

CONTROLLERS DESIGN OF AN ACTIVE SUSPENSION  
SYSTEM BASED ON HALF CAR MODEL

ISMAIL ABUBAKAR UMAR

UNIVERSITI TEKNOLOGI MALAYSIA

CONTROLLERS DESIGN OF AN ACTIVE SUSPENSION SYSTEM BASED  
ON HALF CAR MODEL

ISMAIL ABUBAKAR UMAR

A project report submitted in partial fulfillment of the  
requirements for the award of the degree of  
Master of Engineering (Electrical-Mechatronics and Automatic Control)

Faculty of Electrical Engineering  
Universiti Teknologi Malaysia

JANUARY 2015

Dedicated to my children, Faruq Isma'il Umar (Aiman), Musa Isma'il Umar (Adnan), my parents, my wife, my entire family and to all that believe in me.

## ACKNOWLEDGEMENT

First and foremost, my unlimited gratitude goes to Almighty Allah, the Lord of the heavens and earth for His endless mercies, blessings and guidance through from birth till now and forever. All thanks be to Almighty Allah.

I wish to express my sincere appreciation to my supervisor in person of Dr. Herman Bin Wahid for his thorough and sustained supervision, encouragement and guidance to ensure this research work is successfully completed. I can say with confidence that without his inputs and continued support, this thesis might not have been possible. I would like to also thank all my lecturers for the knowledge they impart, their dedication to duties, hard work and selfless service to humanity. Thank you all.

My deepest thankfulness and appreciation also goes to the Management of Bayero University Kano for the opportunity given to me to go for further studies. To you all, I say thank you.

Last but not the least; I shall forever remain grateful to my parents, my brother/Mentor Dr. Musa A. Auyo and my entire family. I am very proud of my wife Umamatu Uwaisu Muhammad and my children Aiman and Adnan for their understanding, support and encouragement. This is quite memorable to me. I pray to Almighty Allah to reward you abundantly. Thank you very much.

## ABSTRACT

Most automobile suspension systems use passive components which are combination of spring and damper only. These components can only store and dissipate energy, and lack the capability to inject energy into the systems. The parameters of these components are generally fixed and do not vary with operating conditions. In this project, hydraulic actuators are considered in parallel with the passive components to make the systems active suspension so that the system will have the ability to store, dissipate and introduce energy to the system. The mathematical model of the active and passive suspension systems based on half car was derived and represented in state space form for controllers design, analysis and comparison. The control of these active suspensions will improve the load carrying, road handling and ride quality of the car. Three suitable controllers, one state feedback-Linear Quadratic Regulator (LQR), one intelligent-Fuzzy Logic (FL) and Proportional Integral Derivative (PID) controllers were designed and applied to control active suspension in order to improve ride comfort. This was achieved by minimizing the vertical displacement and acceleration of the car body through simulation results. Comparison and analysis of the performance of each controller in relation to ride comfort and road handling was presented. The simulation results were obtained using Matlab/Simulink software with a road profile as input to the system.

## ABSTRAK

Kebanyakan sistem suspensi kereta menggunakan komponen-komponen pasif seperti gabungan spring dan peredam sahaja. Komponen-komponen ini hanya boleh menyimpan dan menyebarkan tenaga sahaja tetapi kurang berupaya untuk menyuntik tenaga ke dalam sistem. Secara umumnya, parameter komponen-komponen ini adalah tetap dan tidak berubah dengan keadaan operasi. Dalam projek ini, penggerak hidraulik dianggap berada dalam keadaan selari dengan komponen-komponen pasif bagi menjadikan sistem suspensi adalah aktif supaya sistem ini akan berupaya untuk menyimpan, menyebarkan dan memperkenalkan tenaga ke dalam sistem. Bagi tujuan mereka bentuk pengawal, analisis dan perbandingan, model matematik bagi sistem suspensi aktif dan pasif telah diperolehi dan dibentangkan dalam bentuk 'state space'. Kawalan terhadap sistem suspensi aktif akan meningkatkan keupayaan membawa bebanan, pengendalian jalan dan kualiti pemanduan kereta. Tiga pengawal sesuai seperti 'State Feedback Linear Quadratic Regulator (LQR)', 'Intelligent Fuzzy Logic (FL)' dan 'Proportional Integral Derivative (PID)' telah direka bentuk dan digunakan untuk mengawal sistem suspensi aktif bertujuan untuk meningkatkan keselesaan pemanduan. Ini dicapai dengan mengurangkan anjakan menegak dan pecutan badan kereta melalui keputusan simulasi. Perbandingan dan analisis prestasi setiap pengawal disampaikan dalam bentuk keselesaan pemanduan dan pengendalian jalan raya. Keputusan simulasi untuk pelbagai jenis permukaan jalan telah diperolehi dengan menggunakan perisian Matlab/Simulink

