

Perp. Sultanah Zanariah, UTM



30000010084530

10360409

IMPLEMENTATION OF MOTION PLANNING AND ACTIVE FORCE
CONTROL TO A VIRTUAL WHEELED MOBILE ROBOT

TANG HOWE HING

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Master of Engineering (Mechanical)

Faculty of Mechanical Engineering
Universiti Teknologi Malaysia

JULY 2004

**To my dearest and loving parents,
brother, sister, and all of my friends
for their unending love, sacrifices, and moral support**

ACKNOWLEDGEMENTS

I would like to take this opportunity to express my utmost appreciation and gratitude to my research project supervisors, Associate Professor Dr. Musa Mailah and Dr. Mohammad Kasim Abdul Jalil for their thorough supervision, advice, guidance and motivation that enable me to achieve the objectives of this project. Without their precious contributions, I would simply be at loss and would not be able to bring the project to completion.

I would also like to thank a number of individuals who have unselfishly contributed their effort and time to provide useful information, materials and equipment related to the project. These include personnel from the Perpustakaan Sultanah Zanariah, Universiti Teknologi Malaysia.

Last but not least, credits are also due to my research colleagues and juniors for their continuous moral support and encouragement during the course of completing the program of study.

ABSTRACT

The research focuses on the development of a virtual wheeled mobile robot (WMR) simulator that integrates the essential aspects of motion planning, motion control and virtual reality (VR) technique. The developed simulator may serve as a virtual testbed for the repetitive experimentation of the proposed mobile robot control scheme within a specified workspace or layout. The motion path planning is based on the A* heuristic search algorithm with a specific reference to the six-elementary jumps graph for the generation of a nonholonomic global collision-free path environment. A robust active force control (AFC) strategy is incorporated as the WMR motion controller that can accommodate effective disturbance compensation control action in order to produce accurate trajectory tracking task even in the wake of the modelled disturbances. A trajectory planner has been deliberately introduced as the interface between the motion planner and the motion controller. Later, a VR technique is applied to create the virtual environment (VE) that effectively integrates the main elements and transforms the system into a virtual WMR simulator with the added features that will enable researcher to perform experimentation of the mobile robot. A case study is furnished in the research study taking into account a computer integrated manufacturing (CIM) layout in which the proposed mobile robot is supposed to navigate. A rigorous simulation study is performed to demonstrate the effectiveness of the proposed system. Results clearly indicate the successful realization as well as implementation of the developed virtual WMR simulator in which the WMR has been conclusively shown to be very stable, robust and accurate in its tracking ability.

ABSTRAK

Penyelidikan ini adalah berkaitan dengan langkah menyepadukan beberapa aspek, iaitu perancangan pergerakan, kawalan pergerakan, dan teknik realiti maya (VR) dalam pembinaan suatu penyelaku maya robot mudah gerak beroda (WMR). Penyelaku yang dibina ini dapat menyediakan satu pentas uji maya bagi penyelidik untuk menjalankan eksperimen secara berterusan dan berulang terhadap skema kawalan robot mudah gerak di dalam suatu ruang kerja yang tertentu. Secara amnya, perancangan pergerakan robot mudah gerak adalah berdasarkan kepada algoritma pencarian heuristik A* yang menggunakan graf enam lompatan asas dalam perancangan secara global untuk memperolehi satu laluan tak holonomik yang bebas daripada segala halangan. Sementara itu, skema kawalan daya aktif (AFC) juga telah disepadukan ke dalam kawalan pergerakan robot mudah gerak supaya kawalan tersebut adalah lasak dan berkeupayaan dalam memampas segala gangguan yang wujud dalam sistem tersebut. Dengan ini, robot mudah gerak dapat mengikut laluan dengan tepat dan berkesan. Dalam kajian ini, satu perancang laluan juga telah dimodel dan diaplikasikan sebagai satu pengantara di antara perancang pergerakan dan kawalan pergerakan. Seterusnya, teknik VR telah digunakan untuk membina satu persekitaran maya (VE) yang menggabungkan kesemua unsur yang terlibat ke dalam model penyelaku maya WMR. Satu kajian kes telah dilakukan dengan melibatkan suatu susunatur pelan bagi sistem pembuatan berbantuan komputer bersepadu (CIM) di mana robot mudah gerak diarahkan beroperasi dalam suasana kerja tersebut. Akhir sekali, kajian simulasi telah dilakukan untuk mengkaji keberkesanan sistem yang dicadangkan. Hal ini dapat diperhatikan hasil daripada keputusan simulasi yang telah diperolehi yang menunjukkan bahawa robot mudah gerak yang diuji dapat beroperasi secara stabil, lasak dan tepat ketika menjalankan tugas menjejak laluan yang direncanakan.

CONTENTS

CHAPTER	SUBJECTS	PAGE
	TITLE PAGE	i
	DECLARATION OF ORIGINALITY	ii
	DEDICATION	iii
	ACKNOWLEDGEMENTS	iv
	ABSTRACT	v
	ABSTRAK	vi
	CONTENTS	vii
	LIST OF FIGURES	xii
	LIST OF TABLES	xiv
	LIST OF SYMBOLS	xv
	LIST OF APPENDICES	xviii
CHAPTER 1	INTRODUCTION	
	1.1 General Introduction	1
	1.2 Reseach Objectives	3
	1.3 Scope of Study	3
	1.4 Problem Statements	4
	1.5 Research Methodology	7
	1.6 Equipments and Tools Requirement	10
	1.7 Research Contribution	10
	1.8 Organization of Thesis	11

CHAPTER 2 THEORETICAL PRELIMINARIES AND REVIEWS

2.1	Introduction	13
2.2	Motion Planning	13
	2.2.1 Model-based Path Planning	14
	2.2.2 Sensor-based Path Planning	21
2.3	Motion Control	22
2.4	Integration of Motion Planning and Motion Control	24
	2.4.1 Hybrid Architecture	25
	2.4.2 Trajectory Planning	25
2.5	Virtual Reality	26
2.6	Definition of Mobile Robot	27
2.7	Physical Design of Mobile Robot	27
2.8	Configuration of Wheeled Mobile Robot	29
2.9	Nonholonomic Constraint	30
2.10	Coordinate Transformation	32
2.11	Kinematic Model of Nonholonomic Wheeled Mobile Robot	33
2.12	Dynamic Model of Nonholonomic Wheeled Mobile Robot	34
2.13	Conclusion	37

