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A Conjugate Gradient Approach to Least Squares Analysis of Cadastral Survey Data

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Abstract

In theory the problem of maintaining the geometrical quality of a digital cadastral database can be solved if measurements are used instead of coordinates as the source data. This approach requires a very efficient least squares approach to solve the large amount of cadastral measurements. Although many least squares programs has been extensively developed, but they are designed specifically to optimize either geodetic or engineering problems. The nature of cadastral networks is unique compared that of the geodetic or engineering networks and hence require a new approach of the least squares computation. An iterative method known as conjugate gradient is being tested for cadastral applications.

1.0 INTRODUCTION

A method universally used for the processing of survey measurements is known as " least squares". It is a rigorous method and the resulting parameters posses two useful properties (Cross 1983) as follows:

(1) they are unbiased: this is best explained by saying that, on average, least squares parameters (e.g. coordinates) are equal to the true value.
(2) they have minimum variance; the variance of a quantity derived from least squares solution is smaller than the variance of the same quantity derived from other linear unbiased parameters.

In addition, the least squares method has many desirable computational advantages: It is simple (unlike, say, least cubes), unique (unlike say semi-graphic method), highly adaptable to modern computers, general and leads to a straight forward assessment of quality.

For that reasons, many governments and private commercial organizations posses highly sophisticated network adjustments and analysis programs, based on the least squares method. They were mainly dedicated to two main type of surveying data, that is either in geodetic computations or in the engineering controls. The TRAV10 Horizontal Network Adjustment program (Schwartz 1978), the HAVAGO Three-Dimensional Adjustment Program (Vincenty 1979), the 3DSUITE

Combined Terrestrial and satellite program (Abdullah 1989) are few examples of the geodetic adjustment computer programs.

The least squares engineering program are designed for monitoring engineering structures and deformation analysis. A few examples of such dedicated programs are the OPTUN: a program system for extended preanalysis and adjustment (Grundig L and J, Bahndorf 1985), and CANDSN: Computer Aided network analysis and adjustement system (Mepham M P 1984) and DETECT (Halim Setan, 1995).

Such programs which were dedicated to geodetic and engineering are not directly applicable to cadastral problems. This paper discuss the least squares approach dedicated to cadastral requirements taking account the uniqueness of cadastral networks.

2.0 UNIQUENESS OF CADASTRAL NETWORKS

The goal of positioning is to determine the location of points of special interest to somebody. It is a fact that points are carrier of information. The basic information common to every point is its position. Other information include cadastral boundary, gravity values, points of specific topographic importance(e.g. hill-tops, saddle points, etc), electric line, building corners, points of geological interest, etc. (Hamilton 1982). In this respect geodetic controls are considered as point position arrays whereas cadastral boundary markers are points information arrays (carry cadastral information beside positions).

The cadastral points are information arrays. The measurements are concerned with boundaries of land parcel. The creation of parcels arised from social and cultural need. The measurements were recorded and adjusted with the creation of those parcels. This is how the measurements were built up in a cadastral system. Thus cadastral data typically involve excessively large amount of data. A relatively small jurisdiction may handle few millions of measurements (Buyong 1992).

The geometry of cadastral parcels give rise to a very sparse system of equation in the least squares computation. The geometry of cadastral parcels are in the form of polygon. Although the number of sides of these polygons vary, the most common shape of parcel is rectangular. The measurements are recorded along these boundary lines.

The coordinate of the parcel corners had been obtained in a piece-meal manner as parcels were created and surveyed. These coordinates which are correct in relation to nearby parcels had served very well the traditional cadastral applications . However, such geometry of the parcels developed in this way were not homogenous and thus limiting its cadastral role in modern land databases.

3.0 BACKGROUND OF CONJUGATE GRADIENT

Conjugate gradient has been known for some time, having been developed independently by E. Steifel and M.R. Hestenes but it received little attention at that time. Lately Reid [Reid 1971] commented that it is difficult to see why this has been so. The method has several very pleasant features especially when it used as an iterative method for solving large and sparse systems of equations.

The method is shown to be capable of solving the least squares problem without having to form normal equation as in the conventional approach. This is the special feature of this technique. It deal directly the observation equations and then solve these equations to get the parameters (Dragomir 1982].

Conjugate gradient has also been tested for a number of application such as finite element in frequency domain for three-dimensional scattering problems(Smith R.D et all 1992]. Another example to quote a few is the test carried out in varying performance of supercomputer [Javier V.E, Reifman J. 1992].

