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PSEUDO RANDOMIZED SEARCH STRATEGY (PRSS*) OF THE AMBIGUITY FUNCTION MAPPING

Azmi Hassan
Center for Geodetic & Geodynamic Studies
Faculty of Geoinformation Science & Engineering
Universiti Teknologi Malaysia

Abstract

A technique has been developed for reliably resolving the GPS carrier phase ambiguity over a medium length baseline. A combination between three methods of optimization, global random search and ambiguity function mapping produced an efficient search algorithm what the author called the Pseudo Randomized Search Strategy (PRSS). The PRSS is a adaptive search technique that can learned high performance knowledge structures in reactive environments that provide information in the form of a objective function. What makes this search is efficient is that the objective function, where all the GPS measurement resides are evaluated periodically to guide the search to a global optimum. Numerical results shows that, in all the test cases, no more than 5% search of the total search space was conducted to determine the correct set of ambiguities. This result shows that the size of the search window does not play an important role in determining the efficiency of the search and therefore suitable for the on-the-fly ambiguity resolution of a single frequency C/A code receiver.

1.0 INTRODUCTION

The Ambiguity Function Mapping (AFM) method was first introduced by Counselman and Gourevitch (1981) in their paper title Miniature Interferometer Terminals for Earth Surveying:Ambiguity and Multipath with the Global Positioning System. The roots of the AFM is believed to be from the VLBI techniques. Remondi (1984) was first to use this method extensively for GPS static positioning and also for pseudo-kinematic positioning (Remondi, 1989). Mader (1990, 1992) also used the AFM for rapid static and kinematic GPS positioning. The most recent used of this method is by Han (1996) where some improvement on the computation time was gained particularly on the grid step size used. But beyond this, the AFM method never gained any ground.

Theoretically, as proven by Lachapelle, Cannon and Erickson (1992), the AFM is equivalent to the Least Squares Search method which is widely used in other search techniques, such as, Fast Ambiguity Resolution Approach (Frei and Beutler, 1990), Cholesky Decomposition method (Landau and Euler, 1992), Least Squares Search (Hatch, 1990) and most recently the Least Squares Ambiguity Decorrelation Adjustment (Teunissen, 1994). Most of the above techniques has been incorporated in commercial GPS processing softwares.

One of the main high point of the AFM compared to other technique is that it is immune to cycle slips. But still most researcher shy away from the AFM. In fact the AFM is the first on-the-fly ambiguity resolution introduced. The main reason of the 'unpopularity' of the AFM is largely due to the computation burden. In order to reduce the computation burden of the AFM, the most obvious step to take is to reduce the mathematical operation needed to locate the position that produces the maximum of the ambiguity function. Since AFM works in the position domain, it needs a good initial coordinates of the unknown point in order to establish the search window. Another problematic point is that the grid for step size needs to be determine beforehand. If the step size is too small then the computation will take longer time and if it is too coarse, the correct position has the possibility of being eliminated. Han (1996) reported that using a certain combination of the L_1 and L_2 frequency, the search area should be within $\pm \lambda$ for six satellites and the step size should be less that one tenth of the observable wavelength. This method works best with a dual frequency receivers since it can provide a good initial position estimates by reducing the effects of the ionospheric. But with only L_1 measurements, it needs a lot of measurements that rendered the use of kinematic useless.

This paper will address the combination of optimization, global random search strategy and AFM to produce a highly efficient and robust technique in determining the ambiguity on-the-fly using only L_1 measurements. A brief review on each of the three strategy will be discussed. It will be followed by the combination strategy used to produced a highly efficient search algorithm and finally numerical results is presented.

2.0 AMBIGUITY FUNCTION

The AFM uses a function to determine the maximum value of a certain position. The function used in practice is as follows :

$$A(x_o, y_o, z_o) = \sum_{i=1}^{m} \sum_{j=1}^{n} \cos\left\{2\pi\left[\phi_{obs}^{mn}(x,y,z) - \phi_{com}^{mn}(x_o, y_o, z_o)\right]\right\} \tag{1}$$

where $\phi_{obs}^{mn}(x,y,z)$ is the double difference observed phase at the true position (x,y,z), $\phi_{com}^{mn}(x_o,y_o,z_o)$ is the computed double difference phase at the trial position of (x_o,y_o,z_o). The summation i and j refers to the total number of epoch m and number of satellite n. The function A will give a maximum value when the trial position (x_o,y_o,z_o) is equal to the true position (x,y,z). In the case of number of epoch m=1 and number of satellite n=1, and assume there are no biases or errors, the maximum of A is 1. As the trial position (x_o,y_o,z_o) is varied under a volume of the search space, a pattern of maximum and minimum will be observed at each trial position and the correct position will be identified as a peak and if enough measurements are available, this peak is distinguishable among other relative maximum.