## TABLE OF CONTENTS

| <b>CHAPTER</b> | <b>TITLE</b>                         | <b>PAGE</b> |
|----------------|--------------------------------------|-------------|
|                | <b>DECLARATION</b>                   | ii          |
|                | <b>DEDICATION</b>                    | iii         |
|                | <b>ACKNOWLEDGEMENT</b>               | iv          |
|                | <b>ABSTRACT</b>                      | v           |
|                | <b>ABSTRAK</b>                       | vi          |
|                | <b>TABLE OF CONTENTS</b>             | vii         |
|                | <b>LIST OF TABLES</b>                | x           |
|                | <b>LIST OF FIGURES</b>               | xi          |
|                | <b>LIST OF SYMBOLS</b>               | xiii        |
| <br>           |                                      |             |
| <b>1</b>       | <b>INTRODUCTION</b>                  | 1           |
|                | 1.1 Introduction                     | 1           |
|                | 1.2 Problem Statement                | 2           |
|                | 1.3 Research Objectives              | 3           |
|                | 1.4 Project Scope                    | 3           |
|                | 1.5 Overview of Research Methodology | 4           |
| <br>           |                                      |             |
| <b>2</b>       | <b>LITERATURE REVIEW</b>             | 6           |

|          |  |    |
|----------|--|----|
| 2.1      | Introduction                                       | 6  |
| 2.2      | Linear Controller                                  | 7  |
| 2.3      | Nonlinear Controller                               | 8  |
| 2.4      | Intelligent Controller                             | 9  |
| 2.5      | Linear Quadratic Regulator Controller              | 10 |
| 2.6      | Fuzzy Logic Controller                             | 11 |
| 2.7      | Proportional Integral Derivative (PID) Controller  | 11 |
| <b>3</b> | <b>MATHEMATICAL MODEL OF THE SYSTEM</b>            | 12 |
| 3.1      | Introduction                                       | 12 |
| 3.2      | Mathematical Modeling                              | 13 |
| 3.3      | Half Car Active Suspension model                   | 13 |
| 3.4      | Dynamic Model of Half Car Active Suspension        | 14 |
| 3.5      | Dynamics with the Spool Valve Hydraulic Actuators  | 19 |
| 3.6      | Road Profile                                       | 23 |
| <b>4</b> | <b>CONTROLLERS DESIGN</b>                          | 26 |
| 4.1      | Introduction                                       | 26 |
| 4.2      | Linear Quadratic Regulator (LQR) Controller Design | 26 |
| 4.3      | Fuzzy Logic (FL) Controller Design                 | 32 |
| 4.4      | PID Controller Design                              | 37 |
| <b>5</b> | <b>RESULTS AND DISCUSSIONS</b>                     | 39 |
| 5.1      | Introduction                                       | 39 |



|          |   |           |
|----------|---|-----------|
| 5.2      | Simulation Performance for Comparison between Active LQR and Passive Suspension         | 40        |
| 5.3      | Simulation Performance for Comparison between Active Fuzzy Logic and Passive Suspension | 46        |
| 5.4      | Simulation Performance for Comparison between Active PID and Passive Suspension         | 50        |
| 5.5      | Simulation Performance Comparison between Active LQR, FL, PID and Passive Suspension    | 54        |
| <b>6</b> | <b>CONCLUSION AND FUTURE WORK</b>   | <b>58</b> |
| 6.1      | Conclusion  | 58        |
| 6.2      | Suggestion for Future Work  | 59        |
|          | <b>REFERENCES</b>   | <b>60</b> |
|          | <b>Appendices</b>   | <b>63</b> |

