Buletin Ukur, Jld. 7, No. 1, ms 1-10, Februari 1996
SPenerbitan Akademik Fakulti Ukur dan Harta Tanah

A Practical Strategy For Detecting Multiple Gross Errors

Halim bin Satan, PhD
Center for Industrial Measurement and Engineering Surveying Fakulti Ukur dan Harta Tanah Universiti Teknologi Malaysia

Abstract

Survey measurements are subject to random, systematic and gross errors. In practice, it is assumed that measurements are random variables, follow the normal distribution and have redundancy. The method of least squares estimation (ESE) is commonly used to process the redundant measurements. In the presence of gross errors, the results of ordinary ESE are corrupted. Consequently, an interactive post-LSE technique of robustified LSE (RLSE) is introduced for the detection of multiple gross errors in uncorrelated surveying data. In RESE. the locations and magnitudes of such errors are recovered simultaneously, and their effect on the solution are areatlv reduced.

1.0 INTRODUCTION

Much of the surveyor's task involves the acquisition and analysis of survey measurements. Such measurements are subject to random, systematic and gross errors (Caspary, 1987; Halim, 1992). In practice, redundant measurements are made to provide quality control and check for errors. The method of least squares estimation (LSE), which minimizes the sum of the weighted squares of the residuals, is commonly used to process the redundant measurements and to obtain a unique solution.

In principle, the method of LSE is valid without any assumption with respect to errors. However, in qualitative analysis and statistical evaluation of LSE results, it is generally assumed that the measurements contain only random errors and are regarded as random variables. These assumptions are important due to the high sensitivity of LSE to both systematic and gross errors (Halim, 1995a).

In reality, the measurements may contain gross errors. The effects of such errors are distributed over the residuals, and lead to questionable results and interpretations. Erroneous measurements do not necessarily have large residuals, or vice versa. LSE alone is therefore not suitable for dealing with gross errors.

For high precision applications such as deformation monitoring, gross errors need to be detected and localized, prior to deformation analysis. Whenever possible, gross errors should be tackled before LSE (i.e. pre-LSE). by means of screening and independent checks (Cooper, 1974, 1987; Secord, 1986).

Gross errors that remain in the measurements can be detected after LSE (i.e. post-LSE) based on the techniques employing the analysis of residuals. Generally (actually not necessarily) large residuals will indicate erroneous measurements. Whenever such residuals are detected, the corresponding measurement is examined to find out if a gross error can be found. That measurement is then deleted or remeasured.

The most popular post-LSE techniques for gross error detection in engineering surveying are Baarda's (data snooping), Pope's (Tau) and the Danish methods. Further details on this aspect are given by Cross and Price (1985), Caspary (1987), Chen et al (1987), Steeves and Fraser (1987), Kubik and Wang (1991), Gao et al (1992) and Schwarz and Kok (1993).

This paper discusses and examines the modification of the Danish method for detection and localization of multiple gross errors in uncorrelated surveying data. The modified method is known as robustified LSE (RLSE) since it uses LSE and the effects of gross errors on the solutions are reduced.

2.0 REVIEW OF LSE AND OUTLIER TESTS

Equations for LSE using observation equations (Halim, 1995a) are shown here without further derivation and explanation. More details are found extensively in surveying literature, for example Mikhail (1976), Mikhail and Gracie (1981), Cross (1983), Vanicek and Krakiwsky (1986), Cooper (1987), Koch (1987) and Leick (1990). The fundamental equations for LSE of full rank (only if Cayley inverse N^{-1} exist) with n observations, m parameters and redundancy r are (Halim, 1995a):

$l=f(x)$ are the functional models (1)

$W=\sigma_o^2\Sigma_l^{-1}=Q_l^{-1}$ where σ_o^2 is the a priori variance factor. (2)

$Ax=b+v$ are the observation equations (3)

$Nx=u$ are the normal equation where $N=A^tWA$ and $u=A^tWb$ (4)

$\hat{x}=N^{-1}u=(A^tWA)^{-1}A^tWb$ (5)

$\hat{x}_a=x_o+\hat{x}$ are the updated parameters (6)

$Q_{\hat{x}a}=Q_{\hat{x}}=N^{-1}=(A^tWA)^{-1}$ are the cofactor matrix (7)

$\hat{v}=A\hat{x}-b$ are the residuals (8)

$Q_{\hat{v}}=Q_l-AN^{-1}A^t=W^{-1}-AN^{-1}A^t$ (9)

$\hat{\sigma}_o^2=\hat{v}^tW\hat{v}/r$ where $r=n-m$ is the number of degrees of freedom or redundancy (10)

$\hat{l}_a=l+\hat{v}$ are the adjusted observations (11)

$Q_{la}=AN^{-1}A^t$ (12)

If observations are uncorrelated, W is diagonal matrix, and weight of observation i is

$w_i=\sigma_o^2/\sigma_i^2$ where σ_i^2 is the variance of observation i (13)

Statistical tests are used to assess the results of LSE. The commonly used tests are test on the estimated variance factor (i.e. global test) and the outlier test (i.e. local test).

It is likely that significant gross errors will increase the value of the estimated variance factor. For this reason, it is usual to adopt a one-tailed global test where

H_o: $\hat{\sigma}_o^2=\sigma_o^2$ and H_a: $\hat{\sigma}_o^2>\sigma_o^2$ (14)

To test H_o, the test statistic T is computed

$T=\hat{v}^tW\hat{v}/\sigma_o^2 \sim \chi^2_{r,\alpha}$ (15)

H_o is accepted (i.e. the test passes) if $T\leq\chi^2_{r,\alpha}$, where α is the significance level (typically 0.05). Otherwise H_o is rejected (i.e. the test fails).

If the test passes, it is assumed that, overall, there are no model or gross errors, and the results of LSE can be accepted. Failure of this test indicates either the model is incorrect/incomplete, the weight is unrealistic, or the existence of gross errors in the measurements. Hence, further investigations or statistical tests (i.e. outlier test) are needed.

The outlier (or local) test examines the standardized residuals $\hat{v}_i'$ (Caspary, 1987)

H_o: $E\{\hat{v}_i'\}=0$ or each $\hat{v}_i'$ is free from gross error (16)

H_a: $E\{\hat{v}_i'\}\neq0$ or one residual contains gross error

The test statistic is

$T_b=\hat{v}_i'=|\hat{v}_i/(\sigma_o\sigma_{\hat{v}i})|\sim N(0,1)$ (17)

or $T_p=\hat{v}_i'=|\hat{v}_i/(\hat{\sigma}_o\sigma_{\hat{v}i})|\sim\tau_r$ (18)

Equation (17) is used when σ_o^2 is known, and it is called un-studentized test. This test assumes that the residuals are normally distributed. Equation (18) (studentized test) is employed when σ_o^2 is unknown and is estimated by $\hat{\sigma}_o^2$. Evaluation of $Q_{\hat{v}i}$ in order to get $\sigma_{\hat{v}i}$ will require extensive computation, as shown in equation (9). For uncorrelated observations, only diagonal elements of $Q_{\hat{v}}$ need to be computed, and expression for $\sigma_{\hat{v}i}$ can be written as (Halim, 1995a)

$$\sigma_{\hat{v}i}=\sigma_i\sqrt{(r_i)} \qquad (19)$$
$$r_i=1-a_iN^{-1}a_i^tw_i$$

where

σ =standard deviation of observation i
r =redundancy number of observation i
a_i=elements of design matrix A for observation i
N^{-1}=Cayley inverse of N
w_i=weight of observation i

3.0 TECHNIQUES FOR GROSS ERROR DETECTION

In practice, the application of the above tests (i.e. equations 17 and 18) with the appropriate standardization procedure is known as data snooping (Schwarz and Kok, 1993), and consists of either Pope's (Tau) or Baarda's (data snooping) methods. Pope's method uses the studentized test and standardizes for Type I error (α) only. Baarda's method uses the un-studentized test and standardizes both Type I and Type II errors (α and β).

Pope's or Tau method (Pope, 1976) assumes σ_o^2 as unknown, and applies the estimated $\hat{\sigma}_o^2$ in computing the normalized residuals. The test statistic (equation 18) is one-dimensional

$$T_p=\hat{v}_i/(\hat{\sigma}_o\sigma_{\hat{v}i})\sim\tau_r \qquad (20)$$

where r is the number of degrees of freedom (redundancy), and α is standardized as

$$\alpha_n=1-(1-\alpha)^{1/n}\approx\alpha/n \qquad (21)$$

where n is the number of the observations. H_o is accepted if

$$|T_p| \le \tau_{r,\alpha o} \qquad \text{where } \alpha_o=\alpha_n/2 \qquad (22)$$

Otherwise (if $T>\tau_{r,\alpha o}$), H_o is rejected, and the corresponding observation must be examined.

The computation of the critical value of τ is given by Pope (1976), together with the listing of a useful Fortran subroutine.

Baarda's method (Baarda, 1968) assumes that σ_o^2 is known a priori, and employs a multi-dimensional test. The test statistics (equation 18) is

$$T_b=\hat{v}_i/(\sigma_o\sigma_{\hat{v}i})\sim N(o,\sigma_{\hat{v}i}) \qquad (23)$$

H_o is accepted if $|T_b| < N_{\alpha/2}$. Given α 0.05 (5%), the critical value of N is 1.96.

In the actual implementation of Baarda's method, both Type I and Type II errors are taken into account. Typical values for standardized α_o and β_o are 0.1% and 20% respectively leading to the critical value $u_{\alpha o}$ of 3.29 (Figure 1). H_o is accepted if

$$|T_b| \le u_{\alpha o} \qquad (24)$$

Otherwise H_o is rejected if $T_b>u_{\alpha o}$. Interpretation of the test is similar to Pope's method.

Baarda's method also provides a measure of reliability, both internal and external. Some expressions for reliability of each observation i (datum independent quantities) are:

internal reliability

$r_i = 1 - a_i N^{-1} a_i^t w_i$ (25)

$MDGE_i = \nabla_i = \sigma_i (\lambda_o / r_i)^{1/2}$ (26)

external reliability

$\delta_i = \lambda_o (1 - r_i) / r_i$ (27)

where r_i=redundancy number of observation i
$MDGE_i = \nabla_i$=size of the marginally detectable gross errors (MDGE) in observation i.
δ_i=influential factor or global distortion parameters of observation i
λ_o=non-centrality parameters, computed from α_o and β_o

A typical value of λ_o (with α_o 0.1% β_o 20%) is 17. Baarda (1968) provides a nomogram for the evaluation of $u_{\alpha o}$ (with respect to type I error) and λ_o (with respect to degrees of freedom and Type I and Type II errors). Figure 1 shows the nomogram for β_o=20%.

Both Pope's and Baarda's methods assume that just one measurement is affected by a gross error. In fact, as it is likely that the measurements contain multiple gross errors, the general procedure for both methods is to eliminate one gross error (and the suspected measurement) at a time, and to repeat the LSE until no gross errors are detected.

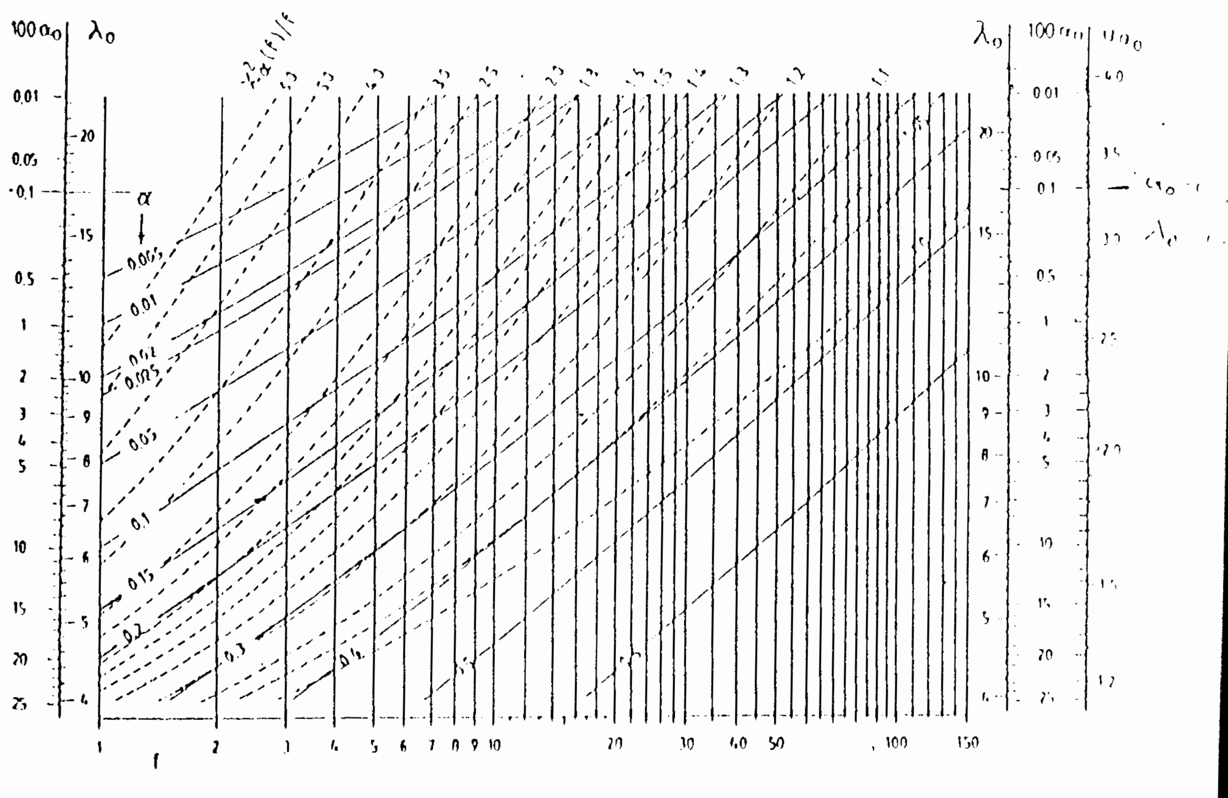

Figure 1. Baarda's nomogram for β_o 20% (taken from Caspary, 1987)

The Danish method was proposed by Krarup in 1980 (Caspary, 1987; Strauss, 1983). In the Danish method, larg[e] (estimated) residuals are associated with gross errors.

The objective of this method is to find observations which are not consistent with the majority and to exclude them from the LSE by reducing their weights (i.e. de-weighting). In this manner, weights are treated as dynamic quantities such that only the consistent measurements are used effectively in the LSE process. It is expected that the effects of gross error on the final estimation will be insignificant.

Application of the Danish method is very simple (Strauss, 1983). After a conventional LSE using a priori weights, the estimation is repeated several times during which the weights of certain measurements are reduced according to their residuals after the preceeding estimation. Weights of the observations with higher residuals are reduced (i.e. low weights), while the weights of observations with lower residuals (i.e. lower than a certain limit) are held stable (equation 28).

With a proper and suitable choice of de-weighting function and constant (factor) c, convergence is achieved. In the final solution, weights of outlying observations will approach zero, effectively removing them from the LSE. Moreover, the observations affected by gross error are found with corrections of the same order of the magnitude of their corresponding residuals but with reversed sign.

However, a proper termination criteria for the iteration and a suitable weighting factor are essential for the successful implementation of the Danish method.

4.0 ROBUSTIFIED LSE

In practice, the de-weighting schemes for the Danish method are based on trial and error, see for example Kubik et al (1985), Jorgensen et al (1985), Strauss (1983) and Kubik et al (1988). However, simulation studies with known gross errors indicate that most of the de-weighting schemes either flagged additional measurements, or were unable to detect some of the errors. The following de-weighting scheme (modified from Caspary, 1987) is found acceptable from experimenting (Halim, 1992) :

$$w_i=1/\sigma_i^2 \tag{28}$$

if $|\hat{v}_i| \leq \text{limit}$; $p_i=1.0$; $w_i'=p_i w_i$ (weight unchanged)
if $|\hat{v}_i| > \text{limit}$; $p_i=e^{-f}$; $w_i'=p_i w_i$ (weight changed)
$\text{limit}=c\sigma_i\hat{\sigma}_o$
$f=|\hat{v}_i|/(c\sigma_i\hat{\sigma}_o)$
c is the weighting factor (usually between 2.0 and 3.0)

If use the standardized residuals (equations 18 and 19)

$$\hat{v}_{i'}=\hat{v}_i/(\hat{\sigma}_o\sigma_i r_i^{1/2}): \tag{29}$$

if $|\hat{v}_i'| \leq c$; $p_i=1.0$; $w_i'=p_i * w_i$ (weight unchanged)
if $|\hat{v}_i'| > c$; $p_i=e^{-f}$; $w_i'=p_i * w_i$ (weight changed)

In principle, the developed technique of robustified LSE (RLSE) uses the above de-weighting scheme (equation 28) and provides a facility for variation of the weighting factor. In addition, global and local tests are used as stopping criteria, together with reliability analysis to determine the capability of gross error detection.

The formulation of the developed interactive procedure for RLSE of uncorrelated measurements can then be summarized as follows:

(a) During and after ordinary LSE (section 2.0), using the weights w_i, compute the estimated residuals $\hat{v}_i$ and the estimated variance factor $\hat{\sigma}_o^2$. The one-tailed global test (equations 14 and 15) is employed. As the global test is not sensitive enough, a local test based on Pope (equations 20 and 22) is also employed. If both tests pass, the LSE results are acceptable, and step (e) (below) can be executed for reliability analysis. Otherwise, a RLSE must be performed, via steps (b), (c) and (d).

(b) Define the de-weighting scheme (equation 28) and weighting factor c. It is recommended that RLSE is begun with a factor c of 3.0 (i.e. bigger factor) to avoid the possibility of additional flagging of good or acceptable measurements. The same factor c is applied during RLSE until no observation weights are changed. Factor c is then reduced interactively by 0.1 (new c becomes 2.9, 2.8 and so on) until the weights of some observations are changed. If c becomes too low, for example less than 1.5, RLSE should be stopped. The use of standardized residuals (equation 29) is also useful.

(c) Compute new weights for all the measurements (i.e. w_i'). In this manner, weights of measurements with $\hat{v}_i$ greater than the limit (i.e. suspect measurements), will be reduced. Otherwise, the previous weights are maintained (equation 28).

(d) A new LSE is carried out using the new weights. This process of re-estimating and de-weighting (a, b and c) is iterated until the solution converges, using the selected c factor in (b). If both global (equations 14 and 15) and local (equations 20 and 22) tests are passed, the procedure is stopped. Otherwise, the c factor is reduced again, and the procedure is repeated until the termination criteria are met (i.e. both global and local tests are passed).

(e) Reliability analysis employs equations 25, 26 and 27.

Once the stopping criteria are met, two options are possible, either direct or indirect use of RLSE.

In the first option, all the changed weights can be drastically reduced close to zero (but not too close in order to avoid numerical instability) in the final computations. The purpose is to minimize or greatly reduce the effects of measurements with gross errors on the final solutions of coordinates, trace, rmse and residuals. Results of the RLSE will be the same as ordinary LSE without the erroneous or flagged observations.

In the second option, results of RLSE are used for checking purposes. Deweighted (i.e. erroneous) measurements need to be carefully examined, and deleted or remeasured as necessary, leading to a new data set. In this case, a new LSE must be carried out with the new data, to arrive at a final result.

The method of RLSE is capable of detecting and localizing gross errors correctly since both global and local tests are used to verify the results. Another important aspect is related to reliability analysis. Redundancy numbers together with MDGE will indicate whether a gross error can be detected or not. If the redundancy number for a particular measurement is very small (i.e. very large MDGE), gross errors in that measurement will be left undetected since there is very little controllability. On the other hand, a redundancy number close to unity indicates that the effects of that measurement (and gross error in it) on the solution are almost insignificant.

Procedure to speed up the computations is discussed in Halim (1995a). In using RLSE the observations should not be de-weighted drastically (to speed up the computation), to avoid the possibility of flagging good observations or non-detection of actual gross errors.

The RLSE has been implemented in program ESTIMATE (Halim, 1995c), which is used for processing the data in section 5.0.

5.0 RESULTS

Simulation tests were conducted using the 3-D test network shown in Figure 2 (Halim, 1995c). To study the capability of gross error detection, 6 gross errors with the magnitudes of 10σ were introduced into the randomized data (see Table 1, column 1). Four cases were examined: ordinary LSE; robustified LSE; Baarda's and; Pope's methods

The computed solutions are displayed in Table 1 (column 3). Ordinary LSE is unacceptable as both global and local tests failed. Applying robustified LSE (with option to speed up the computation), the solutions converged after 13 iterations, with the estimated variance factor of 0.73 and weighting factor c of 2.3. The solution passed both global and local tests. RLSE procedure deweights all 6 observations that contain gross errors. During RLSE, the weights these observations were drastically reduced to zero. Consequently, degrees of freedom for computing variance factor may be reduced by 6, i.e from 34 to 28, and lead to a new estimate of the variance factor of 0.89.

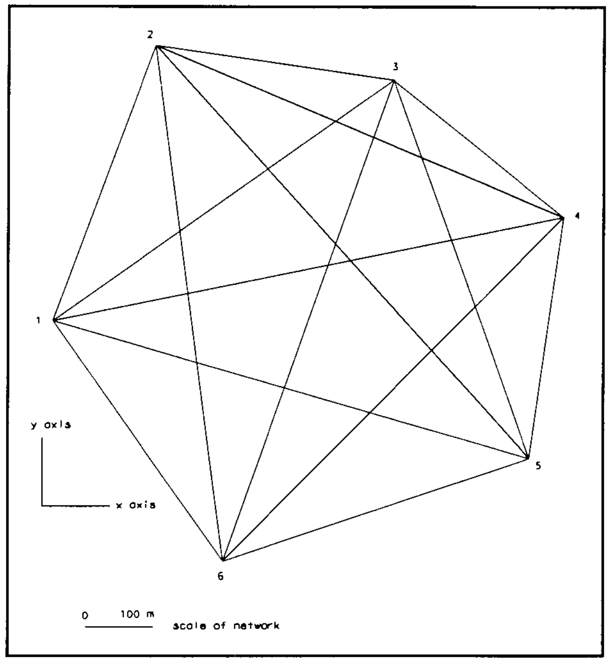

Figure 2. Test network (plan)

The final residuals (with opposite sign) of the deweighted observations indicated that the estimated magnitudes of gross error were close to the simulated gross error (columns 1 and 3 of Table 1). Hence, robustified LSE is able to detect and locate all the gross errors in this network correctly. Also, the redundancy numbers of the deweighted observations are close to unity (Halim, 1995b) showing the effects of gross error in the solution as being negligible.

In Pope's and Baarda's methods, the sequence of LSE with global and local tests followed by successive elimination of suspected observations were repeated until both global and local tests passed. These methods both resulted in variance factors of 0.89, the same as from robustified LSE. As shown in Table 1, all three methods detect the gross errors correctly. However, all these methods lead to a less reliable network as indicated by the reliability analysis (redundancy number, internal and external reliability).

Comparison between the solutions via robustified LSE and Pope's method also revealed that the final coordinates, their standard errors and trace were exactly the same in both cases. Moreover, residuals and reliability measures for non-deweighted observations are also identical. This result shows that by adopting robustified LSE, the solution can be used directly without the need to eliminate the observations (Halim, 1995b).

6.0 CONCLUSIONS

Survey measurements are not free from the following errors: random, systematic and gross. In practice, it is assumed that the measurements contain only random errors and are regarded as random variables. Such assumptions are necessary because LSE is very sensitive to both systematic and gross errors.

	ordinary LSE	robustified LSE	Baarda's method	Pope's method
simulated ge sd 1-2 +50 mm sd 1-5 -50 mm dir 1-3 -50 sec dir 2-1 +50 sec dh 2-3 +50 mm dh 4-6 -50 mm		residuals **[-43.71] **[+40.52] **[+44.90] **[-47.21] **[-54.50] **[+48.21]	 * * * * * *	 * * * * * *
rank analysis	ok	ok	ok	ok
variance factor	10.68	0.89	0.89	0.89
iteration	1	13	1 per run	1 per run
global test	fail	pass	pass	pass
local test	fail	pass	pass	pass
redundancy number	0.32-0.78	0.19-0.78	0.19-0.78	0.19-0.78
internal reliability	23.33-36.53	23.33-47.12	23.33-47.12	23.33-47.12
external reliability	4.77-36.38	4.79-71.83	4.79-71.83	4.79-71.83

Table 1. Test with 6 gross errors for network 1

(note: * indicates deleted observations (one at a time) by Pope's Tau and Baarda's methods
** indicates deweighted observations during robustified LSE
ge indicates gross error)

Although gross errors can be detected at pre-LSE stage via screening, some of them may still remain in the measurements. These undetected gross errors can still be recovered during post-LSE via the analysis of residuals.

The most popular methods for gross errors detection are data snooping, tau and the Danish methods. The method introduced in this paper, an interactive robustified LSE, is a modification of the Danish method, with the incorporation of global and local tests, together with reliability analysis.

Robustified LSE is a practical strategy and suitable for simultaneous detection of multiple gross errors and estimation of their magnitudes. The suspected measurements are not eliminated, however, the effects of gross errors on the final results are insignificant as the erroneous measurements are deweighted drastically.

Practically, gross errors should be tackled prior to LSE, and all available post-LSE techniques for gross errors detection must be employed.

REFERENCES

BAARDA, W. (1968). A testing procedure for use in geodetic networks, Netherlands Geodetic Commission Publications on Geodesy, New Series, 2 (5), Delft.

CASPARY, W.F. (1987). Concepts of network and deformation analysis, School of Surveying, The University of New South Wales, Monograph 11, Kensington, N.S.W., 183pp.

CHEN, Y.Q., KAVOURAS, M. and CHRZANOWSKI, A. (1987). A strategy for detection of outlying observations in measurements of high precision, The Canadian Surveyor 41(4): 529-540.

COOPER, M.A.R. (1974). Fundamentals of Survey Measurement and Analysis. (London: Granada), 107pp.

COOPER, M.A.R. (1982). Modern theodolites and levels. (London: Granada), 156pp.

COOPER, M.A.R. (1987). Control Surveys in Civil Engineering. (London: Collins), 381pp.

CROSS, P.A. (1983). Advanced least squares applied to position fixing, Department of Land Surveying, North East London Polytechnic, Working paper 6, 205pp.

CROSS, P.A. and PRICE, D.R. (1985). A strategy for the distinction between single and multiple gross errors in geodetic networks, Manuscripta Geodaetica 10(3): 172-178.

GAO, Y., KRAKIWSKY, E.J. and CZOMPO, J. (1992). Robust testing procedure for detection of multiple blunders, Journal of Surveying Engineering 118(1): 11-23.

HALIM SETAN (1992). Multiple gross errors detection by Danish method. ESRC Internal report. 35 pp.

HALIM SETAN (1995a). Functional and stochastic models for geometrical detection of spatial deformation in engineering: a practical approach. PhD thesis, ESRC, Department of Civil Engineering, City University, London, 261pp.

HALIM SETAN (1995b). User guide to the deformation detection programs. ESRC Internal report. 21 pp.

HALIM SETAN (1995c). Design and implementation of computer programs for analysis and detection of spatial deformation. Buletin Ukur, Jilid 6, Nombor 2, September 1995.

JORGENSEN, P.C., KUBIK, K., FREDERIKSEN, P and WENG, W. (1985). Ah, robust estimation, Aust. J. Geod. Photo. Surv. 42: 19-32.

KOCH, K.R. (1987). Parameter estimation and hypothesis testing in linear models. (London: Springer-Verlag), 378 pp.

KUBIK, K., WENG, W. and FREDERIKSEN, P. (1985). Oh, gross errors!, Aust. J. Geod. Photo. Surv. 42: 1-18.

KUBIK, K., LYONS, L. and MERCHANT, D. (1988). Photogrammetric work without blunders, Photogrammetric engineering and remote sensing 54(1): 51-54.

KUBIK, K. and WANG, Y. (1991). Comparison of different principles for outlier detection, Aust. J. Geod. Photogram. Surv. 54: 67-80.

LEICK, A. (1990). GPS satellite surveying. (New York: John Wiley & sons), 352pp.

MIKHAIL, E. M. (1976). Observations and Least Squares. (New York: University Press of America), 497pp.

MIKHAIL, E.M. and GRACIE, G. (1981). Analysis and Adjustment of Survey Measurements. (London: Van Nostrand Reinhold), 340pp.

POPE, A.J. (1976). The statistics of residuals and the detection of outliers, NOAA Technical Report NOS 65 NGS 1, National Ocean Service, National Geodetic Survey, US Department of Commerce, Rockville, MD, Washington, 133pp.

SCHWARZ, C.R. and KOK, J.J. (1993). Blunder detection and data snooping in LS and robust adjustments, Journal of Surveying Engineering 119(4): 127-136.

SECORD, J.M. (1986). Terrestrial survey methods for precision deformation measurements, Proceedings of the Deformations Measurements Workshop, Oct.31-Nov. 1, Massachusetts Institute of Technology, Cambridge, MA.

STEEVES, R.R. and FRASER, C.S. (1987). Statistical Post- Analysis of Least Squares Adjustment Results, In: Papers For the CISM Adjustment and Analysis Seminars, ed. by E.J. Krakiwsky, Calgary, Alberta.

STRAUSS, R. (1983). A strategy to trace gross errors. In : Precise levelling: Contributions to the Workshop on precise levelling, ed. by H. Pelzer and W. Niemeier. (Bonn: Dümmler). 490pp.

VANICEK, P. and KRAKIWSKY, E. (1986). Geodesy: the concept, second edition. (Amsterdam: Elsevier), 697pp.