The way the AFM works is that by trial and error under the search area space. For example, if the search space is 1 m x 1 m x 1 m and the step size used for the trial position is 1 cm, for m satellite and n epoch there will a total of $(m * n * 100^3)$ trial positions to be tested. Clearly, for this method to work efficiently, a good initial position is needed so that a small search area can be constructed. If the method of trial and errors are used than a dual frequency measurements have a better computation time compared to single frequency because the availability of a wider search lane.

3.0 SEARCH AND OPTIMIZATION

The optimization theory encompass the quantitative study of optima and methods for finding them. When a optimization process is performed it can be said that we are seeking to improve the performance toward some optimal point or points. The methods used to derive the optimum point are called search techniques.

Basically there are three types of search techniques: calculus-based, enumerative and random. The calculus-based method has been used very extensively and can be categorized into indirect and direct techniques. Indirect techniques seek local extrema by solving the nonlinear set of equations resulting from setting the gradient of the objective function to zero. On the other hand the direct search seek the local optima by hopping on the function and moving in a direction related to the local gradient. The enumerative schemes is a straight forward search method where it begins in a finite search area and the search algorithm starts looking at objective function values at every point in the space. Lastly, the random search method uses probabilistic methodology to guide the search for a optimum point.

The calculus-based method if used together with AFM does not provide any real advantage. The reason is that this method is very local in nature. The optima they seek are the best in the neighborhood of the current point. This means that the GPS phase measurements must have a very low noise or biases. For a single frequency measurements this assumption is highly improbable since it is corrupted with multipath and ionospheric delay errors. Secondly, this method depends heavily on the existence of derivatives (well defined slope values) and this requirement is adding the burden to AFM since computation of derivatives is very expensive. The current AFM uses the enumerative search scheme to conduct it optimization process. Improvement in computation time has been made for example by Remondi (1989), Mader (1992) and Han (1996). The enumerative search works well under dual frequency and good initial trial position but does not work well for single frequency that are corrupted with multipath. This leaves to the choice of using the random search method in the AFM.

There are various methods of random search method that can be used to improve the robustness and efficiency of the AFM. But one method stood up among the rest is the evolutionary algorithms. The main reason the author choose this particular method is that it is based on the collective learning process within a population of individuals, each of which represents a search point in the space of potential solutions. One particular point that interest the author is that the population can be arbitrarily initialized. This means that, if this method is applied to AFM, the initial trial coordinates can be initialized arbitrarily and the search space does not have to be defined beforehand. This algorithm couple by randomization process of *selection*, *mutation* and *recombination* evolves toward better and better regions of the search space. The process of *selection* existed both in calculus-base and enumerative search but they are very deterministic in nature, but the process of *mutation* and *recombination* is exclusive to the evolutionary algorithms. Basically the environment of the evolutionary algorithm induces quality information (in this case the ambiguity function value) about the search points, and the *selection* process favors those trial points of higher ambiguity function value to reproduce more often than those of lower function value. The *recombination* mechanism allows the mixing of the main trial position information while passing it to a new set of trial position. The process of *mutation* introduces innovation into the population.

4.0 EVOLUTIONARY SEARCH ALGORITHM

A skeleton of the evolutionary search algorithm is shown in Figure 1 below.

```
t = 0;
initialize P(t);
evaluate A(x,y,z) at P(t);
while (not termination condition)
{
    t = t + 1;
    select P(t) from P(t-1);
    recombine P(t);
    mutate P(t);
    evaluate A(x,y,z) at P(t);
}
```