**LIST OF TABLES**

| <b>TABLE NO.</b> | <b>TITLE</b>   | <b>PAGE</b> |
|------------------|--|-------------|
| <b>3.1</b>       | Parameter values for Half Vehicle Suspension (Huang and Lin, 2004) | 22          |
| <b>4.1</b>       | Rule Base for Membership function                                  | 36          |
| <b>4.2</b>       | PID Tuning Parameters  | 37          |

## LIST OF FIGURES

| <b>FIGURE NO.</b> | <b>TITLE</b>   | <b>PAGE</b> |
|-------------------|--|-------------|
| <b>1.1</b>        | Project process flow chat  | 5           |
| <b>3.1</b>        | Active suspension for half car model.                            | 14          |
| <b>3.2</b>        | Road profile that represent double 5cm and 2.5 cm<br>bump height | 25          |
| <b>4.1</b>        | Input variable for front body displacement.                      | 33          |
| <b>4.2</b>        | Input variable for front body velocity.                          | 33          |
| <b>4.3</b>        | Input variable for rear body displacement                        | 34          |
| <b>4.4</b>        | Input variable for rear body velocity                            | 34          |
| <b>4.5</b>        | Output variable for front force                                  | 35          |
| <b>4.6</b>        | Output variable for rear force                                   | 35          |
| <b>5.1</b>        | Front body displacement  | 41          |
| <b>5.2</b>        | Front body velocity  | 42          |
| <b>5.3</b>        | Rear body displacement   | 42          |
| <b>5.4</b>        | Rear body velocity   | 43          |
| <b>5.5</b>        | Front wheel deflection   | 43          |
| <b>5.6</b>        | Rear wheel deflection  | 44          |
| <b>5.7</b>        | Front body acceleration  | 44          |
| <b>5.8</b>        | Rear body acceleration   | 45          |
| <b>5.9</b>        | Front suspension travel  | 45          |
| <b>5.10</b>       | Rear suspension travel   | 46          |

|             |  |    |
|-------------|--|----|
| <b>5.11</b> | Front body displacement  | 47 |
| <b>5.12</b> | Rear body displacement   | 48 |
| <b>5.13</b> | Front suspension travel  | 48 |
| <b>5.14</b> | Rear suspension travel   | 49 |
| <b>5.15</b> | Front wheel deflection   | 49 |
| <b>5.16</b> | Rear wheel deflection  | 50 |
| <b>5.17</b> | Front body acceleration  | 51 |
| <b>5.18</b> | Rear body acceleration   | 51 |
| <b>5.19</b> | Front suspension travel  | 52 |
| <b>5.20</b> | Rear suspension travel   | 52 |
| <b>5.21</b> | Front wheel deflection   | 53 |
| <b>5.22</b> | Rear wheel deflection  | 53 |
| <b>5.23</b> | Front body displacement  | 55 |
| <b>5.24</b> | Rear body displacement   | 55 |
| <b>5.25</b> | Front suspension travel  | 56 |
| <b>5.26</b> | Rear suspension travel   | 56 |
| <b>5.27</b> | Front wheel deflection   | 57 |
| <b>5.28</b> | Rear wheel deflection  | 57 |
| <b>6.1</b>  | Simulink Block for LQR Controller With Passive<br>Suspension           | 65 |
| <b>6.2</b>  | Simulink block diagram for fuzzy logic controller                      | 66 |
| <b>6.3</b>  | Simulink block diagram for PID controller                              | 67 |
| <b>6.4</b>  | Simulink block diagram for LQR, FLC and PID with<br>passive suspension | 68 |