CHAPTER 3 GLOBAL COLLISION-FREE PATH PLANNING

3.1	Introduction	38
3.2	Definition of Collision-free Path Planning	38
3.3	Artificial Intelligence	39

3.4	Overview of Problem Solving by <i>Searching</i>	39
3.5	Complexity Hierarchy of Path Planning	40
3.5.1	Known Stationary Environment	41
3.5.2	Initially Unknown Stationary Environment	41
3.5.3	Perfectly Predictable Time Varying Environment	42
3.5.4	Imperfectly Predictable Time Varying Environment	42
3.6	Procedure in Global Collision-free Path Planning	43
3.7	Nonholonomic Global Collision-free Path Planning	44
3.8	Configuration Space	45
3.9	Elementary Jump Graph Generation	47
3.10	A* Heuristic Search	49
3.11	Simulation Parameters	51
3.12	Simulation Results and Discussion	54
3.13	Conclusion	58

CHAPTER 4 NONHOLONOMIC WHEELED MOBILE ROBOT MOTION CONTROLLER

4.1	Introduction	59
4.2	Nonholonomic Motion Controller Modelling	60
4.2.1	Active Force Control	62
4.2.2	Resolved Acceleration Control	64
4.3	Simulation	65

4.3.1	Simulation Parameters	65
4.3.2	Reference Trajectory Model	67
4.3.2.1	Straight Line Trajectory Formulation	67
4.3.2.2	Circular Trajectory Formulation	69
4.3.3	Applied Disturbances	70
4.3.3.1	Constant Disturbance Torque	70
4.3.3.2	Harmonic Disturbance Torque	71
4.4	Simulation Results and Discussion	73
4.4.1	Effect of Increasing τ_a on Straight Line Trajectory Tracking Task	73
4.4.2	Effect of Changing the Phase Angles for τ_h on Straight Line Trajectory Tracking Task	75
4.4.3	Effect of Increasing τ_a on Circular Trajectory Tracking Task	76
4.4.4	Effect of Changing the Phase Angles for τ_h on Circular Trajectory Tracking Task	78
4.5	Conclusion	79

CHAPTER 5 VIRTUAL WHEELED MOBILE ROBOT SIMULATOR

5.1	Introduction	81
-----	--------------	----

5.2	Robot Simulator	81
5.3	Overall Architecture of Virtual WMR Simulator	82
5.4	Trajectory Planning	84
5.5	Scene Graph of Virtual Environment	87
5.6	Simulation Setup	90
5.7	Simulation Results and Discussion	92
5.8	Conclusion	97
 CHAPTER 6 CONCLUSION AND RECOMMENDATIONS		
6.1	Conclusion	98
6.2	Recommendations	99
 REFERENCES		101
 APPENDICES		
Appendix A – H		109 – 148
 LIST OF PUBLICATIONS		149

LIST OF FIGURES

FIGURE. NO	TITLE	PAGE
1.1	Implementation procedures for the research	8
2.1	Computing algorithm for a simple W -potential	16
2.2	A generalized Voronoi diagram	17
2.3	A visibility graph	18
2.4	Formation of quad tree graph through recursive map decomposition	18
2.5	Quad tree representation of the map	19
2.6	Elementary jumps of the mobile robot in motion planning	19
2.7	Flow chart of Dijkstra's method	21
2.8	AFC elements of a dynamic system	24
2.9	Various physical structures of WMR	28
2.10	Configuration of a WMR	29
2.11	A local Cartesian system and a global Cartesian system	32
3.1	Overview of problem solving by <i>searching</i>	40
3.2	Flow chart of global collision-free path planning	43
3.3	Feasible and non-feasible paths for the nonholonomic mobile robot	45
3.4	Configuration space for a cylindrical mobile robot	47
3.5	A bitmap representation of the map	47
3.6	Eight elementary jumps in the selection of potential successors	48

3.7	Six elementary jumps in the selection of potential successors	49
3.8	Flow chart of the A* heuristic search algorithm	51
3.9	Pre-defined topographical map for the workspace	52
3.10	Mobile robot and the segments of the path	53
3.11	Collision-free path planning from (20,20) to (20,40)	55
3.12	Collision-free path planning from (20,20) to (160,160)	55
3.13	Collision-free path planning from (10,10) to (190,190)	56
3.14	Collision-free path planning from (10,10) to (190,10)	56
4.1	A block diagram for the proposed motion controller	60
4.2	Incorporation of AFC into the motion controller	62
4.3	Straight line reference trajectory for the mobile robot	68
4.4	Reference circular trajectory for the mobile robot	69
4.5	The generation of constant disturbance torque	71
4.6	Three models of harmonic disturbance torque	72
4.7	Effect of increasing τ_a on trajectory tracking task	74
4.8	Effect of changing the phase angles for τ_h on trajectory tracking task	76
4.9	Effect of increasing τ_a on trajectory tracking task	77
4.10	Effect of changing the phase angles for τ_h on trajectory tracking task	79
5.1	Interlinking of motion planning, motion control and VR	83
5.2	A block diagram of the virtual WMR simulator	84
5.3	Hierarchical structure of the scene graph for a VE	88
5.4	Procedures of VE rendering for a hierarchical structured scene graph	89
5.5	Performance of virtual WMR simulator under various loading conditions	95
5.6	Several views of the constructed virtual WMR simulator	96

LIST OF TABLES

TABLE. NO	TITLE	PAGE
3.1	Computation time for the global collision-free path planning	54
4.1	The characteristics of the harmonic disturbance models	71

LIST OF SYMBOLS

a_1	-	3 rd order coefficient for cubic spline function
a_2	-	2 nd order coefficient for cubic spline function
a_3	-	1 st order coefficient for cubic spline function
a_4	-	0 th order coefficient for cubic spline function
A	-	Nonholonomic constraint
AFC	-	Active force control
AI	-	Artificial Intelligence
A_p	-	All of the possible transformation
b	-	Distance between driving wheels and axis of geommetry
C	-	Configuration space
C_{en}	-	Center of circular trajectory
C_{free}	-	Free space
C_{obs}	-	Obstacle space
d	-	Distance between P_o and P_c
D	-	Effective radius of mobile robot
DOF	-	Degrees of freedom
f	-	Cost function
F_{child}	-	Total cost of the <i>Child Node</i>
F_v	-	Total cost of V
$F_{v-child}$	-	Cost of the edge from V to adjacent vertex
g	-	Travelling cost
G	-	Gravitational vector
G_f	-	Arbitration function of C -potential
h	-	Heuristic function
I	-	Total moment of inertia for WMR
IN	-	Inertia matrix