Figure 1 : Evolutionary Search Algorithm

During iteration *t*, the algorithm maintains a set of trial positions *P*(*t*) of structures $\{a_1^t, a_2^t, \ldots\ldots, a_N^t\}$ where a_1^t is the trial position 1 under iteration *t* and *N* is the total number of trial position used in each iteration. The size of *N* remains fixed for the duration of the search. Each position a_1^t (in this case (*x,y,z*)) is evaluated by computing *A(x,y,z)* at a_1^t. The value of the ambiguity function provide a measure of fitness of the evaluated structure. When each position in the trial set has been evaluated, a new set of trial positions is formed in three steps. First, the structures in the current iteration are selected to reproduce on the basis of their ambiguity function value. That is, the *selection* algorithm chooses structures for replication by a stochastic procedure that ensures that the expected number of new positions associated with a given structure a_i^t is $A(x,y,z)/\mu(P,t)$, where *A(x,y,z)* is the ambiguity function value at position (*x,y,z*) of the structure a_i^t and $\mu(P,t)$ is the average performance of all structures in that particular set of trial positions. What this mean is that the structures that performed well may be chosen several times for replication and structures that performed poorly may not be chosen at all. Using only this type of mechanism (most cases for other search algorithm), it would cause the best performing structures in the initial set of positions to occupy a larger and larger proportion of the trial sets over time. This is where the process of *recombination* and *mutation* comes into place.

Next the selected structures are combine to form a new set of structures for evaluation using the *recombination* process. This procedure will combine two trial positions, say, a_i^t and a_k^t which contains the trial position x_i^t, y_i^t, z_i^t and x_k^t, y_k^t, z_k^t for set i and k and iteration t respectively to produce a new and better positions of a_i^{t+1} and a_k^{t+1}. This process operates by swapping corresponding segments of a string representation of the position a_i^t and a_k^t.

In generating new structures for testing, the *recombination* process draws only on information present in the structures of the current iteration set. If specific information is missing, then it unable to produce new structures that contain it. The *mutation* process that arbitrarily alters one or more components of a selected structure provides the means for introducing new information into the position set.

5.0 OPTIMIZED AMBIGUITY FUNCTION METHOD

The evolutionary search algorithm as described above are used in the AFM. But instead of using position as the search parameter, the ambiguity integer number of the carrier phase was used as the parameter to be search. The main advantage is that since the search algorithm works on the binary coding (0,1) of the parameter itself, therefore it is easier to work on the ambiguity number than the position itself. Another advantage is that ionospheric correction term can be computed based on the ambiguity number. Lets take an example of a 3 double difference ambiguity of a 4 satellite configuration. Table 1 below shows the coding mechanism used in the search algorithm for set i and k.

Satellite Pair	Ambiguity	Coding Mechanism
SV# 2-7 :i	431 550	0000001110101001011 0
:k	431 425	10000011101010010110
SV# 2-15:i	454 520	00011110111101110110
:k	454 524	01101110111101110110
SV# 2-26:i	155 356	10111011011110100100
:k	155 340	10111011011110100100

Table 1 : Coding Mechanism

Table 1 only shows one set of ambiguity number which can derived one initial position. The three ambiguities are considered one string since they are concatenated together. In this paper a range from 5 - 50 set of ambiguities are used for each iteration and therefore for each run of iteration there will be at least 5 concatenated ambiguities string.

To show an example of how the process of *recombination* is performed, lets take an example for ambiguity of SV 2-7 of set i and k from Table 1: In a one-point recombination that has been implemented in this search, a point is chosen at random (using roulette wheel mechanism) to swap ambiguity i and k to produce a new set for the next iteration.

To make sure that diversity exist and most importantly to prevent a premature convergence of a local optimum, the process of *mutation* in implemented. This process is a random alteration on a particular string of ambiguity where the point chosen will be change from 0 to 1 and vice versa. Figure 2 shows the process of a one bit *mutation* of the SV# 2-7 set i ambiguity.

set i 0000001*1101010010110 0000001*1101010010110

* is the recombination point

set k 1000001*1101010010110 1000001*1101010010110

Figure 1 : Recombination Process

0000001 1 1010100 10110 0000001 0 101010010110

Figure 2 : One Bit Mutation

6.0 RESULTS AND ANALYSIS

To show the validity of this search algorithm, static data of only L_1 measurements was used. Three data sets was used which comprise the short (3 km), medium (12 km) and long (21 km) baseline. Since the main purpose of this research is to validate the search algorithm of the optimized AFM via the ambiguity search suite, therefore the validity here shown is based on the correct set of ambiguity numbers and not the correct position as usually accustomed for the AFM.

The 'true' ambiguities of these measurements was determined by processing all available epoch of measurements using the *ASHTECH*^{TM} *GPPS* processing software. The processing results are as shown in Table 2 below.

