DETERMINATION OF WATER QUALITY IN UNIFORM CHANNEL USING HOMOTOPY ANALYSIS METHOD

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Especially to my beloved father and mother who inspired me a lot throughout the journey of my life;

Baharom Bin Yahaya

&

Siti Normadiah Bt Mohd. Yassin

And special dedication to my beloved husband and daughter

Mohamed Izwan Bin Isa

&

Nur Izara Khalisya Bt Mohamed Izwan

Both of you are the precious gift ever. Thanks for everything

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ABSTRACT

In recent years, the interest in preserving the quality and quantity of water is gaining widespread attention worldwide with current increasing trends in population growth and sosio-economic development. So, it is important for us to check the water quality by knowing the dispersion of pollutant in water. The Homotopy analysis method (HAM) is used for solving one-dimensional convection-diffusion equation with variable coefficients arising in the mathematical modeling of dispersion of pollutants in water. By solving the convection-diffusion equation, the Homotopy analysis method (HAM) is considered for assessment of the chemical oxygen demand (COD) in a river. The HAM is used to compute the concentration of the pollutant for variable inputs. From the result obtained, we can see that the COD concentration along a uniform channel is decreasing. Therefore, the COD concentration can be used in order to check the water quality along the river.

ABSTRAK

Sejak akhir-akhir ini, perhatian terhadap kepentingan dalam pemuliharaan kualiti dan kuantiti air semakin perhatian meluas di seluruh dunia dengan pertumbuhan penduduk dan pembangunan sosio-ekonomi yang semakin meningkat. Jadi, adalah penting bagi kita untuk memeriksa kualiti air dengan mengetahui penyebaran bahan pencemar di dalam air. Kaedah analisis homotopi (HAM) telah digunakan untuk menyelesaikan persamaan satu dimensi perolakan-resapan dengan pekali pembolehubah yang diterbitkan dalam model matematik penyebaran pencemaran di dalam air. Dengan menyelesaikan persamaan perolakan-resapan, kaedah analisis homotopi (HAM) akan digunakan untuk menilaian permintaan oksigen kimia (COD) di dalam sungai. HAM ini digunakan untuk mengira kepekatan pencemar bagi input berubah-ubah. Berdasarkan keputusan yang diterima ini, didapati bahawa kepekatan COD di sepanjang saluran seragam adalah berkurang. Oleh itu, kepekatan COD akan digunakan dalam menilai kualiti air sepanjang sungai.

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LIST OF SYMBOLS

HAM	Homotopy Analysis Method
COD	Chemical Oxygen Demand
DO	Dissolve Oxygen
FDM	Finite Difference Method
CDE	Convection –Diffusion Method
EM	Effective Microorganism
EMA	Effective Microorganism Agent
CSTR	Continually stirred tank reactor
PFR	Plug flow reactor
BOD	Biochemical Oxygen Demand
$u_0(r,t)$	initial approximation
L	auxiliary linear operator
$H(r,t)$	auxiliary function
$C(x)$	concentration of COD
U	flow velocity in x direction
D_x	diffusivity
Q	increasing rate substrate concentration due to a source
N	nonlinear operator
$u(t)$	unknown function
t	independent variable
Q	embedding parameter

CHAPTER 1

INTRODUCTION

1.1   Background of the study

Water pollution is the contamination of water bodies (e.g. lakes, rivers, oceans, aquifers and groundwater). Water pollution occurs when pollutants are directly or indirectly discharged into water bodies without adequate treatment to remove harmful compounds. Water pollution affects plants and organisms living in these bodies of water.

Water pollution from human activities, either industrial or domestic, is a major problem in many countries. Every year, approximately 25 million persons die as a result of water pollution. Developing models to enable us to understand how to control and predict water quality is of crucial interest. When assessing the quality of water in a river, there are many factors to be considered such as the level of dissolved oxygen (DO), the presence of nitrates, chlorides, phosphates and heavy metals, the level of suspended

solids, environmental hormones, chemical oxygen demand (COD) and the presence of bacteria.

The dispersion of pollutants in the river can be known by the assessment of the chemical oxygen demand (COD) concentration in a river. This research devotes a mathematical model for solving the dispersion of pollutants in a river. The Homotopy analysis method for assessment of the chemical oxygen demand (COD) concentration in a river is considered. Pochai *et al.* (2006) addressed a mathematical model of water pollution using the finite element method.

Pochai (2009) implemented the finite difference method (FDM) to the hydrodynamic model with constant coefficients in the uniform reservoir and stream. This model requires the calculation of the substance dispersion given water velocity in the channel. In 1992, Liao developed the basic ideas of the homotopy in topology to propose a general analytical method for nonlinear problems, namely Homotopy analysis method.

1.2   Statement of problem

Nowadays, people are very concern about water quality and quantity which to meet the objective of sustainable water supply and prevent potential deterioration. In recent years, the large amounts of polluted water are discharged into rivers and causing serious problem in the water quality.

Water pollution assessment problems arise frequently in environmental science. There are many parameters to be considered when assessing the quality of water in a river. One of the parameter that can be considered is the level of chemical oxygen demand (COD). One dimensional convection-diffusion equation with variable coefficients arising in mathematical model for the dispersion of pollutants in a river is used. The Homotopy analysis method (HAM) for assessment of the chemical oxygen demand (COD) concentration in a river is considered. By knowing the concentration of COD, it is then used to optimize pollution treatment cost.

1.3   Objective of research

The objectives of this study is;

   i.   To solve convection-diffusion equation using HAM.

  ii.   To determine the concentration of chemical oxygen demand (COD) in a river by using Homotopy analysis method (HAM).

1.4   Significance of the study

The significance of the study is to determine the dispersion of pollutants in a river. By knowing the concentration of the chemical oxygen demand (COD) we can use it to compute the concentration of pollutant for variable inputs using Homotopy analysis method (HAM). Hence, through this research, the model can be used as a tool to look well into the dispersion of pollutants in river by the calculation of the substance dispersion given water velocity in the channel.

1.5   Scope of the project

The scope of this project is to solve one dimensional mathematical equation of the dispersion of COD in a uniform channel by using the Convection-diffusion equation (CDE). The model equations consist of a system of the non linear differential equations. The Homotopy analysis method (HAM) is used for solving the one-dimensional Convection-diffusion equation with variable coefficients arising in the mathematical modeling of dispersion of pollutants in water. HAM is used to solve and obtain numerical COD concentration and also the graph.

1.6   Project overview

This study contains five chapters started with introductory chapter. First chapter describe briefly about the research background, problem statements, objectives, scope and significance of this study.

Literature review of this study will be considered in the next chapter 2. This chapter explained briefly about previous study on mathematical modeling of river pollution and previous approximate analytic technique for solving nonlinear problems until they found Homotopy Analysis Method (HAM).

Then, the chapter three will discuss methodologies and procedure in completing this study. Next, results and discussion of data will be considered in the fourth chapter. Finally, some conclusion of the study will be discussed in chapter five.

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