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Calibration Tests For Underwater Acoustic Transponders Using Kalman Filter

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Abstract

Two tests were carried out for the calibration of underwater acoustic transponders using Kalman filter. Both tests used the Long Baseline Acoustic Positioning method. The first test is known as the relative calibration test and uses observables only from underwater acoustic transponders. This test is for the adjustment of the shape and scale of the transponder array. It requires at least three coordinates to be fixed. The second test, known as the absolute calibration test, involves the observables from four systems (i.e. DGPS, underwater acoustic system, range-range system and hyperbolic system) integrated together to give geodetic position and orientation to the transponder array. The results of both tests are presented.

1.0 INTRODUCTION

Long Baseline Acoustic Positioning System provides local control and high position repeatability independent of water depth for the surface or sub-surface vessel, particularly when the site is out of the range of a suitable surface positioning systems. In this technique, the position of the survey vessel is related to the transponder array on the seabed. Hence, the only practical method of accurately determining the surface and sub-surface positions beyond the line of sight is with referenced to an array of bottom moored acoustic transponder. During operation each transponder in interrogated from the navigating vehicle. The resulting range measurement, to each of the transponders are processed to determine the position of the vehicle relative to the transponder array. However, navigation of surface and sub-surface vessels in a transponder array depends on a prior precise determination of the transponder coordinates (i.e. calibration).

There are several methods commonly employed. Early methods were based on clover-leaf and baseline crossings techniques. These methods require large amounts of ship time and critical ship path. Later methods have been developed based on range data from a number of random ship positions which acoustic markers are then positions from an interative least squares fitting procedure. The method employed in this paper is based on Kalman filter.

2.0 KALMAN FILTERING

The theory of mathematical filters has mainly been developed by statisticians and electrical engineers. The description of the filter type most commonly used presently for scientific and engineering applications is by Kalman (1960). The filter which is now call the Kalman filter can be described as a computation techniques that enables real-time estimation of quantities of interest such as satellite's or ship's position. The filter may be applied to any situation where the desired parameters vary with time. It is used to estimate the various states of a random process from a set of discrete measurements having a known linear connection to these states.

Kalman filter provides a set of algorithms for the estimation of the state vector (i.e. unknown parameters) at any point in time. Its filtering process follows a recursive sequence of mathematical models which remember past data, receive present data and calculate the best estimates of present and probable future positions, based upon the combination of past and present information. It produces optimal estimates of the state vector with well defined statistical properties. The estimates of the state vector are unbiased and have minimum variance, so long as observations and model are normally distributed.

2.1 State Vector

The state vector is a vector of desired parameters. It must include not only those parameters which we wish to estimate but also other parameters necessary to model the dynamic behaviour of the vessel. The elements of this vector might be position, velocity and acceleration.

2.2 Mathematical Models

Filtering makes use of information available from two sources i.e observations and some prediction on how the vessel is expected to move.

This leads to the use of two functional models :

i) Measurement model

ii) Dynamic model

• Measurement Model

Let us assume that at time *i* we make some measurements l_i which are related to the state vector by the functional relationship:

$$F_i(\bar{x}_i) = \bar{l}_i \qquad \text{. . . (1)}$$

If the relationships between measurements and state vectors are non-linear, then the relationships have to be linearised for use in the Kalman filter. The linearised functional model for observations at time *i* is given by:

$$A_i x_i = b_i + v_i \qquad \text{. . . (2)}$$

where A_i ... is the Jacobian matrix or simply $\delta F_i / \delta x$

b_i ... is the 'observed minus computed' quantities
v_i ... is the vector of residuals

• Dynamic Model
This model is based on some knowledge on how the state vector is expected to vary with time. In its most general form, the functional relationship of the dynamic model may be represented as:

$$F_{i-1,i}(\bar{x}_{i-1}, \bar{x}_i, t_{i-1}, t_i) = 0 \qquad \text{... (3)}$$

where

$\bar{x}_{i-1}$ ... is the true state vector at time t_{i-1}

$\bar{x}_i$ ... is the true state vector at time t_i

The linearised dynamic model relating the state vector at times *i-1* and and *i* is given by :

$$X_i = M_{i-1,i}\ x_{i-1} + y \qquad \text{... (4)}$$

where $M_{i-1,i}$ ... is the transition matrix or dynamic matrix describing, approximately, how the state vector changes from time *i-1* to *i*

y ... is the vector of unknown true errors in the model and are assumed to be randomly distributed about a zero mean

For most practical problems, it is convenient to consider the noise vector y as being given by:

$$y = Tg \qquad \text{... (5)}$$

where g ... is the vector of the quantities which cause the model to be in error with covariance C_g, i.e. $g \sim (0, C_g)$

T ... is the coefficient matrix which describes how g propagates into the state vector.

Thus, equation (4) becomes :

$$x_i = M_{i-1,i}\, x_{i-1} + T_g \qquad \text{... (6)}$$

The vector g is not actually known but its covariance, C_g, can be estimated. As a result, the covariance matrix of y can be computed using Gauss's propagation of error law :

$$C_y = T C_g T^T \qquad \ldots (7)$$

2.3 Kalman Filter Algorithms

The derivation of the Kalman filter from standard least squares requirement is found in Cross (1987). The Kalman filter consists of the following parts :

i) Time update equations (Prediction)
ii) Measurement update equations (Filtering)
iii) Smoothing equations (Smoothing)

• Time Update Process

The time update is the prediction of the state vector and its covariance (error). The prediction equation of the state vector can be derived directly from the dynamic model of (4). However, since the vector y is not actually known, we make an assumption that it is zero. Thus, the prediction equation of the state vector is given by :

$$\bar{x}_i(-) = M_{i-1,i}\,\bar{x}_{i-1}(+) \qquad \ldots (8)$$

where the symbols - (i.e. bar) denotes an estimated quantity, and the symbols (-) and (+) following a vector denote the value of that vector at the instant in time before and after a measurement update.

The transition matrix, $M_{i-1,i}$, allows calculation of the state vector at some time *i*, given complete knowledge of the state vector at *i-1*, in the absence of the dynamic model noise, *y*. Thus, equation (8) gives the state vector at time *i* predicted using information up to time *i-1*.

The predicted covariance matrix of the state vector can be obtained from :

$$C_{\bar{x}_i}(-) = M_{i-1,i}\,C_{\bar{x}_{i-1}}(+)\,M^T_{i-1,i} + C_y \qquad \ldots (9)$$

where C_y ... is the covariance matrix of dynamic model errors.

This prediction might be made to some time at which the state is required to be known but at which time there are no measurements. In this case, the same equations (i.e (8) and (9)) may be used again ad again to compute the predicted state at any number of epochs until another set of measurements is available.