## LIST OF SYMBOLS

|          |   |  |
|----------|---|--|
| $A$      | - | System matrix of the active suspension system              |
| $A_c$    | - | Closed-loop system matrix of the active suspension         |
| $A_p$    | - | System matrix of the passive suspension system             |
| $B$      | - | Input matrix of the active suspension system               |
| $B_p$    | - | Input matrix of the passive suspension system              |
| $B_f$    | - | Damping coefficient of the front car damper (Ns/m)         |
| $B_r$    | - | Damping coefficient of the rear car damper (Ns)            |
| $C$      | - | Output matrix of the system                                |
| $Co$     | - | Controllability matrix                                     |
| $D$      | - | Feedforward matrix of the system                           |
| $F$      | - | Disturbance matrix of the active suspension system         |
| $J$      | - | Performance index of the LQR controller                    |
| $J_y$    | - | Centroidal moment of inertia for the car body ( $kg/m^2$ ) |
| $K$      | - | State feedback gain of the LQR controller                  |
| $K_d$    | - | Derivative gain of the PID controller                      |
| $K_i$    | - | Integral gain of the PID controller                        |
| $K_p$    | - | Proportional gain of the PID controller                    |
| $K_f$    | - | Stiffness of the front car body spring (N/m)               |
| $K_r$    | - | Stiffness of the rear car body spring (N/m)                |
| $K_{tf}$ | - | Stiffness of the front car tire (N/m)                      |

|            |   |   |
|------------|---|---|
| $K_{tr}$   | - | Stiffness of the rear car tire (N/m)  |
| $M_b$      | - | Mass of the car body (kg)   |
| $M_{wf}$   | - | Mass of the front wheel (kg)  |
| $M_{wr}$   | - | Mass of the rear wheel (kg)   |
| $P$        | - | Constant auxiliary matrix of the LQR controller   |
| $Q$        | - | An 8 by 8 diagonal weighting matrix of the LQR controller   |
| $R$        | - | A 2 by 2 diagonal weighting matrix of the LQR controller  |
| $Z$        | - | State variable matrix of the active suspension system   |
| $Z_b$      | - | Vertical displacement of the car body at the centre of gravity (m)                                    |
| $Z_{bf}$   | - | Vertical displacement of the car body at the front location (m)                                       |
| $Z_{br}$   | - | Vertical displacement of the car body at the rear location (m)  |
| $Z_f$      | - | An irregular excitation from the road surface at the front car (m)                                    |
| $Z_p$      | - | State variable matrix of the passive suspension system  |
| $Z_r$      | - | An irregular excitation from the road surface at the rear car (m)                                     |
| $Z_{wf}$   | - | Vertical displacement of the car wheel at the front wheel (m)   |
| $Z_{wr}$   | - | Vertical displacement of the car wheel at the rear wheel (m)  |
| $\theta_b$ | - | Rotary angle of the car body at the centre of gravity (rad)   |
| $a$        | - | Distance of the front suspension location with reference to the centre of gravity of the car body (m) |
| $b$        | - | Distance of the rear suspension location with reference to the centre of gravity of the car body (m)  |
| $f_f$      | - | Front actuator force  |
| $f_r$      | - | Rear actuator force   |
| $r_y$      | - | Radius of gyration (m)  |

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Introduction**

The importance of car suspension systems cannot be over emphasized. This is because, it provides steering stability and takes care of friction between the tires and the road surface. It also gives support to the vehicle and equally ensures maximum comfort of the passengers. Typically, the performance of the suspension systems is measured using three criteria; ride comfort, suspension deflection and the road handling. This means that, reducing the angular acceleration and the numerical body axis of the vehicle will improve the quality of ride comfort. The utility of suspension deflection in the restricted rattle space is the major target of all suspension systems designed to achieve ride comfort. Good handling prevents the car from excessive rolling and pitching during maneuvers while passengers comfort isolates vehicle passengers from road disturbances (Readman et al, 2008).

A suspension system is a mechanism that separates the car body from the car wheel. It is also a combination of spring, shocks absorbers and linkages that connect a vehicle body to its wheel. The suspension systems are of three types; passive, semi-active, and active.

A passive suspension possesses the capability to stored energy using spring and also dissipates the energy using damper. The parameters of the passive suspension are fixed and chosen to arrive at a certain level of compromise between

road handling, load carrying and the ride comfort. If the suspension is being controlled externally, it is referred to as semi-active or active suspension.

Meanwhile, active suspension also has the capability to store, dissipate as well as inject energy to the systems and its parameters may vary depending on operating conditions.

## **1.2 Problem Statement**

The open loop control nature of the passive suspension systems makes it difficult to be adjusted by any mechanical part. This means that in the design consideration if it is critically damped (the suspension is too hard), a lot of energy will be transferred to road input. That is to say the car will be thrown on unevenness on the road. While if it is lightly damped (suspension is very soft), the stability of the car is reduced and consequently will make the vehicle to change lane or even swing on the road. Therefore, it can be stated that the performance of the passive suspension depends on various road profiles.