I_c	-	Moment of inertia for the platform excluding the driving wheels and the motor rotor
I_w	-	Moment of inertia for the wheels and the motor rotor about the wheel axis
I_m	-	Moment of inertia for the wheels and the motor rotor about the wheel diameter
K_c	-	AFC constant
K_d	-	Derivative constant
K_p	-	Proportional constant
K_t	-	Motor constant
L	-	Lagrange equation
m	-	Total mass of WMR
M	-	Inertia vector
\bar{M}	-	Simplified inertia vector
m_c	-	Mass of the platform excluding the driving wheels and the motor rotors
m_w	-	Mass of the driving wheels
N_G	-	Goal <i>Node</i>
N_i	-	Current <i>Node</i>
N_s	-	Start <i>Node</i>
O	-	Global axis reference point
p	-	Potential function
ps	-	Position of WMR
pt	-	Control point in the mobile robot
P_c	-	Mobile robot's reference point
P_o	-	Middle point of the axis of rotation for wheels
q	-	Current states of mobile robot
Q	-	Segments for path curve
q_c	-	States command
q_r	-	Reference states
r	-	Radius of wheels
RAC	-	Resolved acceleration control
rad	-	radius of circular trajectory
S	-	Unit cycle dimension

t_{start}	-	Simulation start time
t_{stop}	-	Simulation stop time
U	-	C-potentials
v	-	Linear velocity
V	-	Coriolis vector
\bar{V}	-	Simplified coriolis vector
VE	-	Virtual environment
Vel	-	Tangential velocity of mobile robot
VP	-	W -potentials
VR	-	Virtual reality
WMR	-	Wheeled mobile robot
WTK	-	WorldToolKit®
X	-	Forward kinematic map of WMR
x	-	Current position of mobile robot in x -plane
y	-	Current position of mobile robot in y -plane
β	-	Angle of elementary jump
\mathcal{G}	-	Forbidden region
λ	-	Lagrangian multiplier
ϕ	-	Current orientation of mobile robot
θ	-	Wheel rotation
\mathcal{H}	-	Dimension
τ_a	-	Constant disturbance torque
τ_c	-	Control torque
τ_d	-	Disturbance torque
τ_d^*	-	Estimated disturbance torque
τ_h	-	Harmonic disturbance torque
τ_l	-	Actuation torque on left wheel by left motor
τ_r	-	Actuation torque on right wheel by right motor
ω	-	Angular velocity
\mathcal{W}	-	WMR workspace

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A	Scene graph of the virtual environment for the the CIM facility	109
B	C++ source code for the main program of virtual WMR simulator	114
C	C++ source code for the header file "Astar.hpp"	121
D	C++ source code for the header file "Solution.hpp"	133
E	C++ source code for the header file "Model.hpp"	143
F	M-code for the interface of parametric cubic spline interpolation and ODE solution	144
G	M-code for parametric cubic spline interpolation	146
H	M-code for the ODE solution	147

CHAPTER 1

INTRODUCTION

1.1 General Introduction

Recent improvements in manufacturing technology related to computer integrated manufacturing (CIM) environment result in higher demand and expectation for more sophisticated automated components including the utilization of automated guided vehicle (AGV), autonomous transporter or simply mobile robot. The mobility and manoeuvrability of this particular robot provides an added flexibility to the manufacturing system in terms of its capability to effectively improve the manipulating or handling of materials within the bounded workspace of the CIM layout. The limitation of the workspace for the static manipulator has actually failed to meet all these requirements, and thus the robot should be granted with the mobility (freedom to roam or move) feature. However, the expansion of the mobile robot's workspace on the other hand has also increased the risk of having the robot to collide with obstacles or other 'disturbances' along its navigation path. It has been proven that robots were absolutely unsafe and easy to collide among themselves or with the environmental obstacles [1]. Therefore, it is important that the mobile robotic system should be endowed with the appropriate capabilities and intelligence with respect to motion planning and control. With drastic advances in current electronic and computing technology, the research area on motion path planning has expanded tremendously while at the same time, considerable efforts have been devoted for the synthesis and analysis of the mobile robot control strategies [2].

Actually, motion planning is just one of the components of the more general paradigm of robot control which consists of both motion planning and control components [3]. Generally, motion planning breaks down the high-level control tasks into the low-level motion execution commands [4]. At a higher-level, the motion planning is responsible for the generation of a collision-free nonholonomic trajectory with reference to the time optimality and the workspace layout [5]. Motion planning involves various disciplines and subfields such as map building, search space generation, graph searching, path refining, and trajectory planning. It is presumed that the motion planning is performed in advance before the mobile robot is actually operated in the workspace [6]. Meanwhile at a lower-level, a motion controller is designed in such a way that the high-level 'motion plan' is refined and converted into a series of motor execution commands which drives the mobile robot in the local environment.

Usually, the workspace for the mobile robot is not ideal, i.e. it is full of disturbances and uncertainties. These undesired forces tend to perturb the mobile robot away from the desired course and consequently result in the degradation of the performance of the mobile robot. Thus, appropriate control mechanism must be incorporated into the robot system so that these unwanted features could be effectively accommodated or compensated in such a way that the actual performance is virtually unaffected.

With the emergence of virtual reality (VR), simulation study is no longer limited to the analysis of graphs and numbers. In fact, VR has provided a new tool for the effective virtual and graphical presentation of the simulation results which are easier to be interpreted and understood. Besides, in some specific cases, the mobile robot experimentation can be so complex that it is either too dangerous for the human operators or too costly for the expenditure overheads. As a solution for these situations, VR technique is sometimes applied, particularly during the simulation process in order to cut down the experimentation cost or reduce the safety risk which might be encountered by the human operators. Meanwhile, VR also provides an artificial testbed which is very close to the reality for the simulation study and thus, the results obtained through the simulator are more promising and reliable.

It is clear from the above description that the proposed research centres on the three aspects related to motion planning, motion control and the application of virtual reality technique. In this research project, a virtual environment (VE) of a CIM facility at the Industrial Automation Laboratory, Faculty of Mechanical Engineering (FME), Universiti Teknologi Malaysia (UTM) is modelled in advance prior to being utilized as the VR-based testbed for the autonomous mobile robot motion control simulator. Among other means of control strategies, active force control (AFC) as a type of acceleration feedback control scheme is proposed and applied to the system under study to effectively compensate the disturbances and uncertainties. Likewise, a number of motion planning algorithms will be explored and used in the study. The coordination and implementation of all the components shall be done according to the objectives and scope outlined in the thesis.

1.2 Research Objectives

The aim of this project is to develop a virtual wheeled mobile robot (WMR) simulator that comprises three elements (motion planning, motion control, and VR) that are fully integrated and linked. The research is carried out towards approaching the following objectives:

- To endow the WMR system with basic capabilities and intelligence in collision-free motion planning.
- To incorporate AFC scheme into the nonholonomic WMR system for a precise and robust trajectory tracking.
- To investigate the application of VR in mobile robot simulation and control.

1.3 Scope of Study

The scope of the study shall encompass the following aspects.

- Implementation of a collision-free global path planning for the WMR in a known stationary environment.
- Reference trajectory for the motion controller is generated from geometrical collision-free path through parametric cubic spline interpolation.
- Simulation is implemented with Microsoft Visual C++ which is assisted with MATLAB software package.
- Application of VR using WorldToolKit as a means of a graphical representation of the simulation results.
- The construction of virtual wheeled mobile robot simulator as a testbed for the case study of the motion planning and control scheme of the WMR in the CIM facility of the Industrial Automation Laboratory, FME, UTM.