Although the conjugate gradient has been successfully applied in several scientific and engineering applications, its potential has been overlooked by the surveying community. The method was only mentioned in passing (Ashkenazi 1968), and has not been used in cadastral computations. However, this method has a number of mathematical advantages over direct methods such as Cholesky decomposition and is considered highly suitable for processing very large cadastral measurements (Hintz 1994).

4.0 THE ALGORITHM OF THE CONJUGATE GRADIENT

Following the step given in [Dragomir 1982], each cadastral measurements can be expressed in term of coordinate unknown as follows;

$$V = A\,X + L \qquad (1)$$

Where :

A = Coefficient of design matrix
v = Residual
L = Observed-Computed
X = Corrections to Coordinates

The A matrix is the partial derivatives of each observation with respect to the unknown (coordinates and other biases). The L is a vector that contain the difference between the measured value and the computed value from the provisional coordinates.
For the first iteration, since X=0 then equation (1) become V=L .

Let $R = A^T V$

For first iteration V=L, then $R=A^TL$

Let $H = -R$

$$q_1 = \frac{R^T R}{(AH)^T (AH)}$$

$$x = x_0 + q_1 H$$

$$v = v_0 + q_1 (A\,H).$$

The unknown and the residual is updated and continue with the next iteration.

For the second and the rest of iteration the algorithm is as follows:
Using the residuals obtained from the previous iteration and using the A matrix.

$$R = A^T V$$

$$e = \frac{R_1{}^T R_1}{R_0{}^T R_0}$$

$$H_2 = -R + e H_1$$

$$q_2 = \frac{R_1{}^T R_1}{(AH_2)^T (BH_2)}$$

$$x_2 = X_1 + q_2 H_2 \qquad (2)$$

$$V_2 = V_1 + q_2(AH_2) \qquad (3)$$

The unknown (i.e. coordinates in most cases) is updated in equation (2) and the residual in equation (3). The computation is iterated until the value of the updates(unknown and residual) is insignificant.

The above operation consider each measurement as having equal weight of unity . For weighted measurements, the values of A matrix and vector L is premultipled by by weight (W) i.e.

$$A = A * W$$
$$L = L * W$$

and the rest of the computation follows exactly the above algorithm.

5.0 REMARK AND ADVANTAGES OF THE ALGORITHM

The operation in a least squares computation usually involved a sequence of the following steps:-

(i) computing the coefficient matrix (A)
(ii) computing of the into normal equation (N)
(iii) solving normal equation (mainly by Cholesky decomposition method)
(iv) update the unknown and repeat the process until the correction is insignificant.

The mathematical operation of the above steps which is referred to as a traditional method, is longer compared to the conjugate gradient method. Step(ii) and step(iii) are replaced by the conjugate gradient technique.

The conjugate gradient method directly manipulate the coefficient matrix (A) to get the unknowns. This is a unique way to solve least squares problem.

The manner in which the weight matrix is treated is slightly different between the two approaches. The weight is introduced in the A matrix and L vector. The traditional least squares introduce the weight during step (ii) i.e. the process of transforming matrix (A) into normal equation.

The iteration also takes a different form. The values of matrix A is updated in each iteration in the traditional approach until the correction to coordinates is insignificant. On the other hand the conjugate gradient method carries iteration without having to update the A matrix provided the provisional values do not differ significantly from the true value.

6.0 SOME INITIAL WORKS AND RESULTS

The method of conjugate gradient have been implemented in a computer program using FORTRAN Language. The program takes cadastral measurements in the form of distances, bearings angles and coordinates. These measurements are accompanied with their respective precision. The program compute design matrix for each measurements and solve for the update to the provisional coordinates and residuals.

A simulated network consisted of 550 points with 1000 distances and 700 bearings and two known points was tested. The time taken was 15 seconds using a 486DX personal computer. The results which were computed based on the standard programming algorithm of the conjugate gradient and is expected to improve significantly when the program is optimized.

The speed of the method depends on the quality of the provisional coordinates and the geometrical strength of the network. This requirements favors the cadastral survey. Although the above result looks insignificant, but for very large system of equations, the iterative conjugate gradient has the advantage.

7.0 CONCLUSION

The problem of non-homogenous cadastral coordinates and the problem of updating of the cadastral can be solve by using measurements instead of their graphical maps. This approach require a powerful and efficient measurement processing algorithm. The conjugate gradient is a promising solution for this problem. The method has a number of mathematical advantages and specially suitable for solving such cadastral problems. Work is underway to materialize this method.

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