AN INTELLIGENT CONSTRUCTION OF CONTROL LYAPUNOV FUNCTION

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I wish to dedicate this thesis to my beloved father, mother, father and mother in laws, wife, kids and friends who have encouraged, guided and inspired me throughout my journey of education.

To John

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ABSTRACT

Lyapunov stability theorem has made an important contribution in the world of control engineering. The Lyapunov theory provides a connection between the presence of a control Lyapunov function and the stability of a control system. This research presents a novel approach to construct control Lyapunov function named Intelligent Control Lyapunov Function (ICLF) to test the stability of single input single output (SISO) nonlinear dynamic system using Artificial Neural Network (ANN). The proposed approach is considered the error signal in SISO nonlinear dynamic system as an input signal to Intelligent Control Lyapunov Function (ANN model) to determine the stability of the control system by training the ANN model using target signal. However, the proposed target function is satisfied the stability conditions of second method of the Lyapunov stability theory [Control Lyapunov Function V(x)]. The novel trained model (ICLF) shows the unique characteristics of Artificial Neural Networks in adapting itself to nonlinear dynamic system changes. The Intelligent Control Lyapunov Function has proven that the existence of smooth uniform control Lyapunov function is a condition for satisfying the robustness and stability of a nonlinear dynamic system.

ABSTRAK

Kestabilan teori Lyapunov telah memberikan sumbangan yang amat penting dalam dunia kejuruteraan kawalan. Teori Lyapunov telah mengait rapat hubungan antara kehadiran persamaan kawalan Lyapunov dan kestabilan sistem kawalan. Kajian ini membentangkan satu pendekatan yang baru dalam membina persamaan kawalan Lyapunov yang dinamakan sebagai Persamaan Kawalan Lyapunov Pintar (ICLF) untuk menguji kestabilan output tunggal input tunggal sistem dinamik SISO tidak linear dengan menggunakan Rangkaian Neural Buatan (ANN). Pedekatan yang disyorkan dengan menggunakan isyarat kesilapan dalam sistem dinamik SISP tidak linear sebagai isyarat masukan kepada Persamaan Kawalan Lyapunov Pintar (Model ANN) untuk menentukan kestabilan sistem kawalan dengan melatih model ANN dengan menggunakan isyarat sasaran. Walau bagaimanapun, persamaan sasaran kaedah kedua yang dicadangkan itu telah memenuhi syarat-syarat kestabilan bagi teori kestabilan Lyapunov. Model terlatih novel (ICLF) telah menunjukkan ciri-ciri yang unik untuk Rangkaian Neural Buatan dalam menyesuaikan diri dengan perubahan sistem dinamik yang tidak linear. Persamaan Kawalan Lyapunov Pintar telah membuktikan bahawa kewujudan lancar kawalan seragam Persamaan Lyapunov adalah signal untuk memuaskan kemantapan dan kestabilan sistem dinamik tidak linear.

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LIST OF ABBREVIATIONS

CLF	-	Control Lyapunov Function
H-I layers weights	-	Hidden-Input layers weights
ICLF	-	Intelligent Control Lyapunov Function
LM	-	Levenberg-Marquardt algorithm
MIMO	-	Multi-Input Multi-Output system
MSE	-	Mean Square Error
NDS	-	Nonlinear Dynamic System
O-H layers weights	-	Output-Hidden layers weights
PID	-	Proportional Integral Derivative
SISO	-	Single-Input Single-Output system

LIST OF SYMBOLS

d	-	Damping constant.
e_1	-	Error1
e_2	-	Error2
e_t	-	Total Error
E	-	Error signal of control system
g_v	-	Gravitational constant
l	-	Length of link.
m	-	Mass of link.
net	-	Artificial Neural Networks model.
q	-	Output signal (angular position of link-controlled variable).
r	-	Reference signal.
t_1	-	Target 1
t_2	-	Target 2
T_s	-	Settling time
T	-	Target signal of ICLF model.
u	-	Input signal (manipulated variable).
$V(x)$	-	Control Lyapunov Function.
W_{ij}	-	Weight between layer i and layer j

CHAPTER 1

INTRODUCTION

1.1 Introduction to the Lyapunov Theorem

Lyapunov stability theorem is an important invention in the world of control engineering. The Lyapunov theory provides a connection between the presence of a Lyapunov function and the stability of a control system. For example, to explain the principle of Lyapunov theory, consider a ball on a surface (illustrated system), as shown in Figure 1.1, where positions A, B, C, D, and E are equilibrium points. Stability in the sense of Lyapunov can be defined as follows: if a small disturbance would cause the ball to diverge from its initial equilibrium position, then A and D are the only stable positions based on the concept of energy and dissipative systems.