1.4 Problem Statements

Over the last two decades, the problems of motion planning and motion control have been studied extensively but the two areas are often studied separately or in an isolated fashion. Very few researches have studied the integration of these two topics. Most of them either aim solely on the construction of the motion planner or thoroughly concentrate on the modelling of the motion controller. Very few studies have been carried out on the direct interaction or link between the motion planner and motion controller. In fact, current researchers in this area have ignored the dynamic effects of the WMR in their works. Extensive reviews on this research will be discussed in Chapter 2.

In the motion planning study, most of the researchers gravitate on the collision-free path planning which gives only a sequence of time-independent continuous configurations from the initial position to the goal position as in [7] and [8]. There are also researchers who solely contribute to the time-indexed collision-free trajectory formulation [9, 10]. Meanwhile, for motion control study, it is usual

that during the simulation process, the mobile robot is required to trace a very simple reference trajectory in the form of either a straight line or a circular path [11, 12, 13]. Thus, the study of the motion controller is somewhat constraint along this line, whereas it is sometimes necessary to drive the mobile robot to execute a very complex trajectory tracking activity. Besides, the simulation also neglects the presence of the obstacles in the workspace and thus, it relies very much on a previously planned collision-free trajectory environment. All these factors contribute to the reduced applicability of the motion planner and controller. To overcome this, an interface between the motion planner and motion controller can be constructed to transform the time-independent feasible geometry points from the motion planner into time-dependent trajectory form. The resulting trajectory obtained from this procedure is then fed into the motion controller as the desired reference trajectory.

The motion of the WMR should be restrained by the nonholonomic constraint that consequently increases the degree of complexity in obtaining the solution for the motion planning task. A nonholonomic constraint is expressed as a non-integrable equation involving the derivative of the configuration parameters, and thus an arbitrary path in the admissible configuration space does not necessarily correspond to a feasible path [14]. Conventional path planning methods always assume that the mobile robot as an omni-directional point mobile robot and travels slowly at constant velocity [15]. In other words, the mobile robot is assumed to have excellent manoeuvrability and it is able to access all of the directions of moving without any constraints. In many applications, this condition is actually not realistic and in fact degrades the performance of the motion controller. Therefore, nonholonomic constraint of the WMR should be taken into consideration during the motion planning process.

Besides, it is a fact that the WMR does not meet Brockett's well-known necessary smooth feedback stabilization condition [16]. Therefore, system with non-integrable velocity constraint such as the WMR can not be stabilized to a point with smooth static-state feedback control law. A more sophisticated motion controller, such as the non-smooth feedback controller, time-varying feedback controller and hybrid controller has to be modelled in order to stabilize the mobile robot to a point. However, all these controllers are still not yet fully generic. In this research, the

mobile robot is stabilized about a reference trajectory instead of a point to ensure that the motion controller is robust to the modelling and also the initial condition position errors [17].

In most cases, the mobile robot is often assumed to operate in an ideal workspace which is free from any disturbances and uncertainties. The existence of these disturbances will usually and significantly degrade the system's stability and performance. In order to solve this problem, a robust control mechanism should be incorporated into the proposed system. Thus, in this study, an AFC scheme is employed in the design of the simulator serving as an explicit disturbance cancellation control technique. Although AFC is renowned for its effectiveness in disturbance compensation, the main drawback of this control scheme is the computation burden of the estimated inertia matrix (denoted as IN) which is required in the AFC feed-forward loop [18] to trigger the control action. Several approaches have been proposed for the appropriate estimation of the IN , such as referring to a look-up table, through the crude approximation method or even intelligent means [19, 20, 21]. In this research, a simple crude approximation method is considered suffice and can be applied for the estimation of IN .

For the development of a virtual robot simulator, Abe *et al.* [22] pointed out that there are several issues which need to be addressed, i.e. equality, variety, synchronism, and interface. Among all these issues, the concept of equality is the most important. The virtual environment should be modelled in such a way that there should not be significant errors between the artificial world and the genuine world. It is possible to express the real world in greater details or complexities, however it takes longer time for the scene rendering to take place. Therefore, for practical considerations, the modelled objects should be simple enough to ensure a fast rendering process, and at the same time the virtual environment does not lose its similarity or equality with the real environment.

For the computer-graphic generation procedure, a computer with massive computing power and memory resources are the most fundamental issues that have to be fulfilled first. As a Personal Computer (PC) is used in this project, it generally has very limited resources for the implementation of virtual reality simulation task.

Therefore, in order to avoid the computer from running out of memory occurrence during the simulation, the memory management aspect should be taken into serious consideration to ensure a wiser use of the computer resources and at the same time improve the virtual environment's rendering capability.

1.5 Research Methodology

Motion planning and motion control of the mobile robot involves various subfields and disciplines. Therefore, the coordination between all these subfields has to be carefully managed and planned in order to ensure a systematic and smooth implementation of the simulation work. With the incorporation of VR into the simulation, this has greatly increased the complexity of the simulation algorithm. Figure 1.1 shows the research procedures applied in this project involving all the three main components, namely the motion planning, motion control and VR.

With reference to Figure 1.1, the research begins with the fundamental study of the nonholonomic WMR, where the kinematic and dynamic models of the WMR are obtained. This is followed by the construction of the workspace for the WMR. A binary map indicating the free and obstacle spaces inside the WMR's workspace is generated. The binary map is then invoked by the motion planner for the generation of search space which is closely related to the configuration space (C -space) of the mobile robot. The formulated search space is named as six elementary jumps graph and from this search space, a collision-free path is found through the implementation of the graph searching algorithm. Several graph searching algorithms or artificial intelligence methods have been introduced in the past, such as breadth first, depth first, Dijkstra's method, and A* algorithm [23]. In this project, A* heuristic algorithm is applied for a near-optimal and short collision-free path searching. The path obtained is generally very rugged and coarse. Therefore, it has to be refined and smoothed first by extracting the checkpoints from the planned path, where the WMR is required to pass through all these checkpoints. Before the modelling of the motion controller, the developed motion planning algorithm is simulated and verified.

