MESHFREE FORMULATION FOR BUCKLING OF COMPOSITE BEAM WITH SLIP

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I begin with the name of Allah the most merciful and the most kind, peace and blessing be upon beloved Prophet (S.A.W) All praise is for Allah.

This thesis is dedicated to my lovely parent. Also to my siblings and my supportive friends.

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ABSTRACT

This study concerns with the formulation of Meshfree methods for the buckling analysis of partial interaction of composite beam. Meshfree formulations of Polynomial Interpolation Method (PIM) and Radial Polynomial Interpolation Method (RPIM) have been developed where multi-quadric radial basis functions have been used as the interpolation functions. Comparison of the results for the buckling problem against analytical and existing numerical works showed good agreement thus validates the formulation. Parametric studies have been conducted to study the effect of the method's parameters to the solution accuracy. The effects of support domain and gauss cell were extensively studied for the PIM method. For the RPIM method, parameters used for the parametric study were the support domain, gauss cell, and shape parameter. Once the optimum values of the parameters were obtained, convergence study was carried out to determine the rate of convergence of the Meshfree. It was found that the use of gauss cell proportional to the number of point gives the best convergence rate for the solution of the buckling load. Parametric studies for the analysis of composite beam interaction and geometric ratio have been also conducted for both finite element and Meshfree methods. It was be observed that partial interaction between top and bottom section occurs for some specific connector stiffness values. The effect of geometric ratio, on the other hand, showed that the partial interaction regime for the composite beam reduced as the geometric ratio increased. For all the parametric study conducted, result obtained from PIM and RPIM showed good agreement with converged finite element solution thus indicating the suitability of the Meshfree methods in the study of buckling analysis of composite beam with partial interactions.

ABSTRAK

Kajian ini melibatkan formulasi kaedah Meshfree bagi analisis lengkokan interaksi separa rasuk komposit. Formulasi kaedah Meshfree PIM (Point Interpolation Method) dan RPIM (Radial Point Interpolation Method) telah dibangunkan yakni fungsi asas jejarian kuadratik berbilang telah digunakan sebagai fungsi interpolasi. Perbandingan keputusan masalah lengkokan terhadap kerja-kerja analitikal dan kerja-kerja berangka yang sedia ada menunjukkan persetujuan yang baik dalam mengesahkan formulasi yang dijalankan. Kajian parametrik dijalankan untuk mengkaji kesan parameter kaedah yang digunakan terhadap ketepatan keputusan. Kesan domain sokongan dan sel gauss dikaji secara meluas untuk kaedah PIM. Untuk kaedah RPIM, parameter yang digunakan untuk kajian parametrik ialah domain sokongan, sel gauss, dan parameter bentuk. Kajian penumpuan mendapati bahawa penggunaan sel gauss berkadar dengan bilangan titik memberikan kadar penumpuan terbaik untuk penyelesaian beban lengkokan. Kajian parametrik analisis interaksi rasuk komposit dan nisbah geometri juga dijalankan bagi kedua-dua kaedah unsur terhingga dan Meshfree. Daripada kajian tersebut, diperhatikan bahawa interaksi separa antara bahagian atas dengan bawah berlaku untuk beberapa nilai tertentu penyambung kekakuan. Kesan nisbah geometri menunjukkan bahawa rejim interaksi separa bagi rasuk komposit berkurang sekiranya nisbah geometri meningkat. Bagi semua kajian parametrik yang dijalankan, keputusan daripada PIM dan RPIM menunjukkan persetujuan yang baik dengan penyelesaian unsur terhingga tertumpu iaitu menunjukkan kesesuaian kaedah Meshfree dalam kajian lengkokan analisis rasuk komposit dengan interaksi separa.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOLS :

a - Concrete Coefficient Vector

a_o - Concrete Coefficient Vector In Radial PIM

A_c - Cross-Sectional Area of Concrete

a - Concrete Coefficient

A_s - Cross-Sectional Area of Steel

b - Steel Coefficient

b - Steel Coefficient Vector

b_o - Steel Coefficient Vector In Radial PIM

c - Composite Beam Coefficient

c - Composite Beam Coefficient Vector

c_o - Composite Beam Coefficient Vector In Radial PIM

d - Deformation Vector

d_c - Nodal Spacing

d_s - Size of Support Domain

D - Constants Matrixes

D_{s1} - Constants Matrixes

D_{s2} - Constants Matrixes

E_c - Young’s Modulus of Concrete

E_s - Young’s Modulus of Steel

F - Internal Force Vector

F - Concentrated Load Vector

G_b - Bending Moment Matrix of Composite Beam

G_c - Axial Moment Matrix of Concrete

G_s - Axial Moment Matrix of Steel

H - Distance Between The Centroidsof Concrete And Steel

h_c - Distance From The Centroidof Concrete To Interlayer Surface

h_s - Distance From The Centroidof Steel To Interlayer Surface

I_c - Second Moment of Area of Concrete

I_s - Second Moment of Area of Steel

J - Jacobian Matrix

k - Section Stiffness

K_{ij} - Local Stiffness Matrix

K_{G} - Geometric Stiffness Matrix

k_s - Stiffness of The Interlayer Surface

k_{con} - Stiffness of Shear Connectors

L - Differential Operators Matrixes

L_{s1} - Differential Operators Matrixes

L_{s2} - Differential Operators Matrixes

m - Number of Axial Nodes In Steel

m - Polynomial Basis In Radial PIM

M - Bending Moment of The Commposite Beam

M_c - Bending Moment In Concrete

M_s - Bending Moment In Steel

n - Number of Axial Nodes In Concrete

N_c - Axial Force of Concrete

N_s - Axial Force of Steel

n_r - Number of Rows of Shear Connectors

nc - Number of Background Cells

ng - Number ofGauss Points

$p_i(x)$ - Monomials of x

Q_s - Shear Force of Shear Connector

Q - Shear Force

q - Vertical Distributed Load
q - Dimensionless Shape Parameters
R - Radial Basis Function of x
$\mathrm{R}_{\mathrm{load}}$ - Load Vector
S - Slip Force
s - Slip
u - Displacements Vector
U - Nodal Displacements Vector
U_b - Deflection And Slope Displacements Vector
U_c - Axial Displacement Vector of Concrete
U_s - Axial Displacement Vector of Steel
u_c - Concrete Axial Displacement
u_s - Steel Axial Displacement
u_{cs} - Relative Displacement
V_c - Shear Force In Concrete
V_s - Shear Force In Steel
W_1 - First Weight Function
W_2 - Second Weight Function
W_3 - Third Weight Function
w - Deflection
$\widehat{w}$ - Gauss Weighting Factor
y - Displacement Vector
z - Number of Bending Nodes In Composite Beam
ϕ - Curvature of The Composite Beam
ε_c - Axial Strain of Concrete
ε_s - Axial Strain of Steel
∂ - Differential Operator
∂_s - Differential Operator
δ - Variation
α_s - Dimensionless Size of The Support Domain

α_c - Dimensionless Shape Parameters

θ - Slope

Ω - Global Problem Domain

Ω_k - Domain of$K$th Background Cell

Φ - Matrix of Shape Functions

φ_b - Shape Function of The Bending Displacements of Composite Beam

φ_c - Shape Function of The Axial Displacements of Concrete

φ_s - Shape Function of The Axial Displacements of Steel

$\mathcal{M}$ - Quadratic Lagrange Shape Function

$\mathcal{N}$ - Hermitian Shape Function

ABBREVIATIONS :

FDM - Finite Difference Method

FEM - Finite Element Method

MFree - Meshfree

PIM - Point Interpolation Method

RPIM - Radial Point Interpolation Function

EFG - Element Free Galerkin

RBF - Radial Basis Functions

MQ - Multi Quadrics

TPS - Thin Plate Spline

SS - Simple-Simple

CC - Clamped-Clamped

CF - Clamped-Free

CS - Clamped-Simple

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Composite structures are commonly produced in structural engineering when two different materials joined together to act as a unit. By joining the materials together, the resulting structure is stronger than the sum of its parts. A composite beam is produced when the concrete slab is connected to the lower steel beam member so that the beam responds to the loads not only as a unit but in a way that it benefits from both constituent materials. For composite beams, the concrete is assumed to take most or all of the compression while the steel takes all the tension.

