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Improving The Heights Derived From Geoid Models Using A Regression Model

Khairul Anuar Abdullah, *PhD*
Jabatan Geodesi and Sains Angkasa
Fakulti Ukur dan Harta Tanah
Universiti Teknologi Malaysia

Abstract

The technique of GPS surveying offers an alternative to spirit leveling technique in height determination. The GPS technique provide position in three dimensional coordinates in terms of Cartesian coordinates X, Y and Z or geodetic latitude (4>), longitude (A) and ellipsoidal height (h). Traditionally, orthometric height which refers to the geoid are normally used in everyday application rather then the GPS derived ellipsoidal height. Information on the geoid is thus required in order to convert the GPS ellipsoidal height into orthometric height. In this study, a four parameter fitting procedure was employed to improve the orthometric height derived from two different solutions, i.e., using a local gravimetric geoid model and a global geopotential model solution. This approach yields an accuracy of about 10 cm in the height estimates.

1.0 INTRODUCTION

The advent of high precision relative positioning by use of the Global Positioning System (GPS) has opened up an alternative to the classical method of height determination. Using GPS, it is now possible to determine routinely, very accurate three-dimensional Cartesian coordinate differences (or *baseline* vectors) between observation points. This three-dimensional capability therefore provides the possibility of using GPS for height determination as well as for (horizontal) positioning.

Conventionally, topographic maps, engineering design and construction project plans, usually depict relief by means of orthometric height which differs from the GPS derived ellipsoidal height. Thus, the application of GPS will be further extended if accurate transformations between GPS ellipsoidal height arid the orthometric height can be realized. This can be accomplished on the condition that we know the geoid height, which relates the orthometric height to the GPS ellipsoidal height and this requires the knowledge of the geoid. For small area of less than 10 km, which is not too hilly and the gravity field is smooth, a plane surface can be used to approximate the geoid using control points with known orthometric heights (Hajela, 1990; and Khairul, 1993).

However, for larger area, the geoid information must be known and this can be obtained by the use of two kind of geoid solutions, namely, using a local gravimetric geoid model and a global geoid model. Khairul (1993) have shown that the accuracy of the height determined by using a global geoid model is about 50 -60 cm while employing a local gravimetric geoid model can give an accuracy of about 30 cm. In order to improved the height estimates, a regression model employing four parameters were tested using a very large network covering Central England. This paper attempts to report on the results obtained from the study.

2. ORTHOMETRIC HEIGHT ESTIMATION

2.1 GPS Heighting

The position of points derived from GPS measurements are usually computed in a three-dimensional Cartesian coordinate system. The basic results of the precise differential GPS survey of a baseline are the Cartesian coordinate differences X, Y and Z. Baselines connecting the observed GPS points are then put through a network adjustment such as L3D-HEIGHT (Khairul, 1994) or GEOLAB (Bitwise,1991). The resulting X, Y, and Z coordinates of the GPS points are then transformed, using a reference ellipsoid, into geodetic coordinates in terms of latitude ( ), longitude ( ) and ellipsoidal height (h). The orthometric height (H) is related to the ellipsoidal height (h) by the following relation :

$$H = h - N \quad \text{or,} \qquad [1]$$

$$H = h_{GPS} - N_{MODEL} \qquad [2]$$

where,

h_{GPS} is the GPS derived ellipsoidal height, and,
N_{MODEL} is the geoidal height derived from a geoid model.

or by the relative approach, the orthometric height difference between two GPS points may be deduced from:

$$H = \Delta h_{GPS} - \Delta N_{MODEL} \qquad [3]$$

From the expressions above, the errors in H in eqn. [2] will depend upon the accuracy of the parameters used in its evaluation. It is generally known that, the differences in h between two points measured simultaneously by GPS are much more precise than h at either of the points. This is because of the presence of systematic errors which, being significantly the same at the two points, cancels in the difference. Similarly, N_{MODEL}, is much more precise than the geoid height at either points. This means that for determination requiring highest precision, the approach implied in eqn. [3] is preferred to that in eqn. [2].