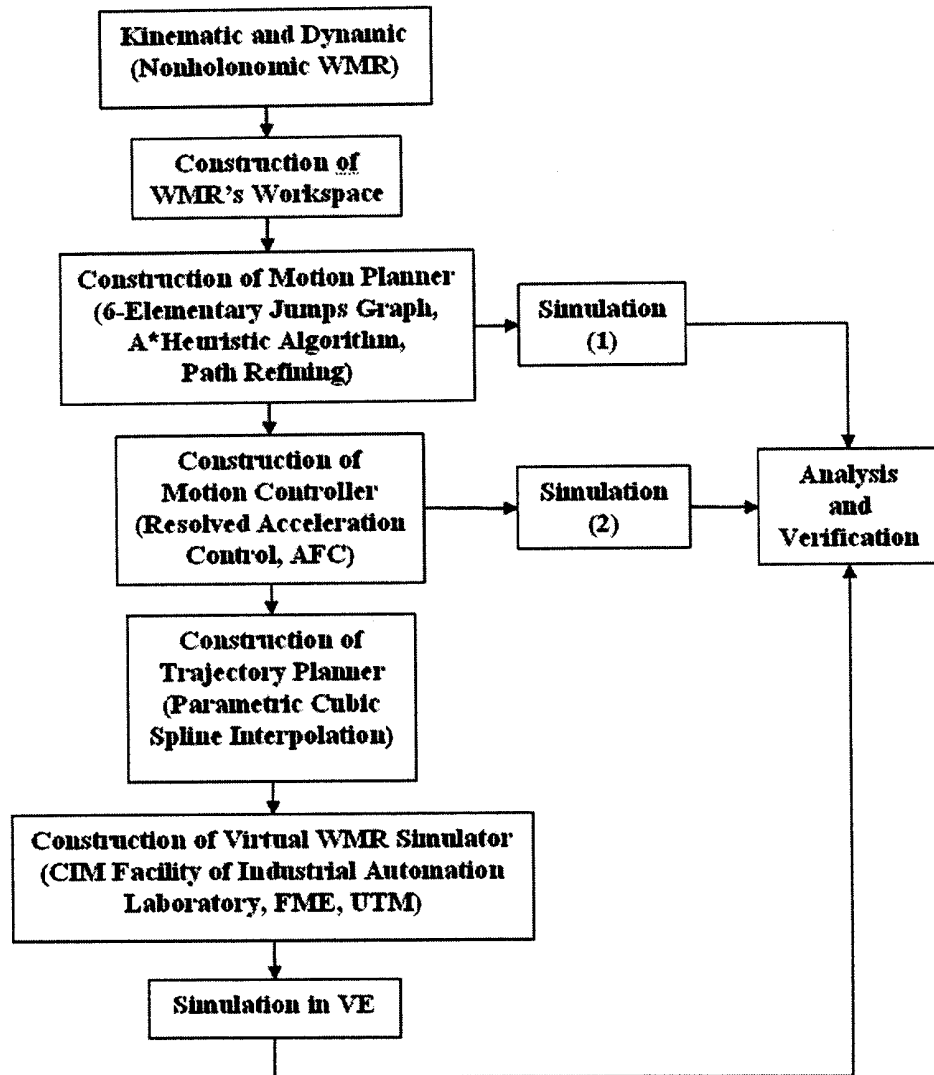


Figure 1.1: Implementation procedures for the research

In order to ensure precise functionality of the nonholonomic WMR, the robot should be well-controlled in such a way that it is able to track the desired trajectory with very small margin of errors. In this research, RAC and AFC have been adopted in the modelling of the motion control algorithm. For the testing of the proposed motion control algorithm, several test and pre-defined trajectories are introduced, namely the linear and circular trajectories. Besides, a number of disturbances or loading conditions in the form of constant friction force, and harmonic disturbance torques are also developed. The modelled trajectories and disturbances are utilised in the motion controller system to determine the robustness and stability of the nonholonomic WMR in trajectory tracking performance.

To link the motion planner with the motion controller, a trajectory planner is developed. This is because the collision-free path generated by the motion planner is in geometric form only. Therefore, the trajectory planner should be modelled in such a way that it is able to transform the geometrical path into the time-indexed trajectory. In this research, the parametric cubic spline interpolation method is used. Through this technique, the trajectory planner is able to generate the spline constants for each arc or segment between the points of the path. With this calculated spline constants, the reference trajectory for the motion controller can be obtained.

After the construction of the motion planner, motion controller and trajectory planner, the virtual WMR simulator is then constructed. Generally, the VE consists of two objects, namely the nonholonomic WMR itself and its working environment which is assumed to be static. The modelling of WMR and all the components within its workspace shall be accomplished using 3D CAD software prior to the created solid models being loaded into the VE. In this research, the CIM facility of Industrial Automation Laboratory, FME, UTM has been chosen as the case study for the application of the proposed virtual WMR simulator. The modelled virtual CIM facility contains various information about the mobile robot's workspace, such as the three dimensional description of the objects' geometry, texture mapping, floor plan layout, viewpoints, and lights.

During the virtual WMR simulation, a topographical map in the form of bitmap is first extracted from the virtual CIM's orthographic view. It is then classified into a binary map that depicts the mobile robot's workspace into either free space or obstacle space. This task is completed by simply stating that all the pixels with different RGB colours that are in contrast with the floor (of CIM) are classified as obstacle space. This binary map is then used by the motion planner in the generation for a geometrical global collision-free path through A* heuristic algorithm. Through the trajectory planner, the planned geometrical path is transformed into time-indexed trajectory which is then fed into the motion controller. From this trajectory, the motion controller generates a series of motor execution commands that drive the mobile robot on track. The controlled motion is then directed to the virtual WMR before the scene of the VE is rendered. Meanwhile, the current position and orientation of the virtual WMR is fed back into the motion

controller for the estimation of the trajectory tracking errors. Towards the end of the research, the simulation results obtained from the virtual WMR simulator are finally analyzed and verified.

1.6 Equipments and Tools Requirements

The implementation of this project requires several equipments for the construction of the virtual autonomous mobile robot simulator. Due to the potential intense graphical processing and computation burden, a PC with excellent computing resources and fast processing capability is required. For the simulation study, Microsoft Visual C++ is chosen as the main programming platform since it is widely supported by other software, such as MATLAB and WorldToolKit. MATLAB library provides the programming-ready mathematical functions which are required during the simulation. Meanwhile, WorldToolKit which is originally based on C functions has greatly simplified the implementation of VR into the simulation.

In this project, the CIM facility in Industrial Automation Laboratory, FME is chosen for the case study to imitate the actual manufacturing environment in which the robot is supposed to navigate. A complete knowledge of the Industrial Automation Laboratory is thus required for the VE modelling. To facilitate VE modelling, 3D CAD software shall be used for the objects modelling tasks in the laboratory. The attributes of all the virtual objects, in terms of the texture mapping, materials designation, and glossiness are assigned before they are loaded into the VE.

1.7 Research Contribution

One of the main contributions of this research work is the construction of a virtual WMR simulator as a platform for the designing, developing and testing of the autonomous mobile robot motion planning and control algorithm. In most cases, real world is far too complex and involves a number of risks for the actual physical

testing to take place. Therefore, a simulation testbed which is designed to mimic the real world and contains all the necessary information is required to serve this purpose, subsequently providing a more reliable and generic experimentation of the newly developed and proposed algorithms. Meanwhile, the interactivity between the autonomous mobile robot with its virtual environment under the operation of this newly designed algorithm can also be analyzed in depth and more importantly the VR testbed is able to provide the realistic scientific visualization of this simulation module. The knowledge learnt through the simulation may potentially be used by the physical vehicle in real world operation.