Despite its higher strength and the better performance, the behaviour of composite beam is more difficult to study as compared to its bare beam counterpart. The difficulty rises from the fact that the interaction between both constituent materials should be considered as partially interact. This is represented by the occurrence of slip which is the relative longitudinal movement between the materials at the interface. Another relative movement at the interface is the uplift. However, latter movement has been shown to be insignificant in a conventional composite beams. Such consideration is ought to be considered if actual behavior is required to be analyzed. This behavior is called as partial interaction at the interface.

Studies related to the behaviour of composite beams started to take shape in 1950's. At the early stage, most theoretical studies of composite beam involved with the derivation and the solution of the differential equation of the problem. The most prominent work was conducted by Newmark et al, (1951). As the problem getting more sophisticated, the resulting differential equation became of higher order and coupled leading to the use of numerical solution. The first work which employed numerical technique was carried by Adekola in 1968 where the work solved the problem of both the slip and uplift by using the Finite Difference Method (FDM).

Although the use of FDM were later continued by Roberts (1985), the robustness of Finite Element Method (FEM) had attracted the interest of composite beam researchers. FEM is well known to engineering community as an efficient numerical technique due to its flexibility especially in the modeling of complex geometry. FEM has been successful in the study of composite beams where it has been applied in various problems such as beam bending, stability and vibration as well as in both linear and nonlinear cases (Porco et al, 1994; Faella et. al, 2000; Salari and Spacone, 2001; Ayoub and Filippou, 2000; Ayoub, 2001; Ranzi et. al, 2006; Silva and Sousa, 2009).

A new numerical technique known as Meshfree method (Meshfree) has been introduced (Liu and Gu, 1999). As opposed to FEM, which is using a set of mesh, Meshfree uses a set of nodes for the discretization of the problem domain. This omits the need for predefined shape functions allowing a higher order polynomial to be used in interpolating the dependent variable. According to Liu and Gu (2005), Meshfree method is a promising numerical technique in providing better approximate solutions and even more convenient for a complex system.

1.2 BUCKLING OF COMPOSITE BEAMS

Buckling phenomenon can exist in a slender structure when it is subjected to axial loading. It is classified as eigenvalue problem as its solution involved only with the homogenous part of the differential equation. For composite beams problem, this chapter has been the major interest of study because it is important to have an established mathematical model to predict the value of the buckling load for the structure. So far, works done by Wu, et.al (2007), Xu and Wu (2007, 2008), Emam and Nayfeh (2009) and Xu and Chen (2012) focused in the provision of analytical solution to the buckling problem of composite beams.

1.3 RESEARCH BACKGROUND AND RATIONALE

Despite the potential of Meshfree, it is still at its infancy (Liu and Gu, 2006). So far, its formulation has been applied to composite beam buckling problems. More study is therefore needed so as to establish the efficiency and robustness of the method. This can be done by deriving its formulation for various engineering problems then the results are validated and verified against analytical solution or established numerical method such as the FEM. So far, there is no Meshfree formulation for the buckling problem of composite beams.

The buckling analysis of composite beams is relatively less studied as compared to its static analysis, both in FEM and Meshfree. Whilst recent works have established the analytical solutions to the problems but it is important to extend the work to involve numerical methods. This is because, whilst analytical solution can be applied to simple configuration and consideration, it is usually unavailable for complex problem.

Therefore, it is the main interest of this study to formulate and validate as well as to verify FEM and Meshfree methods for the buckling problem of composite beams. The work is conducted so as to assist in the establishment of Meshfree as a robust numerical method and to promote its use in structural engineering fields. It is also conducted with the intention to extend the consideration of buckling problem of composite beam using numerical analysis.

1.4 PURPOSE AND OBJECTIVES OF THE STUDY

The purpose of the study is to provide FEM and Meshfree formulations that can predict the occurrence of buckling phenomenon of composite beam by predicting its buckling loads.