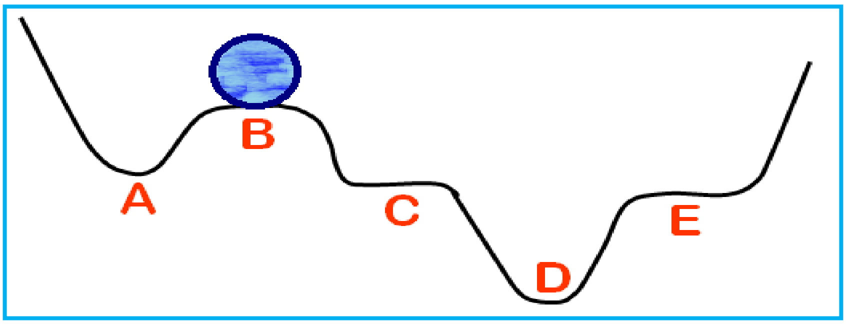

Figure 1.1 Illustrated system.

If a system has no input and the system energy is always decreasing, the system is said to be stable; otherwise, as the system energy increases, the system becomes unstable. The following diagram illustrates the techniques of the Lyapunov stability theory.

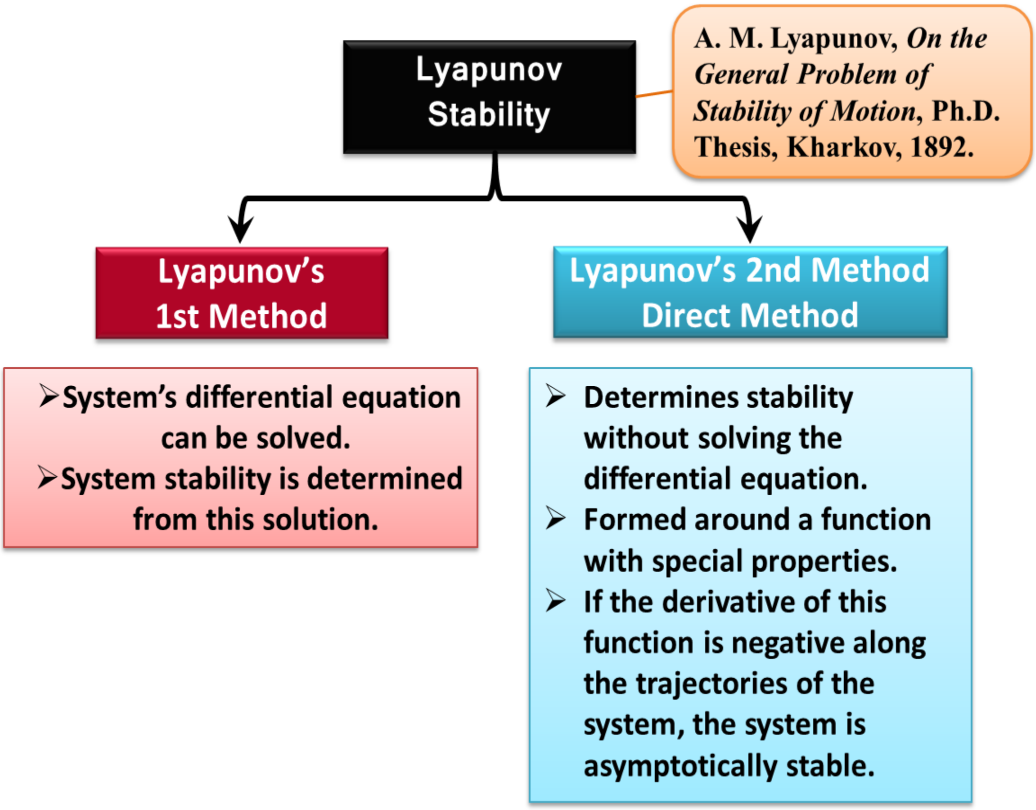

Figure 1.2 Methods of the Lyapunov Theory.