This research shall also incorporate the AFC scheme into the mobile robot motion controller. In real world application, the mobile robot's workspace is not ideal, in the sense that the mobile robot is always disturbed by 'noises' and 'uncertainties', such as the friction forces, wind and gravity effects. It has been proven that conventional controller alone, such as proportional-derivative (PD) controller is not sufficient to ensure a robust and accurate operation of the mobile robot. In fact, the performance of the controller degrades significantly once the mobile robot is operated at high speed or disturbed by external forces [24, 25, 26]. Therefore, AFC is incorporated into this simulator as the disturbance compensation scheme. In order to replicate the real world condition during simulation, a number of disturbances have been modelled and applied such as the constant drag force and the harmonic forces. With the incorporation of AFC into the mobile robot motion controller, the robot should be able to achieve stability and convergence even in the presence of the disturbances.

1.8 Organization of Thesis

The thesis consists of six chapters. Chapter 1 introduces and mainly discusses the basic information about this research project which is related to the project's research objectives, scope, problem statements, research methodology and tools, and contributions. Chapter 2 reviews a number of research issues pertaining to the subject focus of the study, i.e. the development of virtual WMR simulator. This includes the

aspects of path planning, search space generation, graph searching, trajectory planning, motion control, and virtual reality. A theoretical modelling on the kinematics and dynamics of the proposed WMR is also given in this chapter. Chapter 3 provides a brief description about the global collision-free path planning considering a number of appropriate motion path planning algorithms applied to WMR. A simulation study is carried out and the results relevantly discussed. Chapter 4 describes the modelling of the nonholonomic mobile robot motion controller that incorporates the robust AFC scheme. The simulation aspect is also highlighted and the chapter ends with the discussions of the simulation results. Chapter 5 describes the integration of motion planning and the motion control of the system with a particular emphasis on the application of the VR technique. In this chapter, the modelling of the trajectory planner and the creation of virtual element with respect to the case study are introduced followed by the presentation and discussion of the simulation results. Finally, Chapter 6 summarizes the research work and consequently recommends further works that can be done in future to implement the research study.

discarded. In future researches, this problem should be further investigated so that none of these potential paths are neglected. Generally, this involves the study of local path planning which is responsible for the calculation of the mobile robot's desired orientation.

It is observed that the computation time for the simulation is generally very slow. This is because of the intensive calculation during the solution of the ordinary differential equation by MATLAB. As a result, this has reduced the rate of frames rendering per second and thus reduced the feeling of immersive within the virtual environment. In order to solve this problem, it is recommended that the distributed computing method should be applied where the solution for a tedious task is shared over a network of computers.

For effective cancellation of the undesirable disturbances, the estimated inertia matrix of the dynamic system through the AFC scheme has to be properly approximated. In this research, the inertia matrix is obtained through crude approximation method which is rather crude and unsystematic. Therefore, it is recommended that in future, a more proper way of estimating the inertia matrix should be employed, for example through the use of intelligent mechanism such as fuzzy logic, neural networks, iterative learning algorithm or other AI methods.

REFERENCES

1. Cameron, S. Tutorial: Obstacle Avoidance and Path Planning. *Industrial Robot*. 1994. 21(4): 9-14.
2. Cameron, S. Motion Planning and Collision Avoidance with Complex Geometry. *Proceedings of the 1998 IEEE Annual Conference on Industrial electronics Society*. 31 August-4 September, 1998. Aachen, Germany: IEEE. 1998. 2222-2226.
3. Meystel, A. Planning: A Sketch of the theory. *Proceedings of the 1998 International Symposium on Intelligent Control*. September 14-17, 1998. Gaithersburg, MD USA: IEEE. 56-62.
4. Hourtash, A. and Tarokh, M. Manipulator Path Planning by Decomposition Algorithm and Analysis. *IEEE Transactions on Robotics and Automation*. 2001. 17(6): 842-856.
5. Fraichard, T., Langier, C. and Liévin, G. Robot Motion Planning: The Case of Non Holonomic Mobiles in a Dynamic World. *Proceedings of the 1990 IEEE International Workshop on Intelligent Robots and Systems – Towards a New Frontier of Applications*. July 3-6, 1990. Ibaraki, Japan: IEEE. 1990. 757-764.
6. Sharir, M. Algorithmic Motion Planning in Robotics. *Computer*. 1989. 22(3): 9-19.
7. Saab, Y. and VanPutte, M. Shortest Path Planning on Topographical Maps. *IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans*. 1999. 29(1): 139-150.
8. Lozano-Pérez, T. A Simple Motion-Planning Algorithm for General Robot Manipulators. *IEEE Journal of Robotics and Automation*. 1987. 3(3): 224-238.

9. Pfeiffer, F. and Johanni, R. A Concept for Manipulator Trajectory Planning. *IEEE Journal of Robotics and Automation*. 1987. 3(2): 115-123.
10. Lin, C. S., Chang, P. R. and Luh, J. Y. S. Formulation and Optimization of Cubic Polynomial Joint Trajectories for Industrial Robots. *IEEE Transactions on Automatic Control*. 1983. 28(12): 1066-1074.
11. Yamamoto, Y. and Yun, X. Coordinating Locomotion and Manipulation of a Mobile Manipulator. *IEEE Transactions on Automatic Control*. 1994. 39(6): 1326-1332.
12. Fierro, R. and Lewis, F. L. Control of a Nonholonomic Mobile Robot Using Neural Networks. *IEEE Transaction on Neural Networks*. 1998. 9(4): 589-600.
13. Kozłowski, K. and Majchrzak, J. A Backstepping Approach to Control a Nonholonomic Mobile robot. *Proceedings of the 2002 IEEE International Conference on Robotics and Automation*. May 11-15, 2002. Washington, USA: IEEE. 3972-3977.
14. Laumond, J. P., Jacobs, P. E., Taïx, M. and Murray, R. M. A Motion Planner for Nonholonomic Mobile Robots. *IEEE Transactions on Robotics and Automation*. 1994. 10(5): 577-593.
15. Muñoz, V., Ollero, A., Prado, M. and Simón, A. Mobile Robot Trajectory Planning with Dynamic and kinematic Constraints. *Proceedings of the 1994 International Conference on Robotics and Automation*. May 8-13, 1994. San Diego, CA USA: IEEE. 2802-2807.
16. Brockett, R. W. Asymptotic Stability and Feedback Stabilization. In: Brockett, R. W., Milman, R. S. and Sussman, H. J. *Differential Geometric Control Theory*. United States of America: Birkhäuser Boston, Inc. 181-191; 1983.
17. Walsh, G., Tilbury, D., Sastry, S., Murray, R. and Laumond, J. P. Stabilization of Trajectories for Systems with Nonholonomic Constraints. *IEEE Transactions on Automatic Control*. 1994. 39(1): 216-222.
18. Mailah, M. and Rahim, N. I. Intelligent Active Force Control of a Robot Arm Using Fuzzy Logic. *Proceedings of TENCON 2000*. September 24-27, 2000. Kuala Lumpur, Malaysia: IEEE. 291-296.