Baseline	SV Pair	DDN	Epochs	σ(XYZ)
3 km	2 - 15	454530	495	1.30 cm
12 km	2 - 7	40004	514	0.82 cm
21 km	7 - 14	1906348	488	1.27 cm

Table 2 : True Double Difference Ambiguity

From the value of σ for the position, it can be safely assumed that the ambiguities obtained are the true values. The validation test that the search has reached a global optimum will therefore be based on the above true ambiguities values.

The PRSS algorithm depends on two main parameters which are the probabilities values of ***recombination*** (P_{rec}) and ***mutation*** (P_{mut}). In order to find the best combination of the above two parameters, various test was performed. The primary concern with this research is that to minimize the number of measurements used, that is, to use the minimum number of measurements epoch. In this case measurement of only one epoch was used. The parameter range that are used in the test are as shown in Table 3.

Parameter	Test Range
P_{rec}	0.5 - 0.9
P_{mut}	0.0001 - 0.001

Table 3 : Test Parameters

The maximum value of the ambiguity function has been normalized to 1. The iteration are stopped when the ambiguity function reached a value of greater than 1 and also all ambiguities set values shows the same value.

Example of how PRSS iterates and reaches a possible global optimum is discussed for the 3 km baseline. Starting with the first iteration with the best fitness value of 0.270883 with the ambiguities that are generated randomly. The next best fitness value of 0.271546 belongs to iteration 658. This process will continue until it reaches the maximum 1 or the nearest to 1. Theoretically the value 1 cannot be reached with one epoch of measurement because the presence of systematic errors in the carrier phase measurements. Iteration 6164 should be stopped because the fitness value 0.950352 has reached the maximum value by the fact that the ambiguities shown is 454 533 (the true ambiguity is 454 530). The best iteration for this baseline belongs to 7661 with fitness value of 0.988221. Theoretically this should be the best ambiguities set but since some systematic errors present, this fitness value gives a ambiguity value of 454 529 which is better than ambiguity from iteration 6164. A total of 4.368 seconds computation time is needed to reach the 7661 iterations. Table 4, 5 and 6 shows the summary of the 3, 12 and 21 km baselines results.

Epoch	Set Ambiguities Solved	Iterations	Time (sec)	%Search Space
1	454529(7),454528(1)	7661	4.368	0.73
2	454530(8)	6364	3.121	0.61
3	454530(8)	6364	3.121	0.61
4	454530(8)	6364	3.121	0.61

Table 4 : Summary of the 3 km Baseline Processing

Epoch	Set Ambiguities Solved	Iterations	Time (sec)	%Search Space
1	40021(8)	2464	0.872	3.76
2	40004(6), 7236(2)	2482	0.879	3.79
3	40004(5),40020(3)	2317	0.878	3.54
4	40004(5),40020(3)	2317	0.878	3.54

Table 5 : Summary of the 12 km Baseline Processing

Epoch	Set Ambiguities Solved	Iterations	Time (sec)	%Search Space
1	1906368(8)	4331	1.165	0.21
2	1906394(7),1906392(1)	3497	3.363	0.17
3	1906399(6) 1906398(1) 1905886(1)	5845	2.812	0.28
4	1906397(8)	161	0.155	0.01

Table 6 : Summary of the 21 km Baseline Processing

True to its name, the PRSS algorithm is very random in nature. As explained above, this algorithm depends on two parameters which are the probabilities of *recombination* (P_{rec}) and *mutation* (P_{mut}). Various combination are tested for each baseline and there are no clear cut combination of P_{rec} and P_{mut} was found. For the 21 km baseline, the best range for P_{rec} is between 0.9 through 0.5. P_{rec} less than 0.5 will fail the PRSS algorithm. The probability of P_{mut} has a more random value for various P_{rec} since a different P_{rec} will produce a different P_{mut} range. For example P_{rec} = 0.9 will resulted in the best range for P_{mut} between 0.001 through 0.009. But for P_{rec} = 0.8, the value of P_{mut} = 0.001 will fail the search. Instead the best range is from 0.003 through 0.009.

7.0 CONCLUSION

It has been shown that coupled with a pseudo randomized search strategy and the AFM, it can produce a very efficient and robust search of the correct set of ambiguities for baseline length ranging from 3 km to 21 km. Using only one epoch of L_1 measurement derived through a C/A code single frequency receiver, the ambiguities are resolved under four seconds of computation time.

The most prominent advantage of this coupled method is that it does not need a good initial search space to perform the search. The large search space does not create a problem and this can be seen from the number of iteration needed to find the correct set of ambiguities. This search method is very suitable for data derived from the C/A code single frequency receiver since this data tend to be more noisy.

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