2.2 Derivation of Geoidal Height

There are two ways of deriving the geoidal height. The first approach employs a global geoid solution. Global geoid solutions are obtained from global geopotential models which are given as a set of coefficients consisting of a series of spherical harmonic functions. The coefficients of the various terms in the series are determined using a combination of satellite orbit analyses (for the long wavelength geoid features), terrestrial gravity (medium to short wavelength features) and geoid heights measured by satellite altimetry over the ocean (medium to short wavelength features). The geoidal height from a global geoid model N_{GM} is computed from a set of normalised geopotential coefficients using the following equation:

$$N_{GM} = \frac{GM}{r\gamma} \sum_{n_{MAX}}^{n=2} \left(\frac{a}{r}\right)^n \sum_{n}^{m=0} \left[\bar{C}^{*}_{nm} cosm\lambda + \bar{S}_{nm} sinm\lambda\right] \bar{P}_{nm}(\sin\phi) \qquad [4]$$

where,

n_{MAX} is the maximum degree at which the coefficients are known.

$\bar{C}^{*}_{nm}$ are the $\bar{C}_{nm}$ less the xonal coefficients of the normal potential of the selected reference ellipsoid.

G is the gravitational constant.
M is the mass of the earth, including the atmosphere.

a is the earth's equatorial radius.
r is the distance from the earth's centre of mass.
ϕ, λ are the geocentric latitude and longitude.
$\boxed{\overline{P}_{nm}(\sin f)}$ is the normalised associated Lagendre function.
g is the normal gravity.
n, m are the degree and order respectively.

Generally, the more coefficients there are in a model, the more detailed the model usually is since it contains shorter wavelength information of the earth's gravity field. This means that in general, the best solution to use is one that has been determined up to the maximum degree and order of 360, which, theoretically at least, can model features in the geoid with half wavelength of 0.5 degrees or 55 km. In this study, the global geopotential model adopted is the OSU91A which was developed using 30' by 30' mean gravity anomalies derived from terrestrial and altimetric data (Rapp et al., 1991). These data are then combined with GEM-T2 (Marsh et al., 1989) to produce the model complete to degree and order of 360. This model was chosen on the basis that it is the most up-to-date global geopotential model made available.

The second approach involved the use of a local gravimetric geoid model in computing the geoidal height Ngrav. The geoidal heights are basically computed using the Stokes integral, integrating gravity anomalies in principle over the surface of the earth. In practice, it is modified to integrate gravity anomalies over a small spherical cap $\sigma_\cap$:

$$N_{grav} = N_{GM} + \frac{R}{4\pi\lambda}\iint_{\sigma_\cap}(\Delta g - \Delta g_{GM})s(\psi)d\sigma \qquad [5]$$

where;

N_{grav} is the total geoid height,
N_{GM} is computed using eqn. [4],
R is the mean earth radius,
Δg are the gravimetric observations given as free-air gravity anomalies,
Δg_{GM} are the free-air gravity anomalies computed from the global geopotential model using an equation similar to eqn.[4], and,
$s(\psi)$ is Stokes function.

The selection of the 'optimum' cap size is very important. It is preferable to keep the cap size small, since the larger the cap, the longer the computation time. However, as the global geopotential model adopted may have quite large errors in the coefficients of higher degrees, larger cap sizes are required in many cases. For this study, the gravimetric geoids of the British Isles produced by the University of Oxford (hereon refers as OXFORD model) was used. The Oxford geoid was computed using the combination of a high degree global potential model and numerical integration of a modified Stokes' integral similar to that described in eqn. [5] (Featherstone, 1992).The global geopotential model used is the OSU91A and the geoid was computed at 2' latitude and 4' longitude grid spacing corresponding to approximately 4 km by 4 km squares over the British Isles. The gravimetric geoid height is computed at the centre of each grid elements to yield 135,000 points comprising geodetic longitude, geodetic latitude and geoidal height, all referred to GRS80 (ibid.). To derive the geoid height of a point, the method of Bi-linear interpolation described in DMA (1987) can be used. The OXFORD geoid model was selected for this study on the basis that it was made available for this study.