The objectives of this study are summarized as follows:

a) To establish one-dimensional formulation and algorithm for FEM and Meshfree focusing of polynomial PIM and RPIM for the buckling of composite beam with a slip
b) To validate the obtained results with the results of existing analytical models.
c) To verify the performance of Meshfree by conducting parametric studies on various numerical parameters by comparing with a converged FEM result.

1.5 SCOPE OF THE STUDY

The scopes and limitations of the presented formulation in this study are as follows:

1. Materials are assumed to be linear elastic.
2. The deflection of the composite beam is small.
3. The shear forces Q_s of the shear connector are proportional to the relative slip u_s.
4. The shear deformations of concrete-steel composite beam are neglected.
5. The sections of composite beam are assumed equal curvatures thus no transverse separation of elements occurs.
6. The plane sections remained plane after deformation for both concrete and steel sections.
7. The problem is analyzed as 1-dimensional element.
8. The concrete element is assumed as to be uncracked.

1.6 OUTLINE OF THESIS

This thesis comprises five chapters. Chapter 1 is an introductory chapter where it details the present state of composite beam and its technique of solutions as well as the rationale for the present work. Limitation of the work is given at the end of the chapter.

Chapter 2 provides an extensive review on composite beam including its behaviour and types. Review of existing analytical model as well FEM of composite beam is given thoroughly. A review on Meshfree method and its application on various engineering problems are also explained.

Chapter 3 provides the new formulation of both FEM and Meshfree of composite beam with slip and for buckling problem. At the end of the chapter, the obtained results are compared with the analytical solution derived by Newmark's model for validation purposes.

Chapter 4 details out the parametric studies on polynomial PIM and RPIM. The intention is to find the optimal values of several parameters used and its effects on the results. The convergence rates are plotted, compared and analyzed for verification purposes.

Lastly, conclusions of the study are detailed and recommendations for future work are given in Chapter 5. In the Appendix, the Matlab source codes of polynomial PIM and RPIM derived in the study are given for references.

REFERENCES

Adekola, A. O. (1968). Partial interaction between elastically connected elements of a composite beam. *International Journal of Solids and Structures, Vol. 4, 1125-1135.*

Airil, Y. (2007). The Development of Precast Cold-formed Composite Beams. Ph.D Thesis, University of London.

Ayoub AS. (2001). A two-field mixed variational principle for partially connected composite beams. *Finite Element in Analysis and Design*, 37:929–59.

Ayoub, A. and Filippou, F. C. (2000). Mixed formulation of nonlinear steel-concrete composite beam element. *Journal of Structural Engineering, Vol. 126, 371-381.*

C Amadio, M Fragiacomo (2002) Effective width evaluation for steel–concrete composite beams, *Journal of Constructional Steel Research* 58 (3), 373-388

Da Silva, Amilton R.' Sousa Jr., João Batista M. (2009) A family of interface elements for the analysis of composite beams with interlayerslip, *Finite Elements in Analysis & Design*, Volume 45, issue 5, p. 305-314.

Dolbow, J. and Belytscho, T. (1999). Numerical integration of the Galerkin weak form in meshfree methods. . *Computational Methods*. 23, 219-230.

Faella C., Piluso V., and Rizzano G., 2000, "Structural Steel Semi Rigid Connections - Theory, Design and Software".

G. Ranzi, F. Gara, G. Leoni, M.A. Bradford, (2006). Analysis of composite beams with partial shear interaction using available modelling techniques: a comparative study, *Comput. Struct*. 84 930–941.

Girhammar, U. A. and Gopu, V. K. A. (1993). Composite beam-columns with interlayer slip-exact analysis. *Journal of Structural Engineering. Vol. 119. No. 4. pp 1265-1282.*

Girhammar, U. A. and Pan, H. D. (2007). Exact static analysis of partially composite beams and beam-columns. *International Journal of Mechanical Sciences*. Vol. 49. pp 239-255.

Gu, Y. T. and Liu, G. R. (2001). A meshless local Petrov-Galerkin (MLPG) method for free and forced vibration analyses for solids. *Computational Mechanics*, 27, 188-198.

Liu G.R (2003), Mesh Free Methods: Moving Beyond the Finite Element Method. CRC press, Boca Raton, USA.

Liu, G. R., & Gu, Y. T. (2005). *An Introduction to Meshfree Methods and Their Programming.* Netherland: Springer.

