S-TRANSFORMATIONS: A PRACTICAL TOOL FOR DEFORMATION MONITORING

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ABSTRACT

During the process of rigorous least squares estimation (LSE) and deformation detection, it is required to transform the results (i.e. coordinates and cofactor matrix) from one datum to another, i.e. datum re-definition. In this paper, the application of general equation of S-transformations as a tool for datum re-definition in deformation monitoring is presented. In addition, two numerical examples are also included.

1.0 INTRODUCTION

The methods for monitoring deformation in engineering are usually based on the repeated observation of a survey monitoring networks at different epochs, followed by two-step analysis (i.e. independent least squares estimation (LSE) of single epoch and deformation detection between epochs). The detection of deformation employs two-epoch analysis, an absolute monitoring approach and a static model to compare the coordinates between the epochs. Further aspects on deformation monitoring are given in Chen (1983), Fraser and Gruendig (1985), Caspary (1987), Cooper (1987), Niemeier (1987), Biacs and Teškey (1990), Chrzanowski et al (1991) and Halim (1995a).

The three dimensional coordinates and their cofactor matrix obtained from least squares estimation (LSE) of each epoch for deformation detection are datum dependent, and must be referred to a common datum. During the process of deformation detection, it is also required to re-define the datum with respect to a set of stable points. Consequently, a facility to allow for changes of datum or computational base is needed.

This paper discusses the transformation of LSE results and datum re-definition via the general equation of similarity covariance transformation (S-transformation). Aspects on datum re-definition and S-transformations are briefly outlined, followed by the practical computational strategy. Two numerical examples are then described and analysed.

2.0 DATUM RE-DEFINITION

The required variables estimated from LSE of each epoch for the purpose of deformation detection are the estimated three dimensional coordinates ($\hat{x}_a$), their cofactor matrix ($Q_{\hat{x}}$), estimated variance factor ($\hat{\sigma}_0^2$), degrees of freedom (r) and datum defect (d). For simplicity, x and Q_x will be used to represent $\hat{x}_a$ and $Q_{\hat{x}}$ respectively throughout this paper.

Both x and Q_x are datum dependent. LSE can be based on the minimum trace, minimum constraints or partial minimum trace datum (Caspary, 1987). Datum invariant quantities (for example $\hat{\sigma}_0^2$, r, d) remain the same because of their datum independence property.

The concept of datum definition is readily applicable in the monitoring of deformation. The observations at each epoch are processed independently by LSE (via the method of observation equations) to estimate x and Q_x. In general, the monitoring network is treated as free network where all stations are assumed to be unstable a priori, and hence a minimum trace datum is used. In some cases, a set of stable points is known in advance, and in this case a partial minimum trace datum can be used. In practice, the conventional minimum constraints datum is favoured due to its simplicity. In this case, transformation of x and Q_x into either minimum or partial minimum trace datum is needed.

In the initial stage of deformation detection, x and Q_x of any two epochs are differenced to estimate displacement vectors and their cofactor matrix. Theoretically, x and Q_x have to be referred to the same common datum. However, different datum definitions may be necessary for each epoch, possibly because of different defects in the configuration or practical limitations (such as obstruction of the line of sight or destruction of points).

The solution with respect to a common datum can be obtained either from LSE of each epoch where the new datum is defined by a common set of points, or via S-transformations (section 3.0) of x and Q_x of each epoch to the new datum. The S-transformations approach is very useful as it replaces the repeated LSE and inversion of the normal equations coefficient matrix. Moreover, during the localization of deformation (Halim, 1995a), S-transformations are used repeatedly for transforming the displacement vectors and their cofactor matrix with respect to new datums defined by different sets of stable points. This approach is analagous to a partial minimum trace solution.

3.0 CONCEPT OF S-TRANSFORMATIONS

The S-transformation is based on the work of Baarda carried out in the 1950s which was published later in Baarda (1973). A comprehensive formulation for S-transformations is given in Strang Van Hees (1982). The general expression for the S-transformations of any arbitrary x_i and Q_{xi} into x_j and Q_{xj} is (Halim, 1995a)

$$\begin{aligned} x_j &= S_j x_i \\ Q_{xj} &= S_j Q_{xi} S_j^t \\ S_j &= (I-G(G^t I_j G)^{-1} G^t I_j) \text{ or} \\ S_j &= (I-G(C^t G)^{-1} C^t) \text{ if } C = I_j G \end{aligned} \qquad (3.1)$$

In equation (3.1), I_j is a diagonal matrix for defining the computational base after S-transformations. If only some of the points are used for datum definition (partial minimum trace datum), the elements of I_j for datum and non-datum points are one and zero respectively. Dimensions of the S-transformations matrices involving m stations and maximum datum defect (seven) are x_i (3m,1), x_j (3m,1), Q_{xi} (3m,3m symmetric), Q_{xj} (3m,3m symmetric), G (3m,7), I or I_j (3m,3m diagonal), C (3m,7), C^tG (7,7) and S_j (3m,3m generally non-symmetric). Hence, one only need to invert a maximum of a (7 by 7) matrix, which is quite simple.

In engineering surveying, inner constraints matrix G^t (shown in Figure 3.1) is well known. The first three rows of G^t define the translation along x, y and z respectively. The next three rows define the rotations about x, y and z, respectively, while the last row defines the scale of the network. If the observations contain datum information, the rank deficiencies are less than seven (i.e. number of rows of G^t), and the corresponding rows of G^t are omitted. A typical 3-D network comprising of slope distances, zenith angles and horizontal directions will contain three datum elements, and has four datum defect (three translation and one rotation about z-axis).

$$G^t = \begin{bmatrix} +1 & 0 & 0 & +1 & 0 & 0 & \dots & +1 & 0 & 0 \\ 0 & +1 & 0 & 0 & +1 & 0 & \dots & 0 & +1 & 0 \\ 0 & 0 & +1 & 0 & 0 & +1 & \dots & 0 & 0 & +1 \\ 0 & +z_1 & -y_1 & 0 & +z_2 & -y_2 & \dots & 0 & +z_n & -y_n \\ -z_1 & 0 & +x_1 & -z_2 & 0 & +x_2 & \dots & -z_n & 0 & +x_n \\ +y_1 & -x_1 & 0 & +y_2 & -x_2 & 0 & \dots & +y_n & -x_n & 0 \\ +x_1 & +y_1 & +z_1 & +x_2 & +y_2 & +z_2 & \dots & +x_n & +y_n & +z_n \end{bmatrix}$$

Figure 3.1 Full components of matrix G^t for a 3-D network

In general, the square matrix S is non-symmetric, and only symmetric for a minimum trace datum. Two important properties of S-transformations are (Cooper and Cross, 1991):

(i) S is idempotent. For successive S-transformations, only the last transformation determines the resulting x and Q_x.

(ii) Product of SG is zero. This property is useful for checking the computation of the S matrix.

4.0 APPLICATION IN DEFORMATION MONITORING

S-transformations can be applied in both LSE and deformation detection. As will be shown below, the general S-transformations equation can be applied directly.

In LSE the types of solutions can be based on either a minimum trace, partial minimum trace or minimum constraint datum (section 3.0). The transformation equations for transforming x_i and Q_{xi} into x_j and Q_{xj} based on the chosen datum are given by equation (3.1) as

$$x_j = S_j x_i, \quad Q_{xj} = S_j Q_{xi} S_j^t \qquad (4.1)$$
$$S_j = (I - G(C^tG)^{-1}C^t), \quad C = I_j G$$

Let m be the number of stations in the 3-D network, d be the datum defect and mm the number of coordinates chosen for datum definition. The elements of I_j will be unity for datum points and zero for other points. The solutions can be realised as

if mm=3m, minimum trace datum (4.2)
mm=d, minimum constraints datum
mm between d and 3m, partial minimum trace datum

In most cases, it may be necessary to divide the points into two groups, datum and non-datum points. Fraser and Gruendig (1985) and Cooper (1987) adopted the partitioning and ordering of x_i and Q_{xi} with respect to datum and non-datum points as the following:

$$x_i = [x_r \;\; x_e]^t \quad ; \; Q_{x_i} = \begin{bmatrix} Q_r & Q_{re} \\ Q_{er} & Q_e \end{bmatrix} \tag{4.3}$$

where r (retain) refers to datum points and e (eliminate) refers to other non-datum points. Hence, equation (4.1) becomes

$$x_j = Sx_i \text{ and } Q_{xj} = SQ_{xi}S^t \tag{4.4}$$

$$S = I - \begin{bmatrix} G_r(G_r^tG_r)^{-1}G_r^t & 0 \\ G_e(G_r^tG_r)^{-1}G_r^t & 0 \end{bmatrix} \tag{4.5}$$

This partitioning approach requires re-ordering of x, Q_x, G_r and G_e.

Prior to deformation detection, x and Q_x of any two epochs must be referred to a common datum defined by points common to each epoch. Let the main data (coordinates and cofactor matrix) for epochs one and two be x_1, Q_{x1}, x_2, Q_{x2}. Assume that different numbers of stations and datum definition or computational bases are used in each epoch. Let the computational bases for epochs one and two be i and j respectively. It is then required to transform the main data into the new datum defined by common stations (n points) for both epochs.

Initially, it is necessary to determine the common stations, and then re-arrange the data so that the common stations are ordered at the beginning of arrays x and Q_x. By using the partitioning scheme, with common stations ordered at the beginning of arrays x and Q_x, equation (4.3) becomes

for epoch one: $x_1^{(i)}$ and $Q_{x1}^{(i)}$ with computational base i
for epoch two: $x_2^{(j)}$ and $Q_{x2}^{(j)}$ with computational base j

$$x_1^{(i)} = \begin{bmatrix} x_{1r} \\ x_{1e} \end{bmatrix} \; , \; Q_{x1}^{(i)} = \begin{bmatrix} Q_{1r1r} & Q_{1r1e} \\ Q_{1e1r} & Q_{1e1e} \end{bmatrix} \tag{4.6}$$

$$x_2^{(j)} = \begin{bmatrix} x_{2r} \\ x_{2e} \end{bmatrix} \; , \; Q_{x2}^{(j)} = \begin{bmatrix} Q_{2r2r} & Q_{2r2e} \\ Q_{2e2r} & Q_{2e2e} \end{bmatrix} \tag{4.7}$$

where r refers to common stations, and e refers to non-common stations.

The transformation of the LSE results of each epoch into the new datum defined by n common stations is

$$\text{new } x_1^{(n)} = \begin{bmatrix} x_{1r}^{(n)} \\ x_{1e}^{(n)} \end{bmatrix} = S_1 x_1^{(i)}, \ \text{new } Q_{x1}^{(n)} = \begin{bmatrix} Q_{1r1r}^{(n)} & Q_{1r1e}^{(n)} \\ Q_{1e1r}^{(n)} & Q_{1e1e}^{(n)} \end{bmatrix} = S_1 Q_{x1}^{(i)} S_1^t \tag{4.8}$$

$$\text{new } x_2^{(n)} = \begin{bmatrix} x_{2r}^{(n)} \\ x_{2e}^{(n)} \end{bmatrix} = S_2 x_2^{(i)}, \ \text{new } Q_{x2}^{(n)} = \begin{bmatrix} Q_{2r2r}^{(n)} & Q_{2r2e}^{(n)} \\ Q_{2e2r}^{(n)} & Q_{2e2e}^{(n)} \end{bmatrix} = S_2 Q_{x2}^{(i)} S_2^t \tag{4.9}$$

where $S_1 = I - G_1(G_1^t I_n G_1)^{-1} G_1^t I_n$, $S_2 = I - G_2(G_2^t I_n G_2)^{-1} G_2^t I_n$

Values of I_n are 1 and 0 for common (or datum) and non-common (non-datum) stations respectively.

After this transformation, the useful results for deformation detection are the coordinates and cofactor matrix of the common stations in each epoch:

epoch one: $x_{1r}^{(n)}$ and $Q_{1r1r}^{(n)}$
epoch two: $x_{2r}^{(n)}$ and $Q_{2r2r}^{(n)}$

The displacement vector d and its cofactor matrix Q_d for the common stations can be simply computed as

$$d = x_{2r}^{(n)} - x_{1r}^{(n)} \text{ and } Q_d = Q_{1r1r}^{(n)} + Q_{2r2r}^{(n)} \tag{4.10}$$

Equation (4.10) for computing d and Q_d clearly demonstrate the advantage of ordering the common or datum stations at the beginning of arrays x and Q_x. One only needs to extract the upper parts of the arrays, without the need of additional computations, for the purpose of deformation detection.

Depending on whether stable points are known or not in advance, the S-transformations can be used to transform the minimum constraints solution into minimum trace, partial minimum trace or other minimum constraints solutions respectively (Halim, 1995a). In general, any of these LSE solutions can be used for detection of deformation.

Assume that each epoch has same stations but diffferent datum definitions. Let the coordinates and their cofactor matrix for the two epochs be x_1, Q_{x1} (refers to datum A) and x_2, Q_{x2} (refers to datum B), and it is required to be referred to datum C. In this case, d and Q_d can be determined directly

$$x_{1c} = Sx_1; \ x_{2c} = Sx_2; \ d = x_{2c} - x_{1c} = S(x_2 - x_1) \tag{4.11}$$
$$Q_d = Q_{x2c} + Q_{x1c} = S(Q_{x2} + Q_{x1})S^t$$

During the localization of deformation, S-transformations of this type (equation 4.11) are very useful for the iterative transformation of displacement vector and its cofactor matrix into a datum defined by a set of stable points. The S-transformations, used together with a partial congruency test and test of the largest quadratic form, is in effect removing each unstable or suspected point interactively, one at a time. Elements of d and Q_d need to be re-ordered each time a suspected point is removed from the computational base. Details on such procedures are given in Halim (1995a).

Another application of S-transformations is demonstrated by Chen et al (1990a) for identification of stable points via a robust method. Using the general S-transformation, I is interpreted as a weighting factor (weight matrix) for obtaining an iterative weighted similarity transformation of d and Q_d.

The deformation detection procedure developed in Halim (1995a) uses the general S-transformations formulation effectively for simultaneous identification of stable points and estimation of the deformation of unstable points. This is conceptually correct due to the basic property of the S matrix being idempotent. Hence, once the final datum is defined by the stable points, the final transformation determines the resulting d and Q_d.

5.0 COMPUTATIONAL STRATEGY

The general equation for S-transformations of x_i and Q_{xi} to x_j and Q_{xj} (equation 3.1) is simply

$$\begin{aligned} x_j &= S_j x_i \\ Q_{xj} &= S_j Q_{xi} S_j^t \\ S_j &= (I - G(G^t I_j G)^{-1} G^t I_j) \text{ or} \\ S_j &= (I - G(C^t G)^{-1} C^t) \text{ if } C = I_j G \end{aligned} \tag{5.1}$$

Although the above equation looks simple, its direct implementation is not practical because the transformation matrix S is non-symmetric in general and the computation of Q_{xj} is time consuming.

In terms of storage requirements, for a 3-D network of m stations, the major storage areas are occupied by S, Q_{xi} and Q_{xj}. Matrix S is full and non-symmetric (3m,3m), while Q_{xi} and Q_{xj} are symmetric, each requires [(3m)(3m+1)/2] spaces. The main task is in the computation of Q_{xj} as it involves multiplication of large matrices. Another problem is the numerical instability that might occur.

A computational strategy has been formulated (Halim, 1993, 1995a) based on the following criteria in order to produce an efficient implementation of the equation (5.1) into a working computer program:

i. Working with single arrays in most cases and only the triangular matrix Q_x is needed.
ii. Reduction of all approximate coordinates for computing G to their centroid to avoid numerical instability.
iii. Further normalization via Cholesky factorization to achieve numerical stability.
iv. A special procedure by decomposition of S matrix (Biacs, 1989; Biacs and Teskey, 1990) to speed up the computation.

The equations for transforming x_i and Q_{xi} to x_j and Q_{xj} are:

$$x_i = [x_r \quad x_e]^t \quad ; Q_{x_i} = \begin{bmatrix} Q_r & Q_{re} \\ Q_{er} & Q_e \end{bmatrix} \tag{5.2}$$

where

$x_j = [I - GR^t]x_i$
$Q_{xj} = Q_{xi} - GP^t - PG^t + GR^tPG^t$
$P^t = [C^tG]^{-1}C^tQ_{xi}$
$R^t = [C^tG]^{-1}C^t$
$C = I_iG$

In equation (5.2), elements of I_i are one and zero for datum and non-datum points respectively. The partitioning procedure is adopted here to simplify the uses of S-transformations in localization of deformation (Halim, 1995a). However, if the purpose is only to transform results of LSE, there is no need for partitioning, and all the remaining equations are still applicable.

6.0 NUMERICAL EXAMPLES

The following two numerical examples demonstrate some applications of S-transformations in LSE and deformation detection. The first example (Figure 6.1) is a levelling network adopted from Caspary (1987).

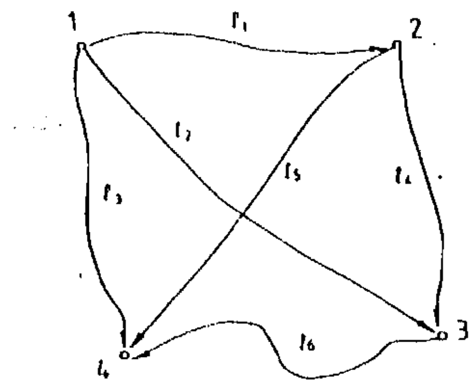

Figure 6.1 Levelling network (from Caspary, 1987)

Assuming all observations have equal weights of I, and using approximate values of parameters as $[10.0 \;\; 11.1 \;\; 11.5 \;\; 11.6]^t$, three types of LSE solutions via the method of observation equations were computed:

#1. Ordinary minimum constraints with station 1 chosen as the datum point.
#2. Minimum trace where all stations are used for datum definition.

#3. Partial minimum trace with stations 2 and 3 as datum points.

The results obtained are similar to Caspary (1987) and are depicted in Table 6.1. Solution #2 gives minimum norm and minimum trace whilst solution #3 minimizes the partial norm and partial trace, with respect to the datum points. The residuals remain unchanged due to their independence on the selection of datum constraints. Although not computed here, other invariant quantities are adjusted observations, cofactor matrices of the adjusted observations and the residuals.

S-transformations of the LSE results in Table 6.1 were performed using equation (5.1) where $G=[1\ 1\ 1\ 1]^t$. Three types of computational bases were chosen:

1. Minimum trace to minimum constraints
Datum point = station 1; $I_p = [1\ 0\ 0\ 0]^t$
2. Minimum constraints to minimum trace
Datum points = all stations (1,2,3,4); $I_p = [1\ 1\ 1\ 1]^t$
3. Minimum trace to partial minimum trace
Datum points = stations 2 and 3; $I_p = [0\ 1\ 1\ 0]^t$

Results of the transformations are summarised in Table 6.2. Comparison of the results of LSE in Table 6.1 with results of S-transformations in Table 6.2 (columnwise) shows that the parameters and cofactor matrices are identical.

The second example demonstrates the application of the S-transformations in deformation detection. The computer programs for numerical processing of the data (i.e. ESTIMATE, DETECT and COMPS) were developed by Halim (1995a, 1995b). Programs DETECT and COMPS use equations (5.2) for evaluation of S-transformations.

Figure 6.2 is a simulated six station 3-D network consisting of 54 uncorrelated observations (12 slope distances with a simulated random error σ of 5 mm, 30 horizontal directions with σ of 5 secs and 12 height differences with σ of 5mm). Deformations were simulated at stations 3, 5 and 6 (table 6.3) to generate data for second epoch.

The rank deficiency is four (i.e. 3 translation and one rotation about z axis), and is removed by fixing coordinates x_1, y_1, z_1 and y_3. The number of parameters is 20 (14 coordinates and 6 orientation unknowns), giving rise to 34 degrees of freedom. The commonly accepted significance level α of 0.05 is used for most statistical testing.

station	simulated deformation (m)		
	dx	dy	dz
1	-	-	-
2	-	-	-
3	-0.050	+0.100	-0.100
4	-	-	-
5	+0.010	+0.050	-
6	-	-	+0.300

Table 6.3 Simulated deformation

	solution #1 minimum constraints datum	solution #2 minimum trace datum	solution #3 partial minimum trace datum
datum defined by stn	1	1,2,3,4 (all)	2,3
parameter			
x_1	10.0	9.925	10.25
x_2	10.6	10.525	10.85
x_3	11.5	11.425	11.75
x_4	12.4	12.325	12.65
norm	0.94	0.93	1.14
partial norm		-	0.35
cofactor matrix			
Q_{11}	0.00	0.1875	0.375
Q_{21}	0.00	-0.0625	0.000
Q_{22}	0.50	0.1875	0.125
Q_{31}	0.00	-0.0625	0.000
Q_{32}	0.25	-0.0625	0.125
Q_{33}	0.50	0.1875	0.125
Q_{41}	0.00	0.0625	0.125
Q_{42}	0.25	-0.0625	0.000
Q_{43}	0.25	-0.0625	0.000
Q_{44}	0.50	0.1875	0.375
trace	1.500	0.750	1.000
partial trace	-	-	0.250
residuals			
v_1	0.6	-0.6	0.6
v_2	0.1	-0.1	0.1
v_3	0.7	0.7	0.7
v_4	0.3	0.3	0.3
v_5	0.3	0.3	0.3
v_6	-0.4	-0.4	0.4

Table 6.1 LSE results for levelling network (unit in mm).

	sol#2 to #1 minimum trace to minimum constraints	sol#1 to #2 minimum constraints to minimum trace	sol#2 to #3 minimum trace to partial minimum trace
datum defined by stn	1	1,2,3,4 (all)	2,3
parameter			
x_1	10.0	9.925	10.25
x_2	10.6	10.525	10.85
x_3	11.5	11.425	11.75
x_4	12.4	12.325	12.65
cofactor matrix			
Q_{11}	0.00	0.1875	0.375
Q_{21}	0.00	-0.0625	0.000
Q_{22}	0.50	0.1875	0.125
Q_{31}	0.00	0.0625	0.000
Q_{32}	0.25	0.0625	-0.125
Q_{33}	0.50	0.1875	0.125
Q_{41}	0.00	-0.0625	0.125
Q_{42}	0.25	-0.0625	0.000
Q_{43}	0.25	-0.0625	0.000
Q_{44}	0.50	0.1875	0.375

Table 6.2 S transformations results for the levelling network (unit in mm).

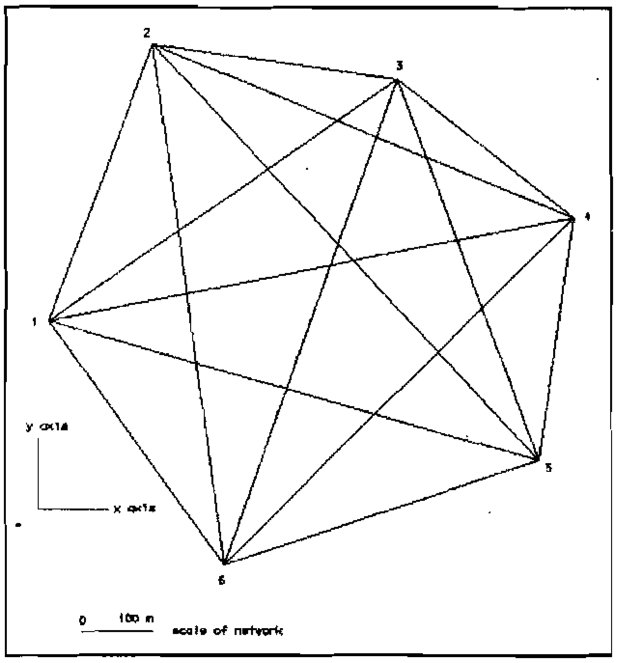

Figure 6.2 Test network (plan view)

LSE for each epoch was carried out, using program ESTIMATE, by minimum constraints, i.e. fixing x_1, y_1, z_1 and x_3. Each solution passed both global and local tests. The estimated variance factors were 0.814 and 0.593 for the first and second epochs respectively.

This was followed by deformation detection using DETECT. As both epochs use the same stations and datum, deformation detection can be proceeded straight away. The failure of the global congruency test confirmed the existence of deformation.

Initially, using an option in DETECT, all stations were used to define the datum. Starting with 6 datum stations, the successive process of removing suspected points from the datum was repeated until a (partial) congruency test passed. This resulted in 3 datum stations (1, 2 and 4) for the final computations. All the datum points passed the single point test and were confirmed as stable. Points 3, 5 and 6 failed the single point test and were suspected as significantly deformed. The estimated deformation obtained via DETECT for each case was very close to the simulated deformation (Table 6.3), as summarized in Table 6.4.

For further verification, COMPS was used to transform the LSE results of each epoch with respect to a new datum defined by stations 1, 2 and 4. The coordinate differences between epochs were computed using DETECT, and the results are the same as the estimated solution in Table 6.4.

The graphical presentation of the solution is depicted in Figures 6.3 to 6.6, and clearly shows that stations 3, 5 and 6 lie outside the ellipses. The deformation trend shown in Figure 6.3 indicates the movement of stations 3, 5 and 6. Figure 6.4 demonstrates movement of stations 3 and 5 in the xy directions, while Figures 6.5 and 6.6 show movement of station 6 in the z direction and station 3 in the xz and yz directions respectively.

station	dx	dy	dz
1	-	-	-
2	0.001	-	-
4	-0.001	-	-
6	0.001	-0.001	0.300
3	-0.050	0.100	-0.100
5	0.012	0.051	-

Table 6.4 Estimated deformation via DETECT
(Datum stations comprised of stations 1, 2 and 4. Unit m)

7.0 CONCLUSIONS

In practice, the coordinates and their cofactor matrix can be transformed from one datum to another via S-transformations, a powerful technique derived from the work of Baarda in the 1950s. In this paper, the general equation of S-transformations is used in all cases, both LSE and deformation detection. For practical implementation of S-transformations, the strategy includes reducing the coordinates to a centroid and normalization of matrix G for numerical stability, and decomposition of matrix S for speeding up the computations.

The results obtained in section 6.0 show that the general equation of S-transformations is applicable for re-definition of datum in LSE and deformation detection. Further applications of S-transformations in deformation monitoring are discussed in Halim (1995a) and Robson et al (1995).

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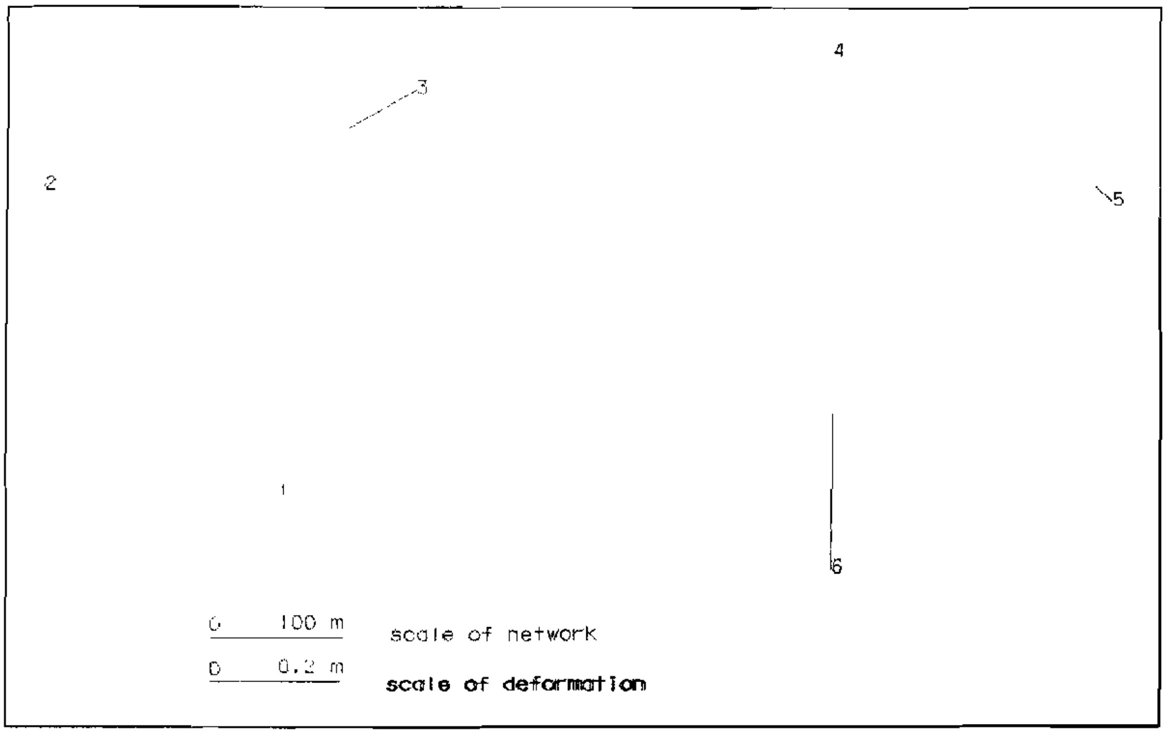

Figure 6.3 Estimated deformation (isometric view)

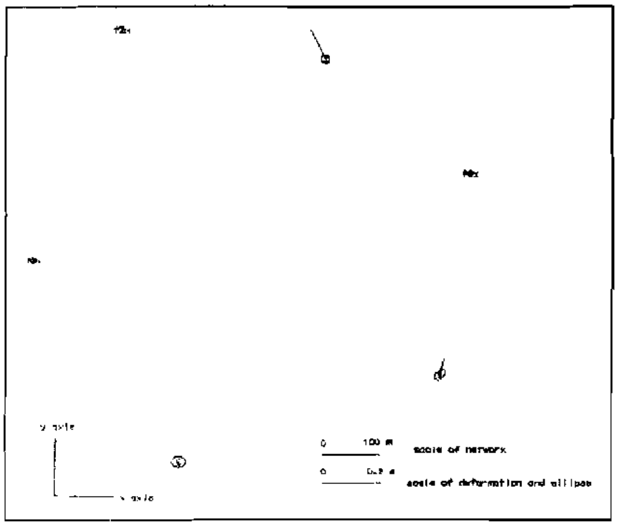

Figure 6.4 Estimated deformation (plan view)

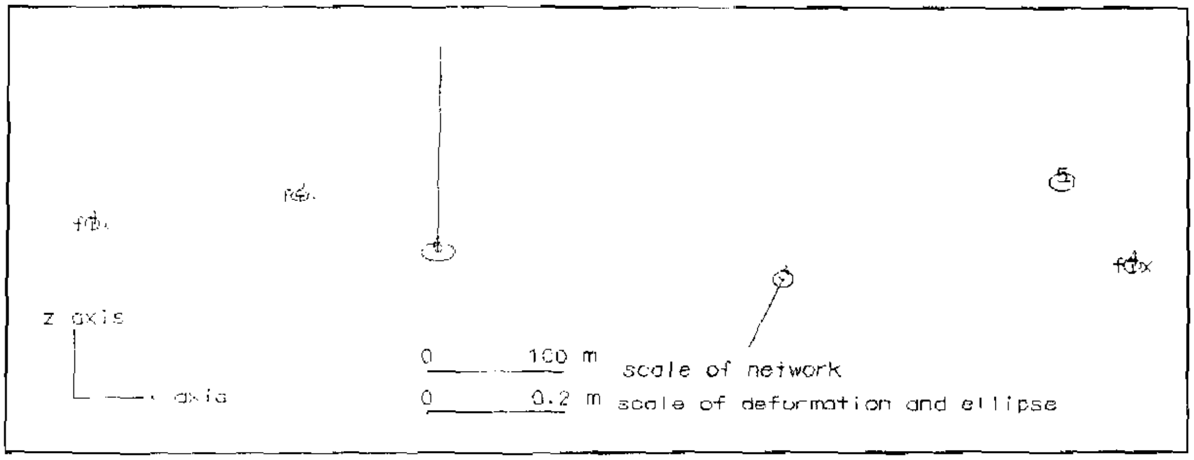

Figure 6.5 Estimated deformation (front view)

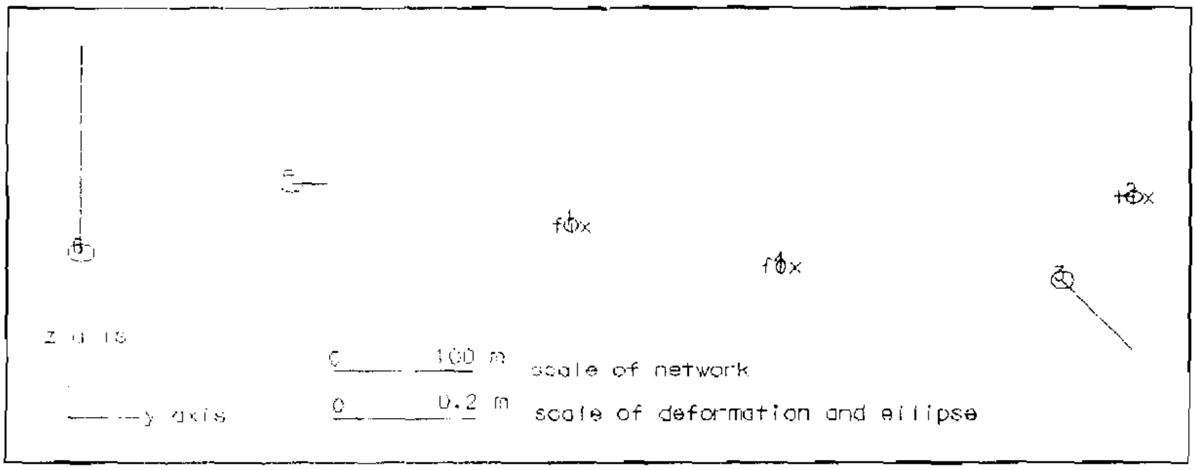

Figure 6.6 Estimated deformation (right view)