However, the closed-loop control nature of the active suspension systems using hydraulic force actuator will provide better performance in terms of stability and ride comfort. This actuator is a mechanical part incorporated inside the systems so as to be controlled and manipulated by controller. The controller with the help of data of road profile provided by sensors as input will calculate to add or dissipate energy from the system. This makes it more stable, efficient and effective.



In such a situation, there is a need for active element inside the system which possesses the ability of absorbing the vertical acceleration of the wheel to allow the frame and car body to ride without any disturbances while the wheels follow the bump on the road. Therefore, this research work is intended to investigate the performance of active suspension system with the application of three different controllers.

### **1.3 Research Objectives**

The following are the objectives of this research project:

- i. To obtain the mathematical model for passive and active suspension systems based on half car model.
- ii. To design a state feedback and an intelligent controllers to control the active suspension system (LQR, FL and PID).
- iii. To compare the performance of each controller with the passive suspension system using simulation results from Matlab/Simulink software with a road profile as input to the system.

### **1.4 Project Scope**

The following are the scope of work for this project:

- i. The research will consider derivation of mathematical equations of passive and active suspension systems for half car model only.

- ii. Controllers will be designed to implement the mathematical model of the passive and active suspension systems.
- iii. Matlab/Simulink software will be used to obtain the simulation results for comparison and analysis.

## 1.5 Overview of Research Methodology

To understand passive and active suspension components:

- i. Literature review on passive and active suspension systems
- ii. Review on the types of suspension systems
- iii. Review on the control strategy.

To obtain mathematical models of passive and active suspension systems for half car model:

- i. Applying physical laws to the suspension components to find the state space equations for passive half car model, then proceed to get the equations for active half car model.
- ii. Manipulating matrix equations to find the state space equations

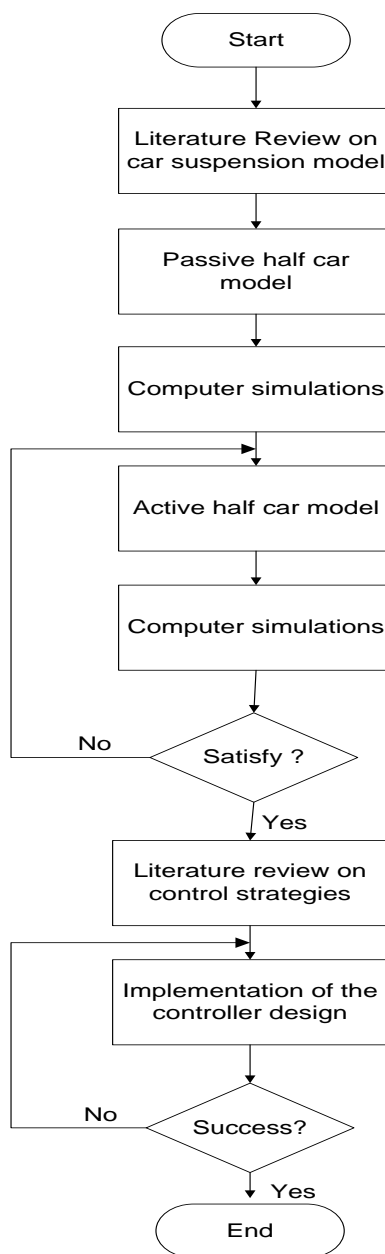
To implement the controller design into systems for performance analysis and comparison:

- i. Literature review on the control techniques
- ii. Comparison of the performance of each controller with passive and active suspension systems

To obtain the computer simulation results:

- i. By changing the state space equations into Simulink diagrams
- ii. Using simulation results for analysis.

Below is the flow chat to show the overall process for this project:



**Figure 1.1** Project process flow chat

## REFERENCES

- Chiou, J. S., and Liu, M. T. (2008). Combining Support Vector Machines with Linear Quadratic Regulator Adaptation for the Online Design of an Automotive Active Suspension System. *Journal of Physics: Conference Series* 96. 1-8.
- Yagiz, N., and Hacıoglu, Y. (2008). Backstepping control of a vehicle with active suspensions. *Control Engineering Practice*. 16, 1457-1467.
- A.M. Kashtiban, et al. (2009). Nonlinear Optimal Control of a Half Car Active Suspension, 2009 Second International Conference on Computer and Electrical Engineering page 460-464.
- Ozgur Demir, et al. (2011). Modelling and control of a nonlinear half-vehicle suspension system: a hybrid fuzzy logic approach, Springer Science+Business Media B.V. 2011.
- Agharkakli A. et al. (2012). Simulation and analysis of passive and active suspension systems using quarter car model for different road profile, international journal of engineering trends and technology, 3(5) page 636-644.
- Yu S. et al. (2013). Full-Car Active Suspension Based on H<sub>2</sub> / Generalized H<sub>2</sub> Output Feedback Control, IEEE International Conference on Control Applications (CCA), page 241-246.
- Sam, Y. M., Ghani, M. R. A., and Ahmad, N. (2000). LQR Controller for Active Car.

Suspension. *Proceedings of the IEEE TENCON 2000*. 441-444.

Cao, J., Liu, H., Li, P., Brown, D., and Dimirovski, G. (2007). An Improved Active Suspension Model for Attitude Control of Electric Vehicles. *Proceedings of the 2007 IEEE International Conference on Mechatronics and Automation*. August 5-8, 2007. Harbin, China, 147-152.

Huang, C. J. and Lin, J. S., (2004). Nonlinear Backstepping Active Suspension Design Applied to a Half-Car Model. *Vehicle System Dynamics*. 42 (6), 373-393.

Li, H., Tang, C. Y., and Zhang, T. X. (2008). Controller of Vehicle Active Suspension Systems Using LQG Method. *Proceedings of the IEEE International Conference on Automation and Logistics*. September 2008. Qingdao, China, 401-404.

Sam, Y. M., Johari H.S. Osman, J. H. S., and Ghani, M. R. A. (2004). A Class of Proportional-Integral Sliding Mode Control with Application to Active Suspension System. *Systems & Control Letters*. 51, 217-223.

Sam, Y. M., and Osman, J. H. S. (2005). Modeling and Control of the Active Suspension System using Proportional Integral Sliding Mode Approach. *Asian Journal of Control*. 7(2), 91-98.

Nan, Y. H., Xuan, D. J., Kim, J. W., Ning, Q., and Kim, Y. B. (2008). Control of an Active Suspension Based on Fuzzy Logic. *International Conference on Computer and Electrical Engineering*. Dec 20-22, 2008. Phuket, Thailand, 303-307.

- Fialho, I and Balas, G. J. (2002). Road Adaptive Active Suspension Design Using Linear Parameter-Varying Gain-Scheduling. *IEEE Transactions on Control Systems Technology*. 10(1), 43-54.
- Lin, J. S., and Kanellakopoulos, I. (1997). Nonlinear Design of Active Suspensions. *IEEE Control Systems Magazine*. 17(3), 45-59.
- Suaib, N. M, and Sam, Y. M. (2008). Modeling and Control of Active Suspension using PISMC and SMC. *Jurnal Mekanikal*. Dec (26), 119-128.
- Chen, H., and Guo, K. H. (2005). Constrained  $H$  Control of Active Suspensions: An LMI Approach. *IEEE Transactions on Control Systems Technology*. 13(3), 412-421.
- Ho, H. F., Wong, Y. K., and Rad, A. B. Adaptive Fuzzy Sliding Mode Control with Chattering Elimination for Nonlinear SISO Systems. *Simulation Modeling Practice and Theory*. 17 (2009), 1199-1210.
- John, E. D. E., Olurotimi, A. D., Jimoh, O. P. (2011). PID Control of a Nonlinear Half-Car Suspension System Via Forced Feedback. *IEEE African 2011-The Falls Resort and Conference Centre*. September, 2011. Livingstone, Zambia, 978-993.
- Rajeswari, K., and Lakshmi, P. (2008). GA Tuned Distance Based Fuzzy Sliding Mode Controller for Vehicle Suspension Systems. *International Journal of Engineering and Technology*. 5(1), 36-47.