Liu, G. R., & Gu, Y. T. (December 1999). A Point Interpolation Method. *Proceeding 4th Asia-Pasific Conference on Computational Mechanics*, (pp. 1009-1014). Singapore.

MohdYassin,A.Y., Vellasco,P.C.G.S., Nethercot, D. A (2005)., Finite Element Modelling of Profiled Composite Beams, *1st International Conference on Advances in Experimental Structural Engineering (AESE 05),* Nagoya, Japan,

Nethercot, D. A. (2003). *Composite construction,* Spon Press.

Newmark, N. M., Siess, C.P. and Viest, I. M. (1951). Tests and analysis of composite beams with incomplete interaction. *Proceedings, Society for Experimental Stress Analysis, Vol. 9. pp.75-92 (84).*

Porco, G., Spadea, G. and Zinno, R. (1994). Finite Element analysis and parametric study of steel-concrete composite beams. *Journal of Cement and Concrete Composites, 16, 261-272.*

Queiroz FD. (2007). Finite element analysis of composite beams: The influence of different levels of shear connection. Ph.D. thesis. London: Imperial College;.

R.Q. Xu, Y.F. Wu, Static, (2007). dynamic, and buckling analysis of partial interaction composite members using Timoshenko's beam theory, *International Journal of Mechanical Sciences*

Ranzi G, Bradford MA. Analytical solutions for the time-dependent behaviour of composite beams with partial interaction. Int J Solids Struct 2006;43:3770–93.

Roberts, T. M., (1985). Finite difference analysis of composite beams with partial interaction. *Computers and Structure,* Vol.21,No.3pp.469-473

Rongqiao Xu and Dequan Chen (2012), Variational Principles of Partial-Interaction Composite Beams *American Society of Civil Engineers*

Rongqiao Xu, Yu-FeiWub (2008), Free vibration and buckling of composite beams with interlayer slip by two-dimensional theory, *Journal of Sound and Vibration* 313 (2008) 875–890

S. Schnabl, M. Saje, G. Turk, I. Planinc (2007), Analytical solution of two-layer beam taking into account interlayer slip and shear deformation, *Journal of Structural Engineering* 133 886–894.

Salari, R. and Spacone, E. (2001). Finite Element formulations of one-dimensional elements with bond-slip. *Journal of Engineering Structures, 23, 815-826.*

Samir A. Emam and Ali H. Nayfeh (2009). Post buckling and free vibrations of composite beams. *Journal of Composite Structures, 88, 636-642.*

Thevendran, V., Chen, S., Shanmugam, N. E., and Liew, J.Y. R.; (1999), Nonlinear analysis of steel-concrete composite beams curved in plan, *Finite Elements in Analysis and Design,*Vol. 32, pp. 125-139.

Wang JG and Liu GR (2002a) A point interpolation meshless method based on radial basis functions. *Int. J. Numer. Meth. Eng.* 54 (11): 1623-1648.

Wang JG and Liu GR (2002c), On the optimal shape parameters of radial basis functions used for 2-D meshless methods .*Computer Method Appl. M.* 191 (23-24): 2611-2630.

Wang JG and Liu GR(2000), Radial point interpolation method for elastoplastic problems. *Proc. of the 1st Int. Conf. On Structural Stability and Dynamics,* Dec. 7-9, 2000, Taipei, Taiwan, 703-708,2000.

Wang JG, Liu GR and Wu YG (2001b), A point interpolation method for simulating dissipation process of consolidation. *Computer Method Appl. M.* 190 (45): 5907-5922

Xu, R. Q. and D. Q. Chen (2012). "Variational Principles of Partial-Interaction Composite Beams." Journal of Engineering Mechanics-Asce **138**(5): 542-551.

Xu, R. and Chen, D. (2012). ”Variational Principles of Partial-Interaction Composite Beams.” *J. Eng. Mech.*, 138(5), 542–551.

Yu-Fei Wu, RongqiaoXu, Weiqiu Chen (2007), Free vibrations of the partial-interaction composite members with axial force, *Journal of Sound and Vibration* 299 (2007):1074–1093