3.0 IMPROVING THE GEOID HEIGHT ESTIMATES

The geoidal height computed using the two solutions described above may contain biases due to several factors, such as the problem arising from the differences in the GPS and geoid model datums. This is especially apparent in the case of using a local geoid computed by combining a global solution with terrestrial gravity data. Here, the biases may consist of long-wavelength errors contributed by geopotential model errors, bad gravity coverage and a bad elevation datum for the gravity observations, since barometric levelling is commonly used. These biases can be reduced or absorbed by implementing some kind of transformation procedure such as that used by Forsberg et al. (1990). The geoid change ($N' - N_{MODEL}$) due to these biases can be expressed in geodetic coordinates in the form of a regression formula (ibid.):

$$N' - N_{MODEL} = a_1 + a_2 \cos\phi\cos\lambda + a_3 \cos\phi\sin\lambda + a_4 \sin\Phi \quad [5]$$

By using at least four known geoidal heights, N', in the above equation, the four coefficients in the regression model can be computed. These coefficients are then used in computing the 'correction' that will be applied to N_{MODEL} in deriving the geoid height and the orthometric heights at the other points.

4.0 THE ROSES/BORDER/LAKES NETWORK EXPERIMENTS

The network used in this study is given the name of Roses/Border/Lakes (RBL) Network because it lies in two blocks of the new National GPS Network established by the Ordnance Survey (see Figure [1]). The RBL network shown in Figure [2] consists of 129 GPS points including thirteen Fundamental Benchmarks (FBM) located in the area. These FBMs are important in this study since they are used as Height Control Point (HCP) and also used in computing the estimated accuracy of each of the methods. The GPS points are placed at intervals of 20-25 km. around urban centres with this spacing being relaxed in rural areas (Christie, 1991). Due to the limited number of FBMs and also their irregular distribution, a more ideal size of the network could not be constructed.

4.1 Observations and Processing

The GPS observations and post-processing work was undertaken by the Ordnance Survey. A total of eight receivers, four dual and four single frequency were used in the observation campaign with the dual frequency receivers deployed at every FBMs (ibid.). A total of 367 baselines vectors provided by the Ordnance Survey were processed using the program L3D-HEIGHT with three control stations (see Figure [2]) being held fixed.

4.2 Orthometric Height Experiments

In the first test, geoidal heights for all the GPS points were computed using the global geoid model, the OSU91A. Using eqn.[3], the orthometric height differences between pair of points were computed and the orthometric height of all the GPS points were reduced. The r.m.s of the estimated orthometric height was found to be about 60 cm which have been anticipated (see Table[1]). The test was further extended by employing a four parameters fitting procedure previously discussed, to see if any improvement can be gained. In this case, the accuracy of the estimation seems to improved by about fifty percent to the 30 cm level as shown in Table[2]. The level of accuracy obtained is still above the requirement of many applications. The same test as above was repeated but this time, the local gravimetric geoid model, OXFORD was used. From the result shown in Table [3], the estimated accuracy which we can expect if the geoidal height computed directly is used, is about 30 cm. This level of accuracy is similar to that obtained using a four parameter fitting procedure on OSU91A. However, using such fitting procedure on OXFORD gives a very encouraging result as shown in Table[4]. This time the level of accuracy obtained was below the 10 cm level which is within the accuracy requirement of a number of applications.

Table 1

Differences between orthometric heights obtained from GPS and OSU91A, and orthometric heights from Levelling at the Roses/Border/Lakes Network (in metres).

Geoid Model	OSU91A				
Station Name	GPS ellipsoidal height, h (GRS80)	Model based geoid height, N (GRS80)	GPS based orthometric height, h-N	Levelled orthometric height, H	Orthometric height differences
93	363.2130	52.3654	310.8476	310.3450	0.5026
111	183.1866	53.4999	129.6867	128.9020	0.7847
119	243.6673	54.0505	189.6168	188.4962	1.1206
84	200.3262	52.3204	148.0058	147.5678	0.4380
106	107.1321	52.8584	54.2737	53.9804	0.2933
69	362.1012	51.1562	310.9450	310.2261	0.7189
49	410.4363	50.1854	360.2509	359.5567	0.6942
51L	308.9492	50.9095	258.0397	257.7624	0.2773
52L	273.2387	50.4889	222.7498	222.5100	0.2398
118L	174.2775	49.6421	124.6354	124.5062	0.1292
n = 10, No. HCP = 0, Average Difference = 51.9 cm, RMS = 59.6 cm					

Table 2

Differences between orthometric heights obtained from GPS and OSU91A plus FIT, and orthometric heights from Levelling at the Roses/Border/Lakes Network(in metres) using 4 HCP.

Geoid Model	OSU91A with 4 PARAMETER FIT				
Station Name	GPS ellipsoidal height, h (GRS80)	Model based geoid height, N (GRS80)	GPS based orthometric height, h-N	Levelled orthometric height, H	Orthometric height differences
93	363.2130	52.4936	310.7194	310.3450	0.3744
111	183.1866	54.5425	128.6441	128.9020	-0.2599
119	243.6673	55.0285	188.6388	188.4962	0.1426
84	200.3262	52.4779	147.8483	147.5678	0.2805
106	107.1321	52.9622	54.1699	53.9804	0.1895
69	362.1012	51.3793	310.7219	310.2261	0.4958
49	410.4363	50.4721	359.9642	359.5567	0.4075
51L	308.9492	51.1628	257.7864	257.7624	0.0240
52L	273.2387	50.7723	222.4664	222.5100	-0.0436
118L	174.2775	49.9709	124.3066	124.5062	-0.1996
n = 10 No. HCP = 4 Average Difference = 14.1 cm RMS = 28.5 cm					

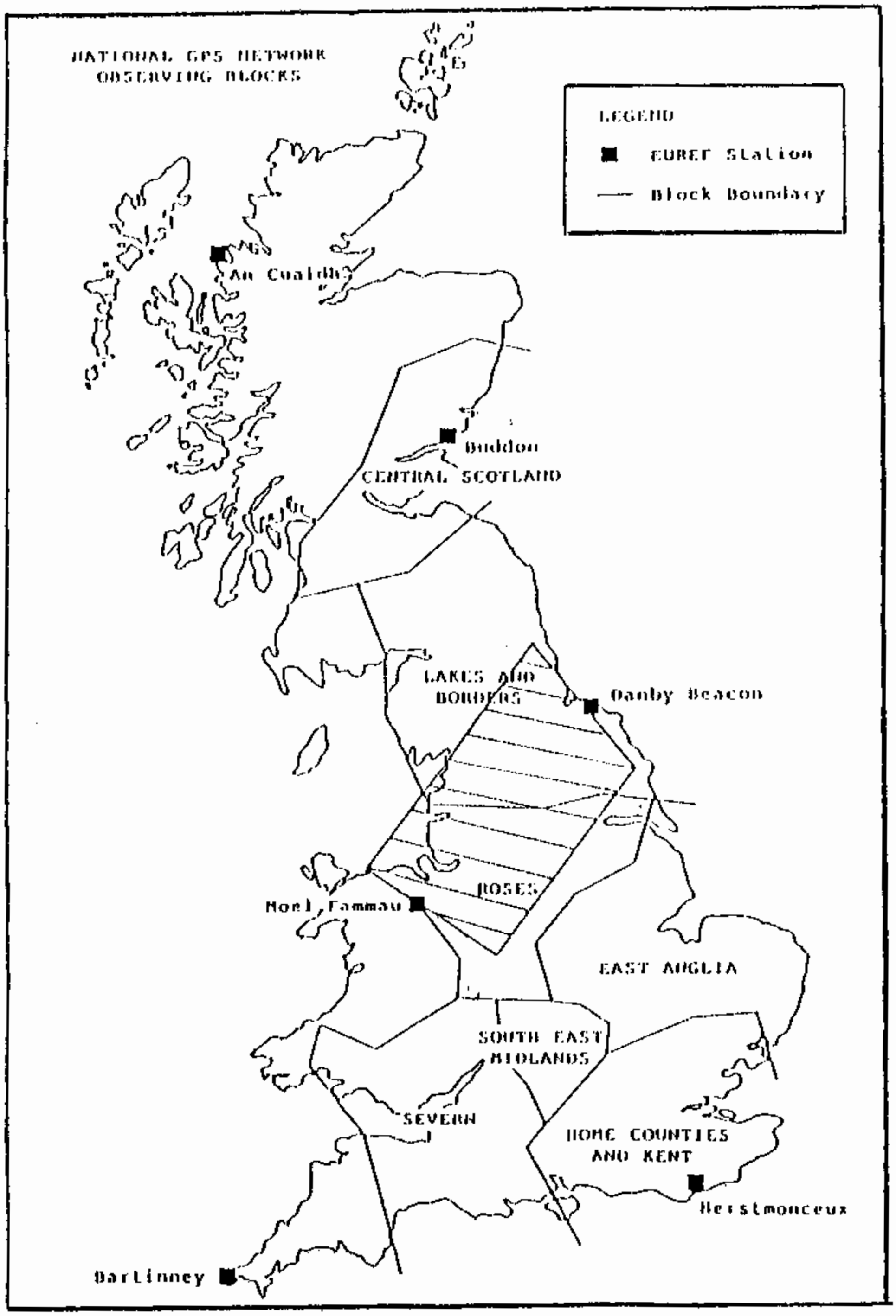

Figure 1 Location of the Roses/Border/Lakes Network (from Christie,1991).

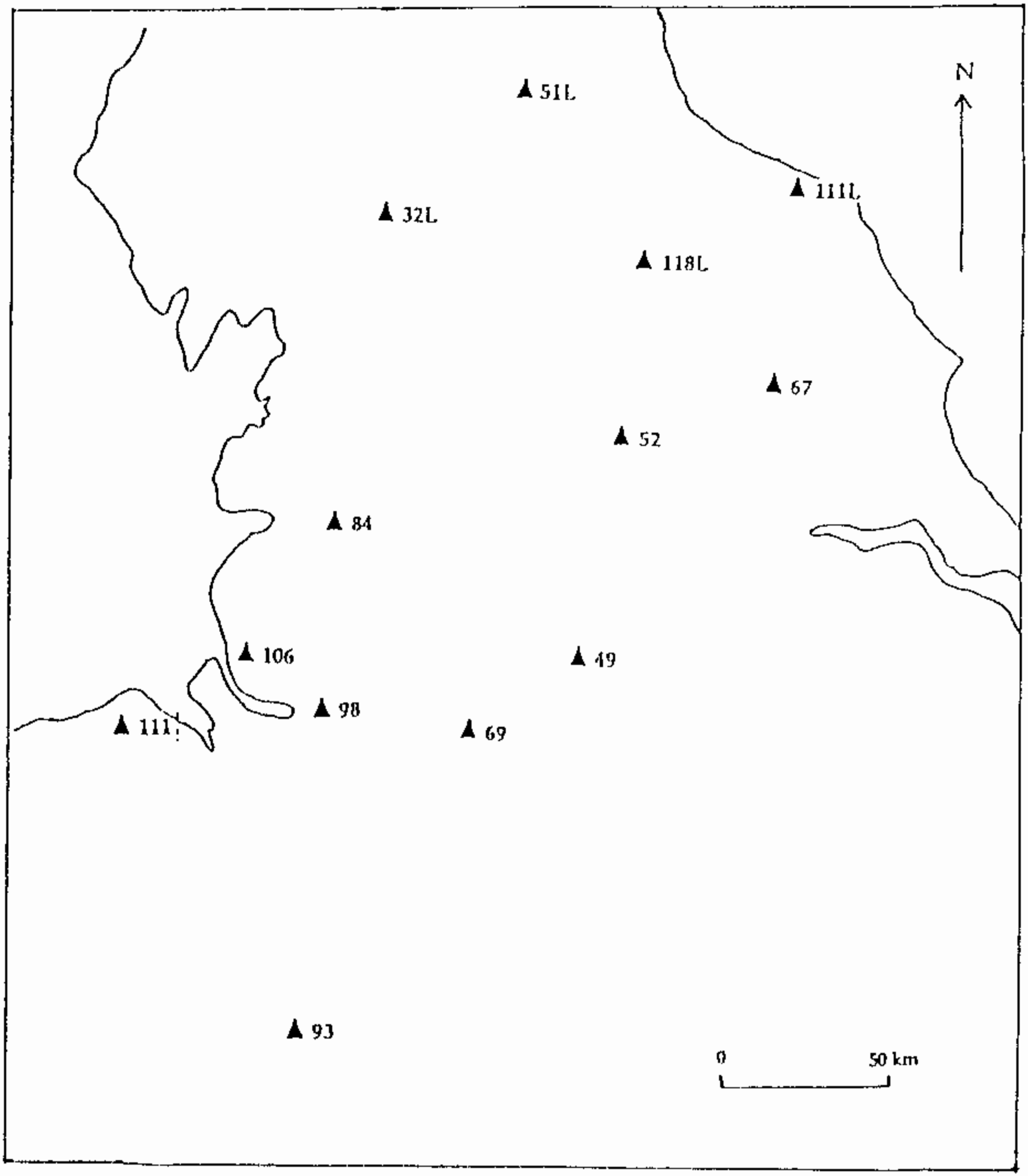

Figure 2 Location of FBM (Fundamental Bench Mark) in the Roses/Border/Lakes Network.

Table 3

Differences between orthometric heights obtained from GPS and OXFORD, and orthometric heights from Levelling at the Roses/Border/Lakes Network(in metres).

Geoid Model	OXFORD				
Station Name	GPS ellipsoidal height, h (GRS80)	Model based geoid height, N (GRS80)	GPS based orthometric height, h-N	Levelled orthometric height, H	Orthometric height differences
93	363.2130	52.6769	310.5361	310.3450	0.1911
111	183.1866	54.0287	129.1579	128.9020	0.2559
119	243.6673	54.8858	188.7815	188.4962	0.2853
84	200.3262	52.5143	147.8119	147.5678	0.2441
106	107.1321	52.8970	54.2351	53.9804	0.2547
69	362.1012	51.5989	310.2023	310.2261	0.2762
49	410.4363	50.6180	359.8183	359.5567	0.2616
51L	308.9492	50.9193	258.0299	257.7624	0.2675
52L	273.2387	50.4290	222.8097	222.5100	0.2997
118L	174.2775	49.5647	124.7128	124.5062	0.2066

n = 10 No. HCP = 0
Average Difference = 25.4 cm
RMS = 25.6 cm

Table 4

Differences between orthometric heights obtained from GPS and OXFORD plus FIT, and orthometric heights from Levelling at the Roses/Border/Lakes Network(in metres) using 4 HCP.

Geoid Model	OXFORD with 4 PARAMETER FIT				
Station Name	GPS ellipsoidal height, h (GRS80)	Model based geoid height, N (GRS80)	GPS based orthometric height, h-N	Levelled orthometric height, H	Orthometric height differences
93	363.2130	52.8578	310.3552	310.3450	0.0102
111	183.1866	54.3628	128.8238	128.9020	-0.0782
119	243.6673	55.3245	188.3429	188.4962	-0.1533
84	200.3262	52.6813	147.6449	147.5678	0.0771
106	107.1321	53.1904	53.9416	53.9804	-0.0388
69	362.1012	51.8583	310.2429	310.2261	0.0168
49	410.4363	50.8490	359.5873	359.5567	0.0306
51L	308.9492	51.2666	257.6763	257.7624	-0.0861
52L	273.2387	50.7454	222.4932	222.5100	-0.0168
118L	174.2775	49.7696	124.5080	124.5062	0.0018

n = 10 No. HCP = 4
Average Difference = -2.4 cm
RMS = 6.8 cm

5.0 CONCLUSIONS

It has been proven that GPS observable in combination with terrestrial geodetic data such as geodetic coordinates or heights, can be used to estimate orthometric heights. Thus, for some applications, this has the potential for replacing conventional spirit levelling for height determination.

The experiments also suggest that using a global geopotential model alone over large area (similar to that described in (6)) without using any point with known orthometric height as control, accuracies of about 50 cm to 1 metre can be expected in the orthometric height estimation. However, an improvement of about 50% on the accuracy can be gained if a four parameter fit procedure is used.

The experiments also indicate that the accuracy obtained from a regional gravimetric geoid model is normally about 50% better than those obtained from a global potential model. This improvements is contributed by the presence of some of the short wavelength features of the local geoid in the regional gravimetric model. The results from the experiments also suggest that the accuracy of the orthometric height obtained from using a regional gravimetric geoid and a four parameter fitting procedure can be expected to be better than 10 cm. Thus, whenever possible, a regional gravimetric geoid model should be used instead of a global geopotential model.

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About the authour

Dr. Khairul Anuar Abdullah is currently a lecturer in the Department of Geodesy and Space Science, Faculty of Surveying and Real Estate, UTM. He has been with the department since 1980. He obtainedhis M.Sc from the University New Brunswick, Canada in 1987, and Ph.D from the University of Newcastle Upon Tyne, U.K. in 1993. His research interest is in the areas of GPS technology application and deformation monitoring survey.