Unforced mass-damper-spring system is considered to explain how to derive the Lyapunov function:

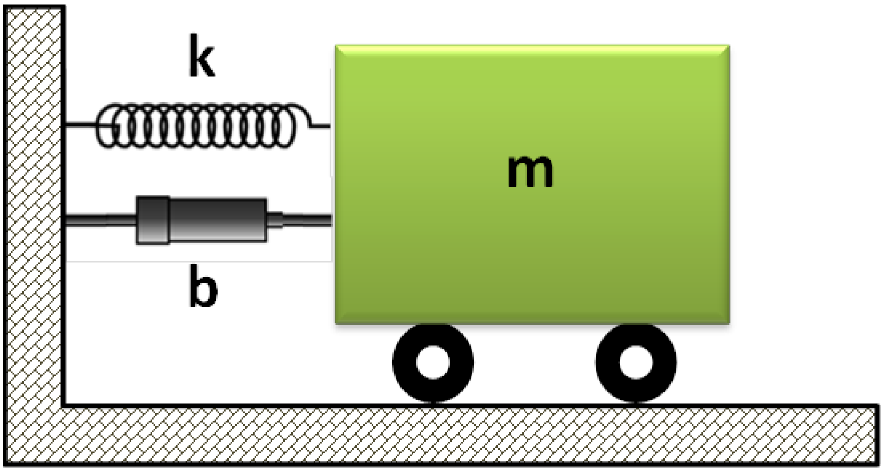

Figure 1.3 Unforced mass-damper-spring system.

The dynamic equation is:

$$m\ddot{x} + b\dot{x} + kx = 0 \tag{1.1}$$

where:

m- cart mass

b- damping constant

k- spring stiffness constant

The state-space model is:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-k}{m} & \frac{-b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{1.2}$$

Consider the energy function of the system equal to:

$$V(x,\dot{x}) = V_{kinetic}(x,\dot{x}) + V_{potential}(x,\dot{x}) = \frac{1}{2}(m\dot{x}^2 + kx^2) \tag{1.3}$$

The derivative of energy function over time will be equal to:

$$\dot{V}(x,\dot{x}) = \frac{d}{dt}\left[\frac{1}{2}(m\dot{x}^2 + kx^2)\right] = m\dot{x}\ddot{x} + kx\dot{x} = -b\dot{x}^2 \quad (1.4)$$

In the evaluation of changes in energy along the trajectory, if b>0, then the value of $\dot{V}(x,\dot{x})$ will be non-positive, and will only be zero when $\dot{x} = 0$ i.e. when the system is motionless. $\mathrm{V}(x,\dot{x})$ is known as a Lyapunov function. If there exist a continuous differentiable function $\mathrm{V}(x,\dot{x})$ and $\mathrm{V}(x,\dot{x})$ satisfies the following conditions:

$V(x_0) = 0$

$V(x) > 0\ \forall\ x \neq x_0$ (positive definite)

then:

$if\ \dot{V}(x) \leq 0,\ \forall\ x$ The system is stable.

$if\ \dot{V}(x) < 0,\ \forall\ x \neq x_0$ The system is asymptotically stable.

$if\ \dot{V}(x) > 0,\quad \forall\ x \neq x_0$ The system is unstable.

Because of the importance of control Lyapunov function in the examination of stability problems, many researchers have studied different methods to construct control Lyapunov function for different kinds of systems in various scientific fields. However, most of them use traditional mathematical approaches by applying many difficult mathematical theories with special assumptions. In this research, a new concept to construct the control Lyapunov function using Artificial Neural Networks has been applied, as Artificial Neural Networks contain unique characteristics.

1.2 Problem Statement

In control theory, a control Lyapunov function $V(x, u, d)$ is a generalization of the concept of Lyapunov function $V(x)$ used in stability analysis. The ordinary Lyapunov function is used to test whether a nonlinear dynamic system is stable or not. However traditional approaches to the construction of control Lyapunov function are very difficult and are comprised of mathematical complexities with more analytical assumptions. Therefore Artificial Neural Networks are used in the construction of control Lyapunov function.

Artificial Neural Networks possess unique characteristics which enable them to cope and adapt efficiently and accurately to nonlinear dynamic system changes. At the same time, Artificial Neural Networks create the possibility of overcoming all the complexities of the mathematical theories in traditional construction techniques. A novel technique like an Intelligent Construction of Control Lyapunov Function guarantees the stability and robustness of a nonlinear dynamic system.

1.3 Project Objectives

Two objectives need to be carried out in this project.

1) To invent a novel approach to construct a control Lyapunov function by Artificial Neural Networks known as Intelligent Control Lyapunov Function (ICLF) used for testing the stability of nonlinear dynamic systems.

2) To simulate the new approach model (ICLF) using Matlab program and to validate the results.

1.4 Project Scopes

In order to ensure that this project is accomplished within the boundary, five major topics are itemized:

1) A nonlinear dynamic system is selected arbitrarily (any nonlinear system).
2) PID controller is designed for the nominated nonlinear dynamic system.
3) The whole control system is simulated to obtain the input-output training pattern for designed Artificial Neural Network model (ICLF).
4) The designed Artificial Neural Network model (Intelligent Control Lyapunov Function) is applied to the nonlinear dynamic system.
5) Stability and Robustness are tested by introducing measurement noise.

1.5 A Glance for Chapter 2

In Chapter 2 (literature review), first, different methods to construct a control Lyapunov function for various types of systems will be investigated. Second, the principles of Artificial Neural Networks, and the unique characteristics of these networks and how they apply will be explored and studied. Finally, a large number of Artificial Neural Networks applications will be explored.

REFERENCES

Anghel, M., et al. 2013. Algorithmic Construction of Lyapunov Functions for Power System Stability Analysis.

Bellman, R. 1962. Vector Lyanpunov Functions. *Journal of the Society for Industrial & Applied Mathematics, Series A: Control* 1(1): 32-34.

Chemachema, M. 2012. Output Feedback Direct Adaptive Neural Network Control for Uncertain SISO Nonlinear Systems using a Fuzzy Estimator of the Control Error. *Neural networks* 36: 25-34. Elsevier.

Chen, Z. and Huang J. 2006. Construction of a Lyapunov Function for Nonlinear Cascaded Systems with Dynamic Uncertainties. Networking, Sensing and Control, 2006. ICNSC'06. Proceedings of the 2006 *IEEE International Conference on, IEEE.*

Clarke, F. H., et al. 1998. Asymptotic Stability and Smooth Lyapunov Functions. *Journal of Differential Equations* 149(1): 69-114.

Danapalasingam, K. A., et al. 2010. A Robust Stabilization using State Feedback with Feedforward. *American Control Conference (ACC)*, 2010, IEEE.

Funahashi, K.-I. 1989. On the Approximate Realization of Continuous Mappings by Neural Networks. *Neural Networks* 2(3): 183-192.

Haddad, W. M. and Chellaboina V. 2008. *Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach,* Princeton University Press.

Hornik, K., et al. 1989. Multilayer Feedforward Networks are Universal Approximators. *Neural Networks* 2(5): 359-366.

Hudon, N. and Guay V. 2013. Construction of Control Lyapunov Functions for Damping Stabilization of Control Affine Systems. *Systems & Control Letters* 62(11): 1009-1017. Elsevier.

Jiang, Z.-P. and Marcels I. 1997. A Small-gain Control Method for Nonlinear Cascaded Systems with Dynamic Uncertainties. *Automatic Control, IEEE Transactions on* 42(3): 292-308.

Jiang, Z.-P., et al. 1999. Robust Control of Uncertain Nonlinear Systems via Measurement Feedback. *IEEE Transactions on Automatic Control* 44(4): 807-812.

Karafyllis, I. and Jiang Z.-P. 2011. A Vector Small-gain Theorem for General Non-Linear Control Systems. *IMA Journal of Mathematical Control and Information* 28(3): 309-344.

Karafyllis, I. and Jiang Z.-P. 2012. Global Stabilization of Nonlinear Systems based on Vector Control Lyapunov Functions.

Kontoyiannis, I. and Meyn S. P. 2005. Large Deviations Asymptotics and the Spectral Theory of Multiplicatively Regular Markov Processes. *Electron. J. Probab* 10(3): 61-123.

Lakshmikantham, V., et al. 1991. *Vector Lyapunov Functions and Stability Analysis of Nonlinear Systems*, Springer.

Lera, G. and Pinzolas M. 2002. Neighborhood based Levenberg-Marquardt Algorithm for Neural Network Training. *Neural Networks, IEEE Transactions on* 13(5): 1200-1203.

Lin, C.-M. and Peng Y.-F. 2004. Adaptive CMAC-based Supervisory Control for Uncertain Nonlinear Systems. *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on* 34(2): 1248-1260.

Meigoli, V. and Nikravesh S. K. Y. 2012. Stability Analysis of Nonlinear Systems using Higher Order Derivatives of Lyapunov Function Candidates. *Systems & Control Letters* 61(10): 973-979.

Mesquita, A. R. and Hespanha J. P. 2010. Construction of Lyapunov Functions for Piecewise-Deterministic Markov Processes. *Decision and Control (CDC)*, 2010 49th IEEE Conference on, IEEE.

Michel, A. N., et al. 2007. *Stability of Dynamical Systems: Continuous, Discontinuous, and Discrete Systems*, Springer.

Nishimura, Y. and Yamashita Y. 2008. Lyapunov Function Design using Quantization of Markov Process. *Proceedings of IFAC World Congress* 2008.

Nishimura, Y. and Yamashita Y. 2008. *Stochastic Lyapunov Function Design using Quantization of Markov Process*. CDC.

Nishimura, Y., et al. 2009. *Construction Method of Lyapunov Functions based on Eigen Analysis*. ICCAS-SICE, 2009, IEEE.

Pillutla, S. and Keyhani A. 1999. Neural Network based Modeling of Round Rotor Synchronous Generator Rotor Body Parameters from Operating Data. *Energy Conversion, IEEE Transactions on* 14(3): 321-327.

Rajewsky, N., et al. 1998. The Asymmetric Exclusion Process: Comparison of Update Procedures. *Journal of Statistical Physics* 92(1-2): 151-194.

Samuelides, M., et al. 1997. Implementing Hebbian Learning in a Rank-based Neural Network. *Artificial Neural Networks—ICANN'97*, Springer: 145-150.

Serpen, G. 2005. Empirical Approximation for Lyapunov Functions with Artificial Neural Nets. Neural Networks, 2005. IJCNN'05. Proceedings. *2005 IEEE International Joint Conference on, IEEE*.

Shariati, O., et al. 2011. Application of Neural Network Observer for On-line Estimation of Salient-pole Synchronous Generators' Dynamic Parameters using the Operating Data. *Modeling, Simulation and Applied Optimization (ICMSAO), 2011 4th International Conference on, IEEE*.

Strozzi, F. and Zaldivar J. 1994. A General Method for Assessing the Thermal Stability of Batch Chemical Reactors by Sensitivity Calculation based on Lyapunov Exponents. *Chemical Engineering Science* 49(16): 2681-2688.

Sun, D. and Meng J. 2006. A Single Neuron PID Controller based PMSM DTC Drive System Fed by Fault Tolerant 4-Switch 3-Phase Inverter. *Industrial Electronics and Applications, 2006 1ST IEEE Conference on, IEEE*.

Tabatabaei, I., et al. 2004. Modeling and Simulation of a Salient-pole Synchronous Generator with Dynamic Eccentricity using Modified Winding Function Theory. *Magnetics, IEEE Transactions on* 40(3): 1550-1555.

Vidyasagar, M. 2002. *Nonlinear Systems Analysis*, Siam.

Vorotnikov, V. I. i. 2005. Partial Stability and Control: The State-of-the-art and Development Prospects. *Automation and Remote Control* 66(4): 511-561.

Zhong, L., et al. 1997. Analysis of Direct Torque Control in Permanent Magnet Synchronous Motor Drives. *Power Electronics, IEEE Transactions on* 12(3): 528-536.