19. Mailah, M., Hewit, J. R. and Meeran, S. Active Force Control Applied to a Rigid Robot Arm. *Jurnal Mekanikal*. 1996. 2: 52-68.
20. Hussein, S.B., Jamaluddin, H. and Mailah, M. An Intelligent Method to Estimate the Inertia Matrix of a Robot Arm for Active Force Control Using On-Line Neural Network Training Scheme. *Jurnal Mekanikal*. 1999. 2: 38-53.
21. Tang, H. H. *Hybrid Intelligent Active Force Control*. B.Sc. Thesis. Universiti Teknologi Malaysia; 2002.
22. Abe, R., Isobe, H. and Yamashita, Y. Simulator in a Virtual Space for Autonomous Robot and Vehicle. *Proceedings of the 1999 IEEE International Conference on System, Man, and Cybernetics*. October 12-15, 1999. Tokyo, Japan: IEEE. 1999, 625-630.
23. Lamothe, A. *Tricks of the windows Game Programming Gurus: Fundamentals o 2D and 3D Game Programming*. United States of America: SAMS. 1999.
24. Mailah, M. *Intelligent Active Force Control of a Rigid Robot Arm Using Neural Network and Iterative Learning Algorithms*. Ph.D. Thesis. University of Dundee; 1998.
25. Mailah, M. and Tang, H. H. A Simulation Study on the Hybrid Intelligent Active Force Control of a Two-Link Planar Robot. *Proceedings of the 2003 19th International Conference on CAD/CAM – Robotics and Factories of the Future (CARS & FOF 2003)*. July 22-24, 2003. Kuala Lumpur, Malaysia: SIRIM. 2003. 73-83.
26. Endra Pitowarno, Mailah, M. and Hishamuddin Jamaluddin. Trajectory Error Pattern Refinement of a Robot Control Scheme Using a Knowledge-Based Method. *Proceedings of the 2001 International Conference on Information, Communication and Signal Processing*. Singapore. 2001.
27. Sheu, P. C. Y. and Xue, Q. *Intelligent Robotic Planning Systems*. Singapore: World Scientific publishing Co. Pte. Ltd. 1993.
28. Hwang, Y. K. and Ahuja, N. A Potential Field Approach to Path Planning. *IEEE Transactions on Robotics and Automation*. 1992. 8(1): 23-32.

29. Barraquand, J., Langlois, B. and Latombe, J. Numerical Potential Field Techniques for Robot Path Planning. *IEEE Transactions on Systems, Man and Cybernetics*. 1992. 22(2): 224-241.
30. Takahashi, O. and Schilling, R. J. Motion Planning in a Plane Using Generalized Voronoi Diagrams. *IEEE Transactions on robotics and Automation*. 1989. 5(2): 143-150.
31. Neus, M. and Maouche, S. Motion Planning Using the Modified Visibility Graph. *Proceedings of 1999 IEEE International Conference on Systems, Man, and Cybernetics*. October 12-15. Tokyo, Japan: IEEE. 651-655.
32. Podszędkowski, L., Nowakowski, J., Idzikowski, M. and Vizvary, I. A New Solution for Path Planning in Partially Known or Unknown Environment for Nonholonomic Mobile Robots. *Robotics and Autonomous Systems*. 2001. 34: 145-152.
33. Graham, R. *C Unleashed*. United States of America: SAMS Publishing. 2000.
34. Choset, H. and Burdick, J. Sensor Based Planning, Part I: The Generalized Voronoi Graph. *Proceedings of the 1995 IEEE International Conference on Robotics and Automation*. May 21-27, 1995. Nagoya, Japan: IEEE. 1649-1655.
35. Faibish, S. and Abramovitz, M. Perception and Navigation of Mobile Robots. *Proceedings of the 1992 IEEE International Symposium on Intelligent Control*. August 11-13, 1992. Glasgow, UK. IEEE. 335-340.
36. Kanayama, Y., Kimura, Y., Miyazaki, F. and Noguchi, T. A Stable Tracking Control Method for an Autonomous Mobile Robot. *Proceedings of the 1990 IEEE International Conference on Robotics and Automation*. May 13-18, 1990. Cincinnati, OH USA: IEEE. 1990. 384-389.
37. Yamamoto, Y. and Yun, X. Coordinating Locomotion and Manipulation of A Mobile Manipulator. In: Zheng, Y. F. *Recent Trends in Mobile Robots*. Singapore: World Scientific publishing Co. Pte. Ltd. 157-181; 1993.
38. Fierro, R. and Lewis, F. L. Control of a Nonholonomic Mobile Robot: Backstepping Kinematics into Dynamics. *Proceedings of the 1995 IEEE Conference on Decision and Control*. December 13-15, 1995. New Orleans, LA USA: IEEE. 1995. 3805-3810.

39. Samson, C. Control of Chained Systems Application to Path Following and Time-Varying Point Stabilization of Mobile Robots. *IEEE Transactions on Automatic Control*. 1995. 40(1): 64-77.
40. Dixon, W. E., Dawson, D. M., Zheang, F. and Erkan, Z. Global Exponential Tracking Control of a Mobile Robot System via a PE Condition. *IEEE Transactions on Systems, Man, and Cybernetics Part B: Cybernetics*. 2000. 31(1): 129-142.
41. Pourboghraat, F. and Karlsson, M. P. Adaptive Control of Dynamic Mobile Robots with Nonholonomic Constraints. *Pergamon Computers and Electrical Engineering*. 2000. 28: 241-253.
42. Miyata, J., Murakami, T. and Ohnishi, K. An Approach to Tracking Motion of Mobile Robot for Moving Object. *Proceedings of the 26th Annual IEEE Conference on Industrial Electronics Society*. October 22-28, 2000. Nagoya, Japan: IEEE. 2000. 2249-2254.
43. Hewit, J. R. and Burdess, J. R. Fast Dynamic Decoupled Control for Robotics Using Active Force Control. *Mechanism and Machine Theory*. 1981. 16(5): 535-542.
44. Tang, H. H., Mailah, M. and Kasim, M. Stabilization of Nonholonomic Wheeled Mobile Robot through Intelligent Active Force Control. *Proceedings of Advanced Technology Congress – Intelligent Systems and Robotics (CISAR 2003)*. May 20-21, 2003. Kuala Lumpur, Malaysia: Institute of Advanced Technology, UPM. 2003.
45. Low, K. H., Leow, W. K. and Ang, M. H. A Hybrid Mobile Robot Architecture with Integrated Planning and Control. *Proceedings of the First International Joint Conference on Autonomous Agents and Multiagent System: Part 1*. July 15-19, 2002. Bologna, Italy: ACM. 219-226.
46. Rabelo, L. C., Avula, X. J. R., Mah, H., Malkani, A. and Tsai, J. S. Planning and Control of A Robotic Manipulator Using Neural Networks. *Proceedings of the First IEEE Conference on Control Applications*. September 13-16, 1992. Dayton, OH USA: IEEE. 306-311.
47. Aydin, S. and Temeltas, H. A Novel Approach to Smooth Trajectory Planning of a Mobile Robot. *Proceedings of the 2002 7th International Workshop on Advanced Motion Control*. July 3-5, 2002. Maribor, Slovenia: IEEE. 472-477.