• Measurement Update Process

The measurement update is the improvement of the prediction (both the state vector and its covariance) which gives the filtered state estimates. The filtered algorithms are given by the following equations:

$$\bar{x}_i(+) = \bar{x}_i(-) + G_i(b_i - A_i\bar{x}_i(-)) \qquad \ldots (10)$$

$$C_{\bar{x}_i}(+) = (I - G_iA_i)\, C_{\bar{x}_i}(-) \qquad \ldots (11)$$

where

$$G_i = C_{\bar{x}_i}(-)A_i^T\,(A_iC_{\bar{x}_i}(-)A_i^T + W_i^{-1})^{-1} \qquad \ldots (12)$$

G is the so-called Kalman gain matrix. The gain matrix performs the role of combining the dynamic model and the observations. It controls the amount by which a particular set of observations affects the predicted state vector.

The updated state vector is obtained after the difference between the actual and predicted measurements has been computed, i.e $b_i - A_i\bar{x}_i(-)$. Equation (10) gives the best estimator of the state at time i using both $\bar{x}_i(-)$ and b_i.

• Smoothing Process

Smoothing process is carried out after the measurements have been completed. It is a requirement that the predicted and updated state vectors and their corresponding covariance have been stored.

2.4 Non-Linear Measurement Model

The measurement update equation listed in (10) is used to solved problems with linear observations equations. However, for offshore positioning most measurement lead to non-linear equations.

As mentioned in the 'measurement model' section if the problems involved non-linear functional model, they have to be linearised as in (2) for used in the Kalman filter.

The design matrix, A_i and the gain matrix, Ç are computed based on the predicted state vector, $\bar{x}_i(-)$. Thus the filtered estimate of the state vector is given by :

$$\bar{x}_i(+) = \bar{x}_i(-) + G_i(l_i - comp(\bar{x}_i(-)))$$

. . . (13)

where

$comp(\bar{x}_i(-))$	...	is the vector of observation computed using the predicted state vector at time i
l_i	...	is the vector of observation at time i.

2.5 Summary of the Kalman Filter Algorithms

A summary of the algorithms used for the linear dynamic model and non-linear measurement model is given below. Here, the estimate $\bar{x}_i(+)$ given in (13) can be improved by repeatedly calculating $\bar{x}_i(+)$, G_i, and $C_{\bar{x}_i}(+)$ and each time linearising about the most recent estimate. As many iterations can be performed as are necessary to reach the point where there is no significant change in consecutive iterates. However, it should be recognised that each iteration contributes to the computation time required to mechanise the filter.

A detailed explaination about the algorithms can be found in Mahmud (1991).

1) Initialise $\bar{x}_{i-1}(+) = x_0$ . . . (14)

and $C_{\bar{x}_{i-1}}(+) = C_{x_0}$ . . . (15)

i.e an a prior estimate of the state vector and its covariance is assumed to be known.

2) Increment i, i := i + 1

3) $\bar{x}_i(-) = M_{i-1,i}\bar{x}_{i-1}(+)$ . . . (16)

4) $C_{\bar{x}_i}(-) = M_{i-1,i}C_{\bar{x}_{i-1}}(+)M^T_{i-1,i} + C_y$ . . . (17)

5) Start iteration

$$G_i = C_{\bar{x}_i}(-)A_i^T(A_iC_{\bar{x}_i}(-)A_i^T + +W_i^{-1})^{-1}$$

. . . (18)

6) $\bar{x}_i(+) = \bar{x}_i(-) + G_i(l_i - comp(\bar{x}_i(-)))$ . . . (19)

7) $$C_{\bar{x}_i}(+) = (I - G_i A_i)\, C_{\bar{x}_i}(-) \quad \ldots (20)$$

8) Return to step 5 (i.e. until a certain stopping criterion is met)

$$\bar{x}_i(-) := \bar{x}_i(+) \quad \ldots (21)$$

and

$$C_{\bar{x}_i}(-) := C_{\bar{x}_i}(+) \quad \ldots (22)$$

9) Return to step 2.

3.0 DATA SIMULATOR

The tests carried out in this paper involved simulated data. Hence it is possible to monitor accurately the performance of the Kalman filter because the true positions of the ship are known. Figure 1 shows the simulated ship's path and the six transponders station. The true coordinates of the six transponders are shown in Table 1. Errors of $\pm$ 100 metres were introduced to all the transponders coordinates except those chosen as fixed coordinates in the relative calibration test.

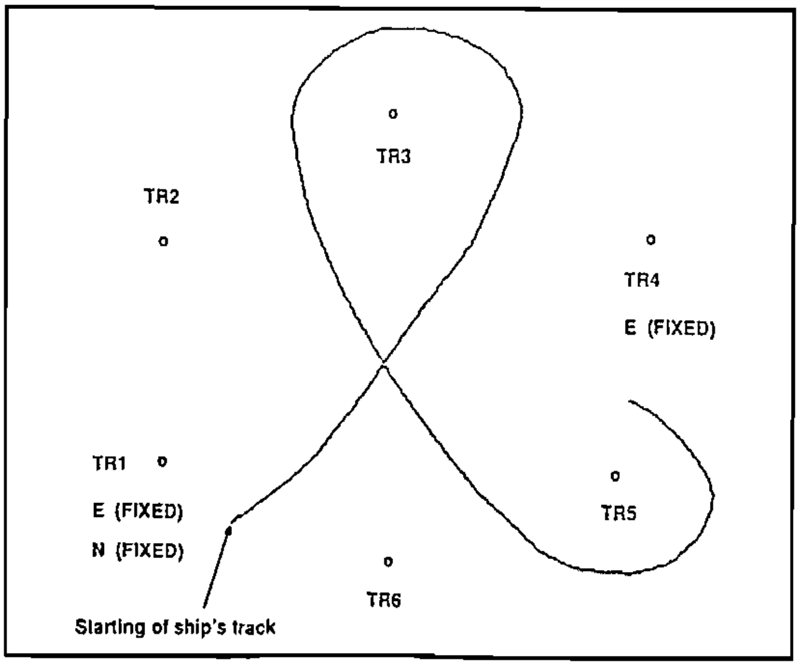

Figure 1. Simulated ship's path and transponders position

Station	Coordinates (metres)	
	Eastings	Northings
TR1	642544.9554	6264513.4918
TR2	642585.8109	6267405.8134
TR3	644967.0925	6269116.0998
TR4	647551.9617	6267446.0043
TR5	647185.9108	6264329.8162
TR6	644905.5715	6263209.8709

Table 1. True coordinates of transponders

4.0 RELATIVE CALIBRATION TEST

4.1 Observables and Provisional Coordinates of Transponders

The provisional coordinates of the transponders are shown in Table 2. Both coordinates for station 1 and the easting coordinate of station 4 were held fixed to give scale and orientation to the array of transponders. Note that in a real situation, the fixed coordinates may be known from previous surveys or arbitrary values can be assigned. In this case the true simulated coordinates were used. Hence it was expected that once all the errors had converged, the coordinates obtained would be the true simulated coordinates. Only the observables form underwater acoustic transponders were used in this test.

