# THE APPLICATION OF DISCRETE HOMOTOPY ANALYSIS METHOD IN ONE-DIMENSIONAL THERMAL PROBLEM

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*To my beloved father, Ooi Seng Chee, mother, Chong Yeat Koon, sisters, brother, and all of my dear friends for their advices, love and support.*

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# ABSTRACT

Many engineering phenomena can be performed in mathematical modelling. Many thermal problems can be presented in Fredholm integral equation. A discretized version of Homotopy analysis method which introduced by Behiry et al (2010) is applied for solving the Fredholm integral equation. Besides that, we used a numerical method which is trapezoidal rule to solve the problem and make comparison with DHAM results. Comparison of approximation results from both methods demonstrates that DHAM is more accurate than trapezoidal rule as the DHAM convergent series solutions are well coincide with the exact solution. The convergence control parameter $\hbar$ in DHAM is able to find the convergence region and control the convergence region of series solution. Moreover, DHAM can help to quicken the convergence of series solution. Therefore, DHAM is a powerful tool to solve the non-linear problem and can be applied to many others linear problems as well. Last but not least, DHAM is a good technique in solving non-linear problems in the science and engineering field.

# ABSTRAK

Banyak fenomena kejuruteraan boleh diwakili oleh pemodelan matematik. Banyak masalah haba boleh dimodelkan dalam persamaan kamiran Fredholm. Kaedah analisis homotopy diskrit yang diperkenalkan oleh Behiry et al (2010) telah digunakan untuk menyelesaikan persamaan kamiran Fredholm. Seterusnya, keadah trapezium digunakan untuk menyelesaikan masalah ini dan perbandingan penyelesaian dari kedua-dua kaedah ini telah dibuat. Perbandingan keputusan anggaran dari kedua-dua kaedah menunjukkan bahawa DHAM adalah lebih tepat daripada kaedah trapezium kerana siri jawapan bagi DHAM boleh menumpu dengan baik dan bertepatan dengan jawapan tepat. Parameter kawalan penumpuan, $\hbar$ dalam DHAM boel membantu untuk mencari rantau penumpuan dan mengawal rantau penumpuan bagi siri jawapan. Selain itu, DHAM boleh membantu untuk mempercepatkan penumpuan siri jawapan. Oleh itu, DHAM adalah kaedah yang sangat baik untuk menyelesaikan masalah tak linear dan boleh digunakan untuk masalah linear yang lain juga. Akhir sekali , DHAM adalah teknik yang baik untuk menyelesaikan masalah tak linear dan boleh diperluas penggunaannya dalam bidang sains dan kejuruteraan.

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# INTRODUCTION

## 1.1 Introduction

In recent year, the concepts of integral equations have been applied in many fields. Its application to diverse branches of engineering, especially to the problems arising in power engineering is possible. In scientific and engineering fields, there have been great and significant developments in the nonlinear phenomena which increase rapidly in many applications. The fields that can be solved by integral equations are like particle physics, fluid dynamics, molecular biology, quantum mechanics, geology, cosmology, oceanography, astrophysics, electricity and magnetism, mathematical economics and others. Therefore, it is important to find the higher accurate solution for the integrals equations. A lot of methods of integral equations for obtaining the numerical solution can be used to solve the problems in science and engineering fields. Thus, we have to choose a suitable method to work out the problem but at the same time we need to try to reduce the size of the system of the equations and also the computation time while maintain its accuracy. With the assist of some computer system or symbolic computation software such as MATLAB, C programming, FORTRAN, MAPLE and so on, it is helping us to compile the numerical result and to get the highly accurate answer within a short time.

## 1.2 Background of the Study

Many problems in science and engineering fields can be modeled by integral equation. In various branches of linear and nonlinear functional analysis, discrete Homotopy is the important method to solve them. In order to find the numerical solution of the linear and nonlinear integral equations, varies methods are applied and have been explored. Nonlinear problems are hard to be solved when compared with linear problems especially analytically. There are two criterions for a satisfactory analytic method of nonlinear problem:

i. Approximation expression can always be provided expeditiously.
ii. The approximation expression is highly accurate in the whole region of physical parameters.

Perturbation technique is a famous method used in science and engineering to assist us to understand more about the nonlinear problems. But there is always an obstacle exist in this technique when solving the nonlinear problem that is perturbation technique depend on small or large physical parameter in the equation. Many nonlinear problems do not contain such kind of perturbation quantities. It is well known that analytic approximations of nonlinear problems often break down as nonlinearity becomes strong and perturbation approximations are valid only for nonlinear problems with weak nonlinearity. From previous papers done by researchers, we can see that many others methods such as Adomian's decomposition method (ADM), variational iteration method (VIM), the *δ-expansion* method, Homotopy perturbation method and so on are used instead of using perturbation technique. These methods are so called non-perturbation method which does not depend upon the small or large physical parameter. However, there also exist some barriers in these non-perturbation methods that is the convergence of the solution series cannot be ensure, the methods difficult to apply and require tedious calculation.

In the past few years, in order to overcome these obstacles, a new analytic method namely ‘Homotopy analysis method (HAM)’ has been introduced. HAM was proposed by Liao in 1992, it was powerful analytic approach to get series solution of strong nonlinear equations. This approach is different with others methods that mentioned before. HAM provided us a simple way to control the convergence region and the rate of convergence of a series solution of nonlinear problem. Many research papers done are shown that through this method, we can get the important and sufficient condition for convergence of series solutions. Besides, a highly accurate solution could be obtained if applied on linear or nonlinear integral equations. Although HAM is widely used to solve nonlinear problems, however in some cases, it is not suitable to apply on. For example, a lot of definite integral need to be computed when solves linear and nonlