

The Design of L3D-HEIGHT Program for Orthometric Height Determination Using GPS And Terrestrial Geodetic Data

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Abstract

The Global Positioning System (GPS) produces, in phase measurement mode, very accurate three-dimensional Cartesian coordinates differences between observation points. Therefore, for many applications, this three-dimensional capability has the potential of replacing costly and time-consuming spirit levelling in height determination, especially when a large or a remote unsurveyed area is involved. For this purpose, a computer program called L3D-HEIGHT was developed to act as a tool in processing for 3D coordinates and orthometric height determination using GPS and other terrestrial geodetic observable. This paper describe the basic structure and primary features of the program.

1.0 INTRODUCTION

The advent of high precision relative positioning by use of the Global Positioning System (GPS) has opened up an alternative to the classical method of height determination. Using GPS, it is now possible to determine routinely, in *phase measurement* mode, very accurate three-dimensional Cartesian coordinate differences (or *baseline vectors*) between observation points. Not only can a high accuracy be achieved in relatively short periods, but it can be produced over long lines, irrespective of terrain, intervisibility or weather. Ashkenazi & Ffoulkes-Jones (1990) have quoted repeatabilities (r.m.s) of about 3 mm, 6 mm, 9 mm and 4 mm for north, east, height and length component respectively, with hardly any component proportional to the baseline (i.e., ppm). This three-dimensional capability therefore provides the possibility of using GPS for height determination as well as for (horizontal) positioning.

The position of points derived from GPS measurements are usually computed in a three-dimensional Cartesian coordinate system, and are then transformed into the more recognisable geodetic latitudes (ϕ), longitudes (λ), ellipsoidal heights (h) or in term of geodetic coordinate differences $\Delta\phi$, $\Delta\lambda$ and Δh . GPS ellipsoidal heights are very useful for deformation and subsidence studies and other applications where the emphasis is not so much in locating a precise point in space as in the relative change of height from one time epoch to another. It is, however, the case that the ellipsoidal heights delivered by GPS are not the same as those historically obtained with geodetic levelling (providing *orthometric heights*). Conventionally, topographic maps, engineering design and construction project plans, usually depict relief by means of orthometric height. Thus, the application of GPS will be further extended if accurate transformations between GPS ellipsoidal height differences and the orthometric height differences can be realised. This can be accomplished on the condition that we know the *geoid height*, or rather, the geoid height difference which relates the orthometric height difference to the GPS ellipsoidal height difference. Hence today, a great deal of interest is being shown in the development of the geoid models which are important to provide the necessary geoid height to transform GPS ellipsoidal heights to orthometric heights.

Several theories and methods have been developed to provide geoid models, each having their inherent strengths and weaknesses. In general, they can be divided into two groups. The first group consists of

methods that utilise gravity data (and in some cases, combined with geometrical data). The extent of the gravity data used for such models may be of global or regional scale or a combination of both. The geoid height of individual points is normally computed directly from the data with the accuracy achieved dependent upon such factors as accuracy and distribution of data and the adopted solution models. Some practical examples of this group can be seen in Engelis et al. (1985) and Sjöberg (1991). The methods in the second group, on the other hand, solve for the geoid (height) based solely on a geometrical model that uses data such as orthometric heights that are available over the area of interest. The accuracy achieved in this case, is also dependent on the accuracy and distribution of data, but the interpolation technique adopted is also crucial. The size and the nature of the terrain of the area are also an important consideration in these methods. Practical examples can be seen in Liddle (1989) and Hajela (1990).

It has been long recognised that one of the best methods of providing an accurate geoid (and geoid height) is by using gravity data with the well-known Stokes' integral technique (see, Strang van Hees (1986), Forsberg & Madsen (1990) and Featherstone (1992)). However, a major drawback is the specialised knowledge needed to compute the geoid (height), which is often beyond the capability of most users of GPS - in terms of both geodetic know-how and computing facilities. Hence, a simple yet effective means of computing the geoid (height) is clearly needed. A comprehensive study on this subject can be found in Khairul(1993).

As mentioned previously, for precise positioning, observations using GPS are normally performed in the differential mode, providing baseline vectors as the main derived observables. In order to determine the three-dimensional coordinates and orthometric heights of observing GPS points, these baseline vectors need to be processed (normally in conjunction with existing terrestrial geodetic data) in a network solution. At the moment, there are two approaches that have been adopted for such purposes.

The first approach is known as the integrated method and has been forwarded by, amongst others, Eissfellar & Hein (1984). In this method, the functional models used in the solution incorporates both geometrical and gravity data. The least squares solution is split into two unknown vectors. The first vector consists of the geometrical unknowns such as point coordinates and the second vector comprises gravity disturbance parameters. Unfortunately, orthometric height does not form part of the estimated parameters and are computed externally using the derived gravity disturbance parameters.

The second approach relates to a purely geometrical method such as that adopted by Sutisna (1988) and Daud (1989). This method uses functional models that relate the observables with the position and bias parameters through a three-dimensional Cartesian coordinate system. This method only estimate the ellipsoidal heights rather than the required orthometric heights. Orthometric heights are then simply derived externally using known geoid heights computed from an adopted geoid model.

It is apparent that a more practical and flexible solution for estimating three-dimensional coordinates and orthometric height using GPS and terrestrial geodetic observables is needed. The estimation of the three-dimensional coordinate and the orthometric heights should be carried out in a **concurrent process**, rather than having to compute orthometric height in an external environment.

This paper describe a computer program that has been developed for this purpose. The program was named L3D-Height since it has the capability of carrying out the estimation and qualitative analysis of 3D coordinates, orthometric heights and transformation parameters, from observed control network data, using both terrestrial and satellite techniques.

2.0 THE DESIGN OF L3D-HEIGHT PROGRAM

2.1 General Features of the Program

The program has two important features. Firstly, it allows satellite derived observables to be combined with existing coordinates for the computation of networks using the least squares statistical technique. Secondly, the program can be used to estimate the orthometric heights of observed points. The estimation of station coordinates and orthometric heights are carried out simultaneously.

The primary operational requirements considered in the design and development of the program were as follows:

- (a) it should be user friendly,
- (b) it should be able to accommodate a large number of stations (minimum of 3000 stations and/or subject to computer system storage capability) in the least squares estimation procedure,
- (c) it should be executed in a batch processing mode in view of its ability to accommodate large number of stations which will require a longer computation (CPU) time,
- (d) program execution should be through a command file containing a list of **keywords** and a complete set of default commands for easy program execution must be provided,
- (e) it should contain extensive error trapping facilities which inform the user when any computing or geodetic regulation is broken,
- (f) it should be able to read observations through multiple data files.
- (g) it should include data snooping and precision analysis procedure, and,
- (h) it should contain several geoid models for the user to choose from in performing the orthometric height estimation procedure.

2.2 General Structure of the Program

The program is written in standard FORTRAN 77, in an attempt to make it portable and independent of computer hardware. It is currently implemented on the mainframe of the University of Newcastle upon Tyne under the UNIX operating system and also a VAX version has been implemented at the British Ordnance Survey, Southampton for their National GPS Network Project (Christie, 1991). Plans are underway to have it installed at the University's mainframe in the near future. Figure [2.1] shows the general structure of the program. The program is designed to be executed using a command file containing the following fourteen keyword inputs:

- LOG FILE
- OUTPUT FILE
- TITLE
- INPUT FILE
- SYSTEM ONE
- SYSTEM TWO
- TRANSFORMATION
- ELLIPSOID ONE
- ELLIPSOID TWO

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- FIXED STATION
- STATION ANALYSIS
- GEOID MODEL
- PRINT
- EXECUTE

The first thirteen keywords relate to the pre-processing procedures which feed in all the necessary information to the program before invoking the last (14th) keyword which will execute the least squares estimation procedures. Further detail on the function of each of the keyword can be found in Khairul(1993). An example of a command file is shown in Figure [2.2].

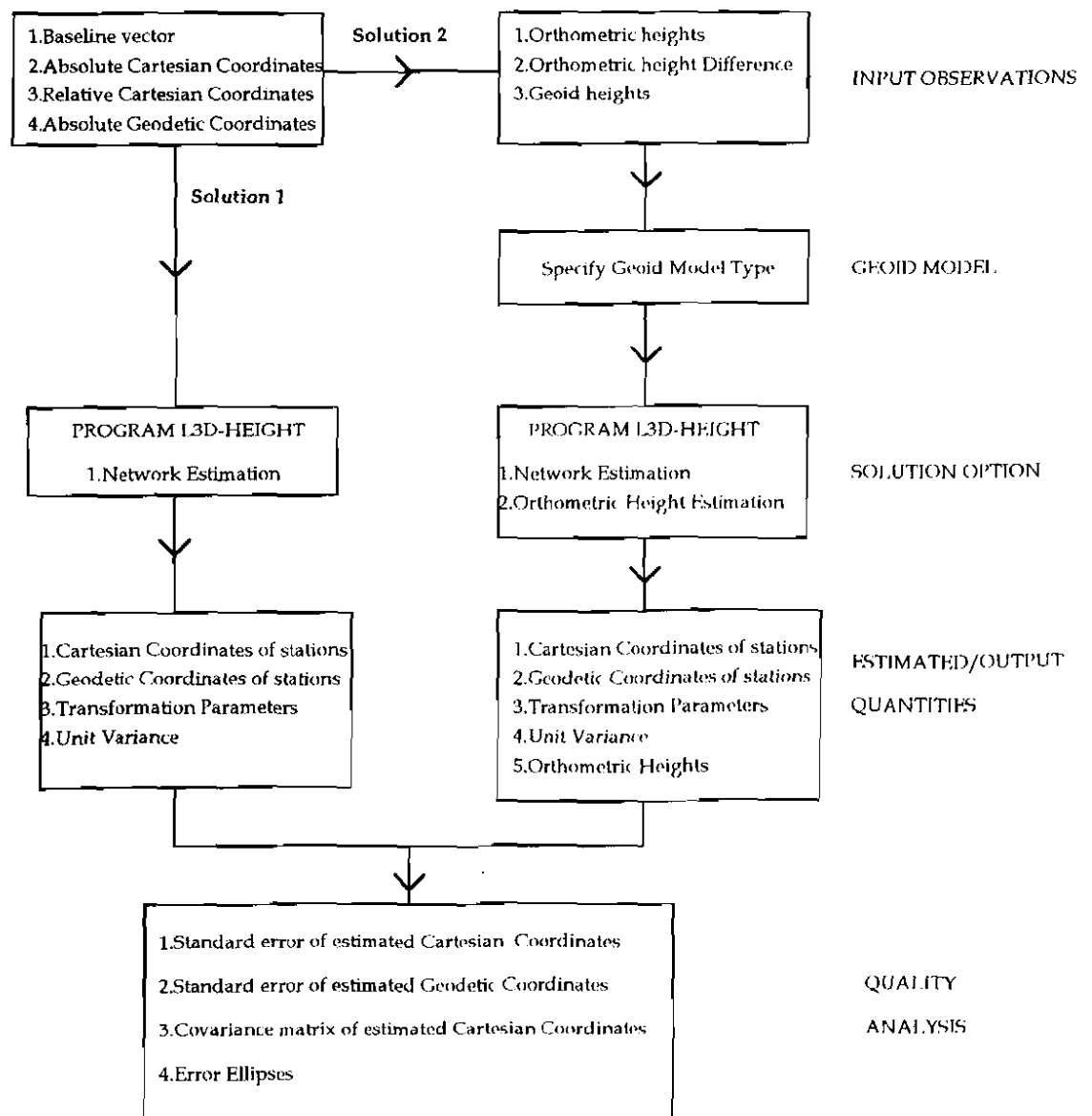


Figure 2.1 Flow diagram showing the general structure of the program

```
LOG FILE:  
job.log  
OUTPUT FILE:  
job.out  
TITLE:  
THIS IS THE TITLE OF THE JOB.  
NO MORE  
INPUT FILE:  
job1.bas  
1.0D0  
job1.rel  
1.0d0  
NO MORE  
SYSTEM ONE:  
ALL  
SYSTEM TWO:  
NONE  
TRANSFORMATION:  
1  
P  
0 1 1 1 0 0 0  
2  
C  
NO MORE  
ELLIPSOID ONE:  
0  
ELLIPSOID TWO:  
0  
FIXED STATION:  
strn.fix  
STATION ANALYSIS:  
strn.anl  
GEOID MODEL:  
1  
PRINT:  
1  
1  
1  
1  
EXECUTE:
```

Figure 2.2 A typical command file.

2.3 Solutions

As shown in Figure [2.1], there two choice of solutions provided by the program. The first is known as the 3D Coordinates only solution while the second is known as the Combined solution.

The 3D Coordinates Only Solution

This option is offered for carrying out tasks which involve only the estimation of the station coordinates in a geodetic network. The types of input data acceptable to this solution comprise of baselines, relative coordinates, absolute Cartesian coordinates and absolute geodetic coordinates. The following quantities are computed in this solution:

- (a) the estimated Cartesian coordinates
- (b) the derived geodetic coordinates
- (c) the estimated transformation parameters
- (d) the estimated unit variance
- (e) the estimated standard errors of the transformation parameters
- (f) variance covariance matrix of estimated coordinates
- (g) the estimated standard errors of Cartesian and geodetic coordinates
- (h) the estimated standard errors of coordinate differences
- (i) the error ellipses
- (j) estimated residuals of observations

The Combined 3D Coordinates and Orthometric Height Solution

If a geoid surface model for the area surveyed can be derived or we have at least three or more GPS points with any of the following known information; orthometric height, orthometric height differences or geoid height; then the combined solution can be used. This solution will provide the following quantities in addition to those described in the preceding section,

- (a) the estimated coefficients of the local geoid model
- (b) the derived geoid heights and their standard errors
- (c) the estimated orthometric heights and their standard errors

The geoid surface models currently implemented in the program consist of five polynomial surface fitting models suitable for relatively small and flat areas as described in Khairul(1993), a global model known as OSU91A1F (Rapp et. al., 1991) and a regional model named OXFORD (Featherstone, 1992). Additional model can be added to the program without much difficulties.

2.4 Data Snooping

Ideally, a rigorous data snooping procedure should be implemented in the program. One such procedure which is commonly adopted by many adjustment programs on the market is described by Pope (1975) which is based on the tau-statistic. Since L3D-HEIGHT is developed primarily to handle large networks where the limit on the maximum number of stations is dependent directly on the storage and memory capability of the computer, such a procedure could not be feasibly implemented in this program. The primary reason is the difficulties in storing and handling a large normal matrix which must be used repeatedly in the above mentioned data snooping procedure. As a compromise, a limited data snooping procedure could only be performed by analysing the following quantities;

- (a) the estimated residuals \hat{v} for each observation,
- (b) the value of $\frac{\hat{v}}{\sigma_{obs}^2}$ for each observation, and,
- (c) the unit variance σ_0^2

2.5 Precision Analysis

Similar problems, as mention in the preceding section, are also faced if precision analysis is performed on a large number of stations. This is due to the difficulties in storing and handling a very large covariance matrix of the estimated parameters. To overcome this problem, instead of analysing all the stations, a reasonable number of stations selected by the user (maximum of 100 stations) are used in the precision analysis procedure. This approach only requires the storing and handling of a covariance matrix of manageable size. The precision analysis procedure is then performed by analysing the following quantities;

- (a) the covariance matrix of the estimated coordinates,
- (b) the standard deviation of coordinates,
- (c) the standard deviation of coordinate differences, and,
- (d) the error ellipses.

2.6 Program Output

Prompted by the keyword 'OUTPUT FILE', the program creates the result file which is named either by default or by the user. After the successful execution of the least squares solution, the output is written in a fixed arrangement and format according to the printer used. Two printing formats are available relating to an 80 column and 132 column printer. The printing of the output is similar in both, according to the following arrangement,

title of the job.

- (b) summary on the stations and observations used.
- (c) information on the reference ellipsoid(s) used.
- (d) estimated transformation parameters and their standard errors.
- (e) the estimated unit variance.
- (f) the estimated Cartesian and geodetic coordinates of stations.
- (g) standard errors of the Cartesian coordinates of the selected stations.
- (h) standard errors of the geodetic coordinates of the selected stations.
- (i) the estimated coefficients of the local geoid surface model and their standard errors if this model is used.
- (j) the derived geoid and orthometric heights and their standard errors.
- (k) the error ellipses in Cartesian coordinates of the selected stations.
- (l) the error ellipses in geodetic coordinates of the selected stations.
- (m) covariance matrix of the estimated Cartesian coordinates of the selected stations.
- (n) residuals of the observations.

From the keyword 'PRINT', the user has the option of printing the quantities (k) to (n). Quantities (i) and (j) will only be printed if the orthometric height estimation option is used.

2.7 Program Limitations

The program is limited in a number of ways:

- (a) the observation data must be purely geometrical,
- (b) the maximum number of stations the program can handle depends directly on the storage and memory capability of the computer system on which the program is implemented,
- (c) the solution of the datum transformation parameters in the program is suitable only for small rotations, and,
- (d) the models available for computing the geoid height needed for estimating the orthometric heights are restricted to those mention in Section(2.3). However, geoid heights computed externally may also be read directly into the program.

3.0 CONCLUDING REMARKS

GPS has opened up an alternative to the classical method of height determination. This alternative approach, however, requires a new tool that can be used in the post-processing of GPS and terrestrial geodetic data in the determination of 3D coordinates and orthometric height. Currently, software available in the market only offers post-processing of 3D coordinates only while the orthometric height are determined separately in another computing environment. Program L3D-HEIGHT dispense with such arrangement and has the capability of giving the 3D coordinates and orthometric height simultaneously. At the moment, L3D-HEIGHT can only be installed on a mini and mainframe computer. A PC version of the program is being planned.

Acknowledgments

This project was jointly funded by the Department of Surveying, University of Newcastle upon Tyne and the Ordnance Survey of Great Britain. Study leaves was granted by the Universiti Teknologi Malaysia. Close supervision of the program development by Prof. Paul A Cross is greatly appreciated.

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