48. Zulli, R., Fierro, R. Conte, G. and Lewis, F. L. Motion Planning and Control for Non-Holonomic Mobile Robots. *Proceedings of 1995 IEEE International Symposium on Intelligent Control*. August 27-29, 1995. Monterey, CA USA: IEEE. 551-557.
49. Hu, H., Gu, D. and Brady, M. Navigation and Guidance of an Intelligent Mobile Robot. *Proceedings of Second EUROMICRO Workshop on Advanced Mobile Robots*. October 22-24, 1997. Brescia, Italy: IEEE. 104-111.
50. Piazzzi, A. and Guarino, L. B. C. Quintic G^2 -Splines for Trajectory Planning of Autonomous Vehicles. *Proceedings of the 2000 IEEE Symposium on Intelligent Vehicles*. October 3-5, 2000. Dearborn, MI USA: IEEE. 198-203.
51. Schmitt, G. N. Virtual Reality in Architecture. In: Thalmann, N.M. and Thalmann, D. *Virtual Worlds and Multimedia*. England: John Wiley & Sons Ltd. 85-97; 1993.
52. Stone, R. J. The Reality of Virtual Reality. *World Class Design to Manufacture*. 1995. 2(4): 11-17.
53. Zheng, J. M., Chan, K.W. and Gibson, I. Virtual Reality. *IEEE Potentials*. 1998. 17(2): 20-33.
54. Kircanski, N. M. Mobile Robotic System. In: Nwokah, O. D. I. and Hurmuzlu, Y. *The Mechanical Systems Design Handbook: Modelling, Measurement and Control*. London: CRC Press LLC. 26.1-26.20; 2002.
55. Bräunl, T. *Embedded Robotics: Mobile Robot Design and Applications with Embedded Systems*. Germany: Springer-Verlag Berlin Heidelberg. 2003.
56. Kolmanovsky, I. and McClamroch, N. H. Developments in Nonholonomic Control Problems. *IEEE Control Systems Magazine*. 1995. 15(6): 20-36.
57. Murray, R. M., Li, Z. and Sastry, S. S. *A Mathematical Introduction to Robotic Manipulation*. United States of America: CRC Press, Inc. 1994.
58. Zomaya, A. Y. *Modelling and Simulation of Robot Manipulators*. Singapore: World Scientific Publishing co. Pte. Ltd. 1992.
59. Barraquand, J. and Latombe, J-C. On Nonholonomic Mobile Robots and Optimal Maneuvering. *Proceedings of the 1989 IEEE International Symposium on Intelligent Control*. September 25-26, 1989. Albany, NY USA: IEEE. 1989. 340-347.
60. Schildt, H. *Artificial Intelligence Using C*. United States of America: McGraw-Hill, Inc. 1987.

61. Barr, A. and Feigenbaum, E. A. *The Handbook of Artificial Intelligence Volume I*. United States of America: William Kaufmann, Inc. 1981.
62. Russell, S. J. and Norvig, P. *Artificial Intelligence: A Modern Approach*. New Jersey: Prentice-Hall, Inc. 1995.
63. Jarvis, R. Distance Transform Based Path Planning for Robot Navigation. In: Zheng, Y. F. *Recent Trends in Mobile Robots*. Singapore: World Scientific Publishing Co. Pte. Ltd. 3-31; 1993.
64. Hague, T. and Cameron, S. Motion Planning for the Oxford AGV. *Proceedings of the 1990 IEEE International Workshop on Intelligent Motion Control*. August 20-22, 1990. Istanbul: IEEE. 1990. 277-282.
65. Tang, H. H., Mailah, M. and Kasim, M. Collision-Free Global Path Planning for a Holonomic Mobile Robot in a Known Stationary Virtual Environment. *Proceedings of Malaysian Science and Technology (MSTC) – Information and Communication Technology*. September 23-25, 2003. Kuala Lumpur, Malaysia: MSTC. 2003. 328-335.
66. Laumond, J-P., Simeon, T., chatila, R. and Giralt, G. Trajectory Planning and Motion Control of Mobile Robots. In: Schweitzer, G. and Mansour, M. *Dynamics of Controlled Mechanical Systems IUTAM/IFAC Symposium Zurich/Switzerland 1988*. Berlin: Springer-Verlag Berlin Heidelberg. 351-366; 1989.
67. Podszędkowski, L., Path Planner for Nonholonomic Mobile Robot with Fast Replanning Procedure. *Proceedings of the 1998 IEEE International Conference on Robotics and Automation*. May 16-20, 1998. Leuven, Belgium: IEEE. 3588-3593.
68. Pearl, J. *Heuristics: Intelligent Search Strategies for Computer Problem Solving*. California: Addison-Wesley Publishing Company, Inc. 1984.
69. Luger, G. F. *Artificial Intelligence: Structures and Strategies for Complex Problem Solving*. 4th edition. United States of America: Pearson Education Limited. 2002.
70. Burdess, J. S. and Hewit, J. R. An Active Method for the Control of Mechanical Systems in the Presence of Unmeasurable Forcing. *Transactions on Mechanism and Machine Theory*. 1986. 21(3): 393-400.

71. Luh, J. Y. S., Walker, M. W. and Paul, R. P. C. Resolved-Acceleration Control of Mechanical Manipulators. *IEEE Transactions on Automatic Control*. 1980. 25(3): 468-474.
72. Foley, J. D., Dam, A. V., Feiner, S. K. and Hughes, J. F. *Computer Graphics: Principles and Practice – Second Edition in C*. 2nd edition. United States of America: Addison-Wesley Publishing Company, Inc. 1996.
73. Cook, C. C. and Ho, C. Y. The Application of Spline Functions to Trajectory Generation for Computer-Controlled Manipulators. In: Aleksander, I. *Computing Techniques for Robots*. London: Kogan Page Ltd. 102-110; 1985.
74. Engineering Animation, Inc. *WorldToolKit® Reference Manual Release 9*. Mill Valley (USA): Product reference manual. 1999.