Station	Coordinates (metres)	
	Eastings	Northings
TR1	642544.9554 (FIXED)	6264513.4918 (FIXED)
TR2	642485.8109	6267305.8134
TR3	645067.0925	6269016.0998
TR4	647551.9617 (FIXED)	6267546.0043
TR5	647085.9108	6264429.8162
TR6	644805.5715	6263309.8709

Table 2 . Provisional coordinates of transponders

4.2 Relative Calibration Results

Tables 3, 4, 5 and 6 show the precision results of epoch 1, 100, 200 and 300 respectively. They show the results of the ship's parameters, error ellipse and relative error ellipse of the transponders position including their coordinates and standard errors. Note that the notation 'NONE' in the tables means that no value is given due to the station coordinates being held fixed and the notation 'DIFF' is the difference between consecutive coordinate results. The coordinates of the transponders shown in the tables can be compared with the true values shown in Table 1. Figures 2, 3, 4, 5 and 6 show the graphs of errors in coordinates against the fix number. The results showed that all the errors of the transponders coordinates converged from ± 100 metres to less than 1.1 metres after 100 epochs except the easting coordinate for transponder 3 (i.e. less than 2.6 metres). Clearly it can be seen that all the transponders coordinates managed to converge satisfactorily as the number of epoch increases. The values of the standard errors for distances and the semi-major and semi-minor axes of the relative error ellipses in Tables 5 and 6 for epoch 200 and 300 respectively, are less than 0.1 metres. These values can be used to decide to end the calibration process in a real situation. For this particular test, it was found that the calibration process could be terminated when the values of standard errors of distances and the semi-major and semi-minor axes of the relative error ellipses were less than 0.1 metres since the error of the transponder coordinates had converged to a satisfactory limit.

SHIP PARAMETERS

EPOCH	EASTING	NORTHING	TIME	HEADING	SPEED (knots)
1	643316.357	6263714.319	2 0 0	49.35	8.095

COORDINATE OF BEACONS

NO.	STN.	EASTING	STD.ER	DIFF	NORTHING	STD.ER	DIFF
1	tr1	642544.955	NONE	NONE	6264513.492	NONE	NONE
2	tr2	642468.943	48.646	16.867	6267378.697	13.543	72.883
3	tr3	645095.897	47.654	18.805	6269073.048	20.112	56.949
4	tr4	647551.962	NONE	NONE	6267436.927	22.620	109.077
5	tr5	647161.587	15.432	75.677	6264444.166	49.108	14.350
6	tr6	644924.820	14.950	119.249	6263277.343	47.527	32.526

ABSOLUTE ERROR ELLIPSES

NO.	STATION	MAJOR	MINOR	ORIENTATION
1	tr1	FIXED STATION		
2	tr2	49.901	7.732	76.97
3	tr3	49.993	13.271	108.27
4	tr4	FIXED STATION		
5	tr5	49.926	12.535	169.27
6	tr6	49.227	7.684	15.29

RELATIVE STANDARD ERRORS

LINE FROM	TO	STD. ERROR OF DISTANCE	STD. ERROR OF BEARING	RELATIVE ERROR ELLIPSES MAJOR	MINOR	ORIENTATION
tr1	tr2	12.515	0.017	49.901	7.732	76.97
tr1	tr3	16.079	0.009	49.993	13.271	108.27
tr1	tr4	11.405	0.003	22.620	0.000	0.00
tr1	tr5	15.858	0.011	49.926	12.535	169.27
tr1	tr6	12.801	0.018	49.227	7.684	15.29
tr2	tr3	56.939	0.014	68.249	19.790	92.25
tr2	tr4	48.778	0.004	50.178	14.261	75.18
tr2	tr5	51.044	0.009	51.476	50.556	78.65
tr2	tr6	39.459	0.012	60.166	38.236	46.97
tr3	tr4	49.058	0.005	50.507	8.158	109.62
tr3	tr5	60.470	0.008	62.268	35.862	138.88
tr3	tr6	51.906	0.007	52.841	48.417	153.72
tr4	tr5	50.916	0.007	54.745	7.597	165.65
tr4	tr6	49.869	0.005	53.977	12.415	9.12
tr5	tr6	36.921	0.024	68.448	16.428	2.29

Table 3. Precision results for epoch 1

SHIP PARAMETERS

EPOCH	EASTING	NORTHING	TIME	HEADING	SPEED (knots)
100	645966.421	6267844.471	2 19 48	14.56	8.127

COORDINATE OF BEACONS

NO.	STN.	EASTING	STD.ER	DIFF	NORTHING	STD.ER	DIFF
1	tr1	642544.955	NONE	NONE	6264513.492	NONE	NONE
2	tr2	642585.713	0.104	0.034	6267405.493	0.065	0.019
3	tr3	644964.550	0.213	0.052	6269115.805	0.093	0.015
4	tr4	647551.962	NONE	NONE	6267445.303	0.093	0.007
5	tr5	647186.369	0.037	0.002	6264330.387	0.163	0.071
6	tr6	644906.614	0.064	0.004	6263209.388	0.099	0.003

ABSOLUTE ERROR ELLIPSES

NO.	STATION	MAJOR	MINOR	ORIENTATION
1	tr1	FIXED STATION		
2	tr2	0.107	0.059	73.31
3	tr3	0.222	0.071	106.72
4	tr4	FIXED STATION		
5	tr5	0.164	0.035	3.35
6	tr6	0.113	0.036	30.15

RELATIVE STANDARD ERRORS

LINE FROM	TO	STD. ERROR OF DISTANCE	STD. ERROR OF BEARING	RELATIVE ERROR ELLIPSES MAJOR	MINOR	ORIENTATION
tr1	tr2	0.065	0.000	0.107	0.059	73.31
tr1	tr3	0.081	0.000	0.222	0.071	106.72
tr1	tr4	0.047	0.000	0.093	0.000	0.00
tr1	tr5	0.035	0.000	0.164	0.035	3.35
tr1	tr6	0.036	0.000	0.113	0.036	30.15
tr2	tr3	0.116	0.000	0.173	0.075	109.50
tr2	tr4	0.104	0.000	0.115	0.053	60.86
tr2	tr5	0.053	0.000	0.188	0.052	31.31
tr2	tr6	0.038	0.000	0.175	0.036	57.26
tr3	tr4	0.208	0.000	0.219	0.046	103.74
tr3	tr5	0.093	0.000	0.215	0.084	76.46
tr3	tr6	0.047	0.000	0.252	0.046	87.68
tr4	tr5	0.128	0.000	0.129	0.036	178.22
tr4	tr6	0.085	0.000	0.096	0.055	30.96
tr5	tr6	0.061	0.000	0.076	0.059	163.86

Table 4. Precision results for epoch 100

SHIP PARAMETERS

EPOCH	EASTING	NORTHING	TIME	HEADING	SPEED (knots)
200	643939.114	6268780.480	2 39 48	172.50	8.205

COORDINATE OF BEACONS

NO.	STN.	EASTING	STD.ER	DIFF	NORTHING	STD.ER	DIFF
1	tr1	642544.955	NONE	NONE	6264513.492	NONE	NONE
2	tr2	642586.240	0.045	0.004	6267405.851	0.040	0.006
3	tr3	644967.302	0.061	0.005	6269115.519	0.053	0.003
4	tr4	647551.962	NONE	NONE	6267445.742	0.070	0.003
5	tr5	647186.568	0.033	0.001	6264329.635	0.071	0.000
6	tr6	644906.393	0.046	0.001	6263209.117	0.048	0.005

ABSOLUTE ERROR ELLIPSES

NO.	STATION	MAJOR	MINOR	ORIENTATION
1	tr1	FIXED STATION		
2	tr2	0.045	0.039	108.86
3	tr3	0.070	0.040	127.55
4	tr4	FIXED STATION		
5	tr5	0.073	0.028	15.41
6	tr6	0.057	0.033	41.79

RELATIVE STANDARD ERRORS

LINE FROM	TO	STD. ERROR OF DISTANCE	STD. ERROR OF BEARING	RELATIVE ERROR ELLIPSES MAJOR	MINOR	ORIENTATION
tr1	tr2	0.040	0.000	0.045	0.039	108.86
tr1	tr3	0.041	0.000	0.070	0.040	127.55
tr1	tr4	0.036	0.000	0.070	0.000	0.00
tr1	tr5	0.032	0.000	0.073	0.028	15.41
tr1	tr6	0.035	0.000	0.057	0.033	41.79
tr2	tr3	0.047	0.000	0.057	0.041	108.83
tr2	tr4	0.045	0.000	0.067	0.036	27.30
tr2	tr5	0.039	0.000	0.079	0.038	40.77
tr2	tr6	0.032	0.000	0.074	0.032	65.06
tr3	tr4	0.053	0.000	0.061	0.040	81.04
tr3	tr5	0.043	0.000	0.076	0.042	74.60
tr3	tr6	0.035	0.000	0.086	0.035	91.78
tr4	tr5	0.046	0.000	0.046	0.033	178.60
tr4	tr6	0.044	0.000	0.051	0.043	146.47
tr5	tr6	0.045	0.000	0.047	0.041	20.44

Table 5. Precision results for epoch 200

SHIP PARAMETERS

EPOCH	EASTING	NORTHING	TIME	HEADING	SPEED (knots)
300	645794.212	6264194.001	2 59 48	146.59	8.016

COORDINATE OF BEACONS

NO.	STN.	EASTING	STD.ER	DIFF	NORTHING	STD.ER	DIFF
1	tr1	642544.955	NONE	NONE	6264513.492	NONE	NONE
2	tr2	642586.083	0.039	0.001	6267405.833	0.037	0.000
3	tr3	644967.239	0.057	0.005	6269115.617	0.048	0.004
4	tr4	647551.962	NONE	NONE	6267445.828	0.064	0.002
5	tr5	647186.582	0.032	0.005	6264329.668	0.065	0.005
6	tr6	644906.374	0.044	0.011	6263209.157	0.043	0.005

ABSOLUTE ERROR ELLIPSES

NO.	STATION	MAJOR	MINOR	ORIENTATION
1	tr1	FIXED STATION		
2	tr2	0.040	0.037	114.98
3	tr3	0.065	0.036	125.64
4	tr4	FIXED STATION		
5	tr5	0.067	0.026	16.47
6	tr6	0.053	0.032	46.14

RELATIVE STANDARD ERRORS

LINE FROM	TO	STD. ERROR OF DISTANCE	STD. ERROR OF BEARING	RELATIVE ERROR ELLIPSES MAJOR	MINOR	ORIENTATION
tr1	tr2	0.037	0.000	0.040	0.037	114.98
tr1	tr3	0.036	0.000	0.065	0.036	125.64
tr1	tr4	0.032	0.000	0.064	0.000	0.00
tr1	tr5	0.030	0.000	0.067	0.026	16.47
tr1	tr6	0.034	0.000	0.053	0.032	46.14
tr2	tr3	0.043	0.000	0.053	0.038	113.10
tr2	tr4	0.040	0.000	0.062	0.033	23.63
tr2	tr5	0.035	0.000	0.073	0.034	38.61
tr2	tr6	0.030	0.000	0.068	0.030	64.85
tr3	tr4	0.050	0.000	0.057	0.039	81.43
tr3	tr5	0.041	0.000	0.072	0.040	74.15
tr3	tr6	0.032	0.000	0.081	0.032	92.12
tr4	tr5	0.044	0.000	0.044	0.032	3.13
tr4	tr6	0.041	0.000	0.049	0.040	138.35
tr5	tr6	0.044	0.000	0.045	0.039	26.01

Table 6. Precision results for epoch 300

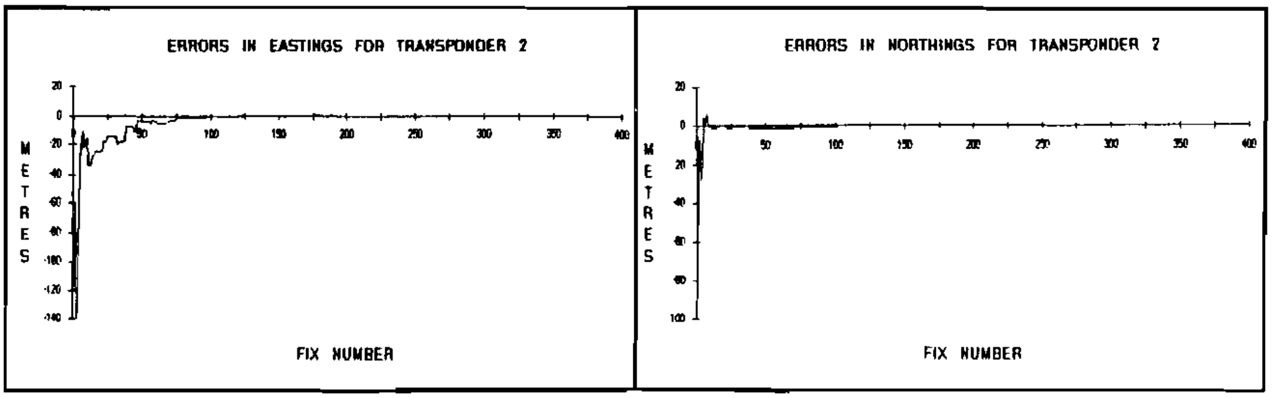

Figure 2. Relative calibration : Errors in coordinates for transponder 2

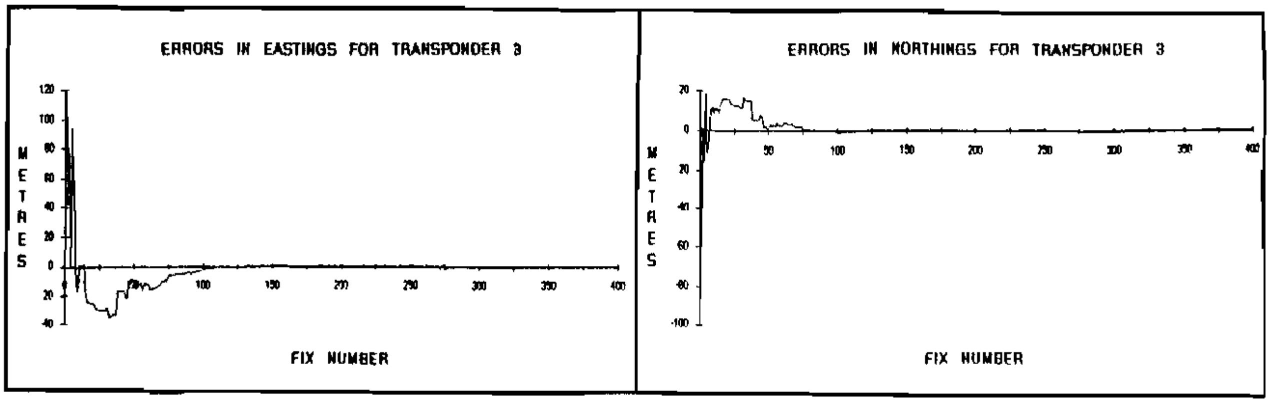

Figure 3. Relative calibration : Errors in coordinates for transponder 3

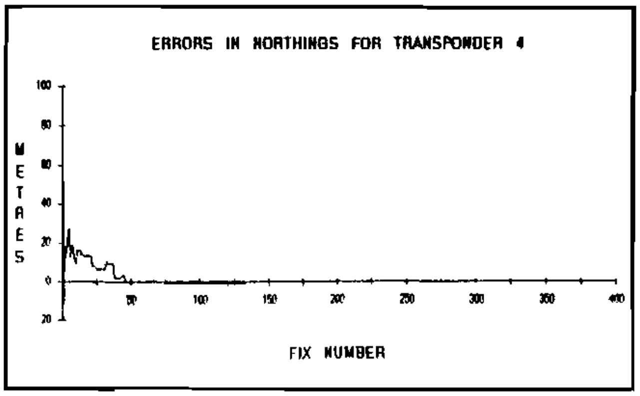

Figure 4. Relative calibration : Errors in coordinates for transponder 4

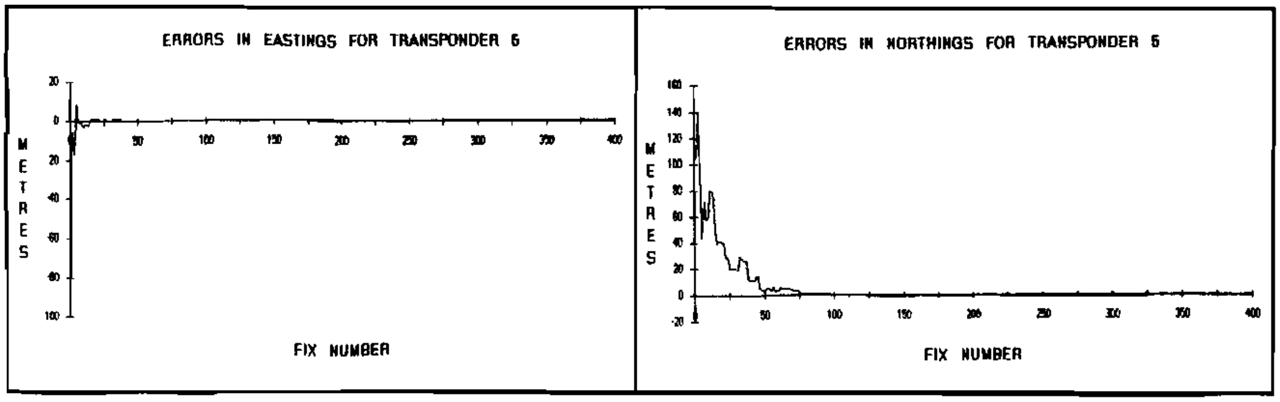

Figure 5. Relative calibration : Errors in coordinates for transponder 5

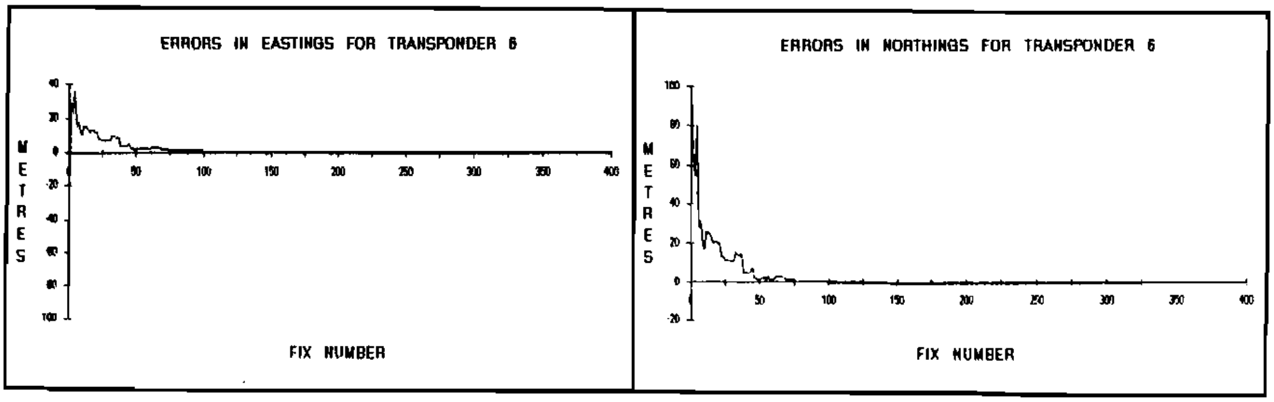

Figure 6. Relative calibration : Errors in coordinates for transponder 6

5.0 ABSOLUTE CALIBRATION TEST

5.1 Observables and Provisional Coordinates of Transponders

The provisional coordinates of the transponders for the absolute calibration test are shown in Table 7. In this test there was no fixed coordinate defined. All the observables from four systems (i.e. DGPS, underwater acoustic system, range-range system and hyperbolic system) integrated together were used in this test.

Station	Coordinates (metres)	
	Eastings	Nothings
TR1	642644.9554	6264413.4918
TR2	642485.8109	6267305.8134
TR3	645067.0925	6269016.0998
TR4	647651.9617	6267546.0043
TR5	647085.9108	6264429.8162
TR6	644805.5715	6263309.8709

Table 7. Provisional coordinates of transponders

5.2 Absolute Calibration Results

Tables 8, 9, 10 and 11 show the precision results for epoch 1, 100, 200 and 300 respectively. The graphs of errors in coordinates against the fix number in Figures 7, 8, 9, 10, 11 and 12 show that all the errors of the transponder coordinates converged from $\pm$ 100 metres to less than 1.1 metres after 100 epochs except the easting coordinate for transponder 3 (i.e. less than 2.7 metres) and the northing coordinate for transponder 4 (i.e. less than 4.9 metres). Clearly it can be seen that all the transponders coordinates managed to converge satisfactorily as the number of epoch increases. Tables 10 and 11 show that the values of standard errors of distances and the semi-major and semi-minor axes of the relative error ellipses for epoch 200 and 300 respectively are less than 0.1 metres. Note that in certain circumstances the above interpretation may be quite different. The small values of the relative error ellipse does not mean that the calibration process has reached its satisfactory limit due to the transponders achieving correct 'relative positioning' but not achieving correct absolute position in the required coordinate system. This can happen when there are more acoustic than other observables. Thus the quality and number of observables received from DGPS, range-range system and hyperbolic system are important to achieve high quality absolute calibration.

SHIP PARAMETERS

EPOCH	EASTING	NORTHING	TIME	HEADING	SPEED (knots)
1	643320.522	6263715.126	2 0 0	49.35	8.095

COORDINATE OF BEACONS

NO.	STN.	EASTING	STD.ER	DIFF	NORTHING	STD.ER	DIFF
1	tr1	642548.142	35.756	96.813	6264513.341	34.664	99.849
2	tr2	642468.898	48.695	16.913	6267378.527	11.347	72.713
3	tr3	645086.510	47.480	19.417	6269075.027	15.654	58.927
4	tr4	647544.399	33.110	107.563	6267450.891	37.444	95.113
5	tr5	647165.758	9.322	79.847	6264444.944	49.076	15.128
6	tr6	644928.456	13.162	122.884	6263276.211	48.008	33.660

ABSOLUTE ERROR ELLIPSES

NO.	STATION	MAJOR	MINOR	ORIENTATION
1	tr1	49.794	0.781	45.89
2	tr2	49.995	0.699	76.91
3	tr3	49.991	0.561	108.24
4	tr4	49.981	0.525	138.52
5	tr5	49.949	0.638	169.27
6	tr6	49.774	0.744	15.31

RELATIVE STANDARD ERRORS

LINE		STD. ERROR OF		RELATIVE ERROR ELLIPSES		
FROM	TO	DISTANCE	BEARING	MAJOR	MINOR	ORIENTATION
tr1	tr2	35.109	0.021	67.994	18.876	61.47
tr1	tr3	48.594	0.010	60.373	36.530	77.28
tr1	tr4	49.325	0.009	51.026	48.731	179.83
tr1	tr5	36.638	0.013	62.097	33.453	17.44
tr1	tr6	18.917	0.025	67.914	18.575	30.61
tr2	tr3	56.509	0.014	68.075	19.101	92.57
tr2	tr4	58.716	0.008	60.721	36.211	107.70
tr2	tr5	48.936	0.009	50.998	48.934	33.71
tr2	tr6	37.678	0.012	60.603	36.125	46.34
tr3	tr4	68.237	0.006	68.238	18.473	123.37
tr3	tr5	59.138	0.008	60.881	35.892	138.71
tr3	tr6	50.434	0.009	51.148	48.593	149.31
tr4	tr5	57.869	0.013	68.132	18.748	153.98
tr4	tr6	49.717	0.010	62.052	33.553	166.73
tr5	tr6	36.845	0.024	68.703	15.898	2.24

Table 8. Precision results for epoch 1

SHIP PARAMETERS

EPOCH	EASTING	NORTHING	TIME	HEADING	SPEED (knots)
100	645965.927	6267845.249	2 19 48	15.01	8.131

COORDINATE OF BEACONS

NO.	STN.	EASTING	STD.ER	DIFF	NORTHING	STD.ER	DIFF
1	tr1	642545.152	0.040	0.003	6264514.157	0.089	0.032
2	tr2	642595.339	0.142	0.024	6267406.397	0.085	0.009
3	tr3	644964.465	0.248	0.129	6269116.797	0.056	0.028
4	tr4	647551.027	0.163	0.224	6267450.824	0.210	0.290
5	tr5	647186.133	0.042	0.004	6264330.703	0.153	0.068
6	tr6	644906.579	0.081	0.016	6263210.355	0.056	0.010

ABSOLUTE ERROR ELLIPSES

NO.	STATION	MAJOR	MINOR	ORIENTATION
1	tr1	0.090	0.039	171.38
2	tr2	0.160	0.044	61.71
3	tr3	0.252	0.030	100.71
4	tr4	0.265	0.026	142.28
5	tr5	0.153	0.041	3.36
6	tr6	0.088	0.045	63.83

RELATIVE STANDARD ERRORS

LINE		STD. ERROR OF		RELATIVE ERROR ELLIPSES		
FROM	TO	DISTANCE	BEARING	MAJOR	MINOR	ORIENTATION
tr1	tr2	0.057	0.000	0.146	0.055	83.83
tr1	tr3	0.067	0.000	0.272	0.063	112.70
tr1	tr4	0.040	0.000	0.316	0.040	149.32
tr1	tr5	0.027	0.000	0.221	0.027	2.34
tr1	tr6	0.033	0.000	0.134	0.033	28.43
tr2	tr3	0.111	0.000	0.180	0.090	119.85
tr2	tr4	0.106	0.000	0.274	0.099	170.93
tr2	tr5	0.048	0.000	0.263	0.048	33.65
tr2	tr6	0.028	0.000	0.232	0.027	60.17
tr3	tr4	0.194	0.000	0.195	0.167	134.16
tr3	tr5	0.092	0.000	0.268	0.088	71.46
tr3	tr6	0.040	0.000	0.304	0.039	91.44
tr4	tr5	0.130	0.000	0.203	0.093	126.89
tr4	tr6	0.076	0.000	0.277	0.066	130.26
tr5	tr6	0.055	0.000	0.128	0.053	160.82

Table 9. Precision results for epoch 100

SHIP PARAMETERS

EPOCH	EASTING	NORTHING	TIME	HEADING	SPEED (knots)
200	643939.342	6268781.252	2 39 48	172.33	8.194

COORDINATE OF BEACONS

NO.	STN.	EASTING	STD.ER	DIFF	NORTHING	STD.ER	DIFF
1	tr1	642545.160	0.039	0.001	6264514.128	0.047	0.005
2	tr2	642585.979	0.052	0.001	6267406.618	0.042	0.002
3	tr3	644967.007	0.078	0.009	6269116.330	0.034	0.000
4	tr4	647552.958	0.052	0.004	6267449.324	0.065	0.005
5	tr5	647186.195	0.040	0.001	6264330.343	0.064	0.001
6	tr6	644906.361	0.057	0.003	6263210.056	0.037	0.001

ABSOLUTE ERROR ELLIPSES

NO.	STATION	MAJOR	MINOR	ORIENTATION
1	tr1	0.054	0.030	145.15
2	tr2	0.053	0.041	73.93
3	tr3	0.081	0.028	105.16
4	tr4	0.079	0.025	143.30
5	tr5	0.064	0.040	4.53
6	tr6	0.057	0.037	85.18

RELATIVE STANDARD ERRORS

LINE		STD. ERROR OF		RELATIVE ERROR ELLIPSES		
FROM	TO	DISTANCE	BEARING	MAJOR	MINOR	ORIENTATION
tr1	tr2	0.037	0.000	0.054	0.036	81.98
tr1	tr3	0.037	0.000	0.090	0.036	113.22
tr1	tr4	0.029	0.000	0.097	0.029	150.01
tr1	tr5	0.026	0.000	0.082	0.026	2.47
tr1	tr6	0.032	0.000	0.058	0.031	34.32
tr2	tr3	0.046	0.000	0.064	0.038	114.22
tr2	tr4	0.042	0.000	0.084	0.041	173.77
tr2	tr5	0.034	0.000	0.092	0.033	30.80
tr2	tr6	0.026	0.000	0.085	0.026	60.37
tr3	tr4	0.053	0.000	0.058	0.053	47.38
tr3	tr5	0.042	0.000	0.091	0.042	65.36
tr3	tr6	0.030	0.000	0.105	0.030	89.02
tr4	tr5	0.044	0.000	0.057	0.944	110.45
tr4	tr6	0.040	0.000	0.087	0.040	120.16
tr5	tr6	0.042	0.000	0.053	0.041	168.05

Table 10. Precision results for epoch 200

SHIP PARAMETERS

EPOCH	EASTING	NORTHING	TIME	HEADING	SPEED (knots)
300	645794.510	6264195.385	2 59 48	146.54	8.016

COORDINATE OF BEACONS

NO.	STN.	EASTING	STD.ER	DIFF	NORTHING	STD.ER	DIFF
1	tr1	642545.181	0.038	0.003	6264514.062	0.044	0.002
2	tr2	642585.878	0.048	0.001	6267406.501	0.040	0.001
3	tr3	644966.920	0.071	0.009	6269116.345	0.032	0.005
4	tr4	647552.807	0.048	0.001	6267448.338	0.061	0.003
5	tr5	647186.227	0.039	0.001	6264330.268	0.060	0.001
6	tr6	644906.387	0.055	0.001	6263210.028	0.035	0.001

ABSOLUTE ERROR ELLIPSES

NO.	STATION	MAJOR	MINOR	ORIENTATION
1	tr1	0.052	0.028	142.84
2	tr2	0.049	0.040	75.80
3	tr3	0.074	0.026	106.00
4	tr4	0.073	0.024	143.80
5	tr5	0.060	0.039	6.50
6	tr6	0.055	0.035	87.33

RELATIVE STANDARD ERRORS

LINE		STD. ERROR OF		RELATIVE ERROR ELLIPSES		
FROM	TO	DISTANCE	BEARING	MAJOR	MINOR	ORIENTATION
tr1	tr2	0.035	0.000	0.050	0.034	82.91
tr1	tr3	0.034	0.000	0.083	0.033	113.81
tr1	tr4	0.027	0.000	0.090	0.027	149.98
tr1	tr5	0.025	0.000	0.076	0.025	2.68
tr1	tr6	0.031	0.000	0.053	0.031	35.88
tr2	tr3	0.042	0.000	0.059	0.036	116.17
tr2	tr4	0.038	0.000	0.079	0.037	174.13
tr2	tr5	0.030	0.000	0.086	0.030	30.72
tr2	tr6	0.025	0.000	0.079	0.025	60.60
tr3	tr4	0.050	0.000	0.053	0.050	43.74
tr3	tr5	0.040	0.000	0.085	0.040	65.14
tr3	tr6	0.028	0.000	0.097	0.028	88.34
tr4	tr5	0.043	0.000	0.053	0.042	109.18
tr4	tr6	0.037	0.000	0.081	0.037	119.97
tr5	tr6	0.041	0.000	0.050	0.040	169.88

Table 11. Precision results for epoch 300

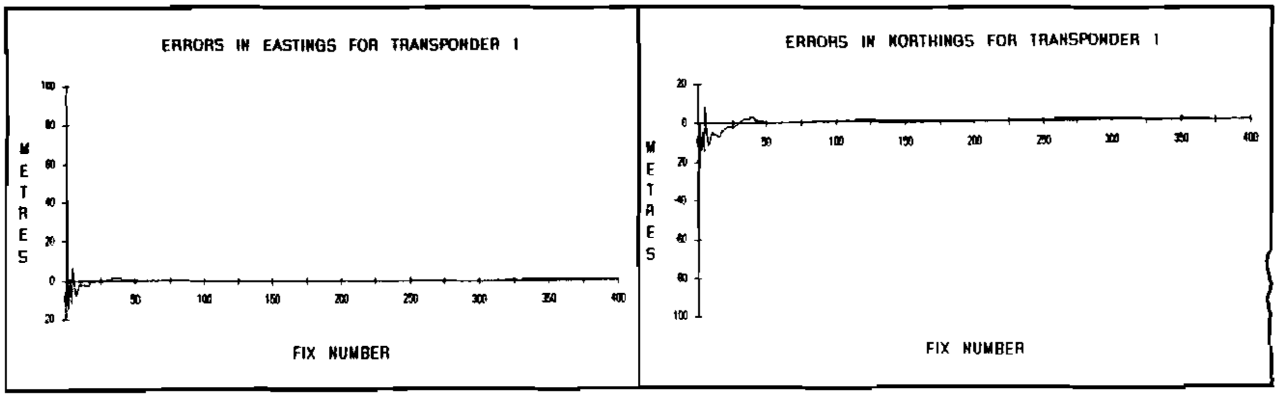

Figure 7. Absolute calibration : Errors in coordinates for transponder 1

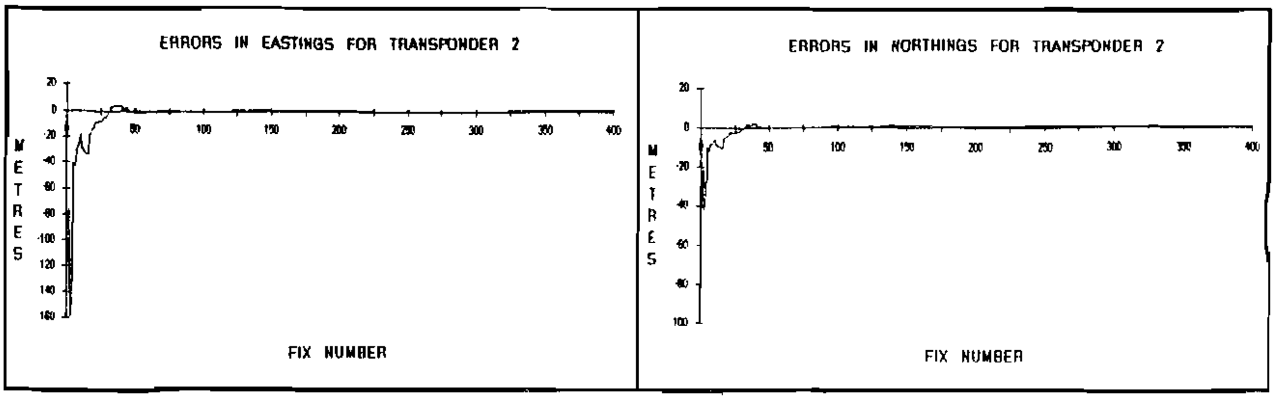

Figure 8. Absolute calibration : Errors in coordinates for transponder 2

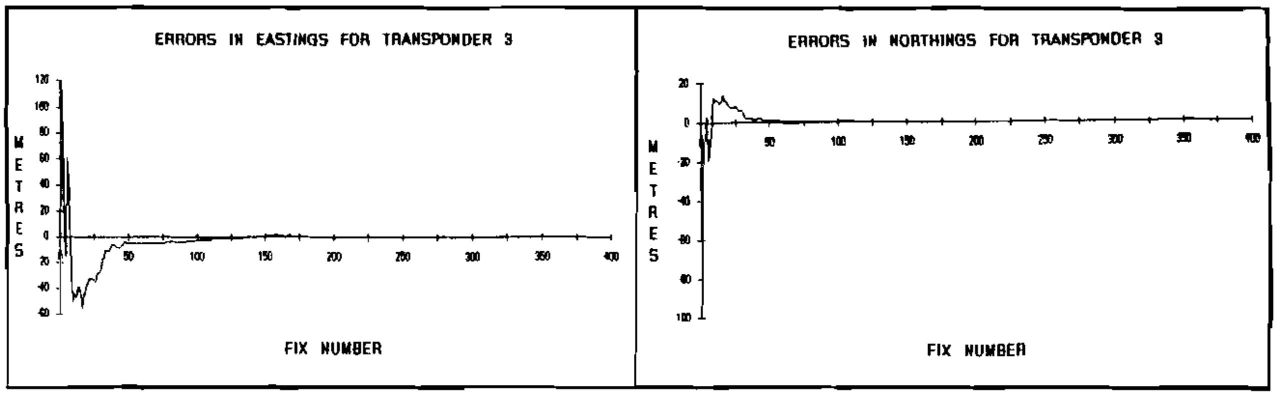

Figure 9. Absolute calibration : Errors in coordinates for transponder 3

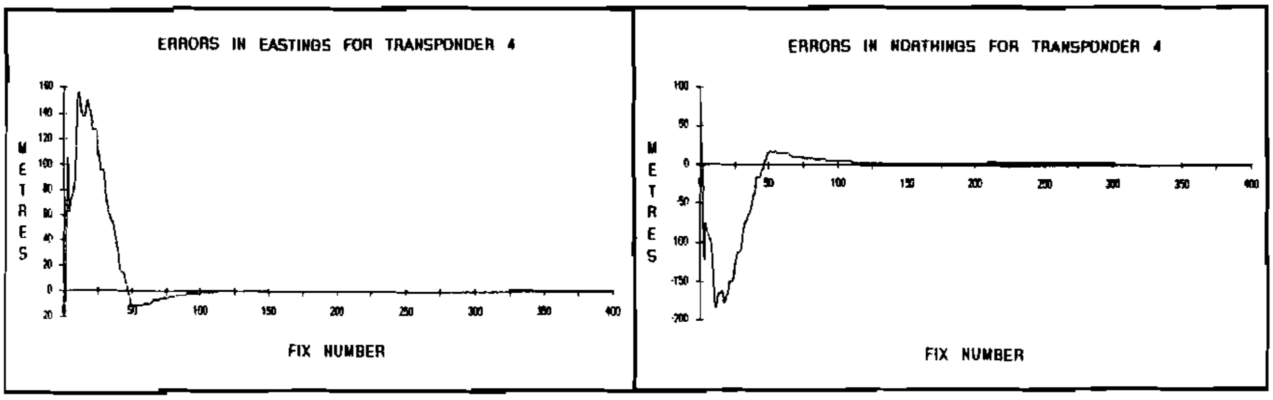

Figure 10. Absolute calibration : Errors in coordinates for transponder 4

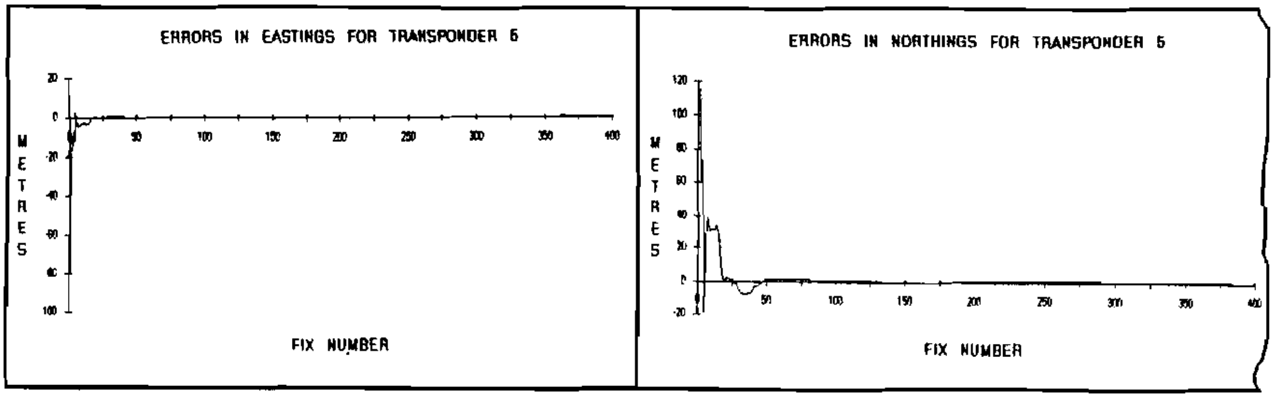

Figure 11. Absolute calibration : Errors in coordinates for transponder 5

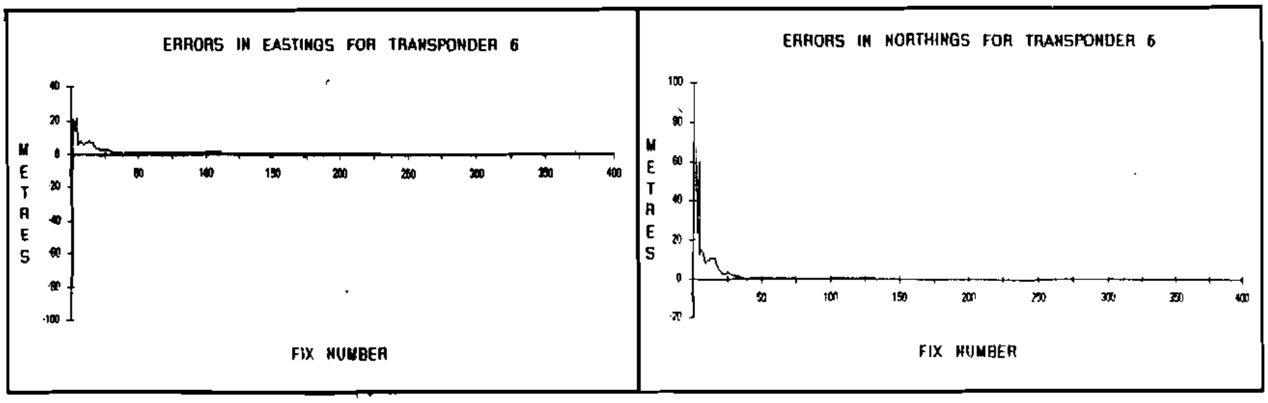

Figure 12. Absolute : Errors in coordinates for transponder 6

6.0 CONCLUDING REMARKS

The tests carried out for relative and absolute calibrations showed that in both cases the errors introduced in the transponders coordinates managed to converge to a satisfactory limit. This was only when the values of the standard errors of distances, semi-major and semi-minor axes of the relative error ellipses were less than 0.1 metres. Both these results suggested that absolute calibration could be carried out without relative calibration if the quantity of observations from the underwater acoustic system are the same or greater than the rest of the positioning systems employed.

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