Edition*. New Jersey: Prentice Hall International, Inc. 2000. Chapter 3, pg 180-218.
17. Norhizan S., M. Idros M.N, M.Latib K.J, M.Nadzrin S., A.Fahmi A.G. *Seminar Fasiliti Keselamatan Jalan Raya*. Jabatan Kerja Raya, Malaysia. 2012.
18. H.E.L.M .*Representing Periodic Functions by Fourier Series*. Workbook Level 2. Version 1: March 18, 2004. Chapter 23, pg 12 – 13.
19. S. H. Sawant, M. V. Belwalkar, M. A. Kamble, PUSHPA B.K and D.D. PATIL. *Vibrational Analysis Of Quarter Car Vehicle Dynamic System Subjected To Harmonic Excitation By Road Surface*. Mechanical Engineering Department, Dr. J. J.Magdum College of Engineering, Jaysingpur. Vol-1 Iss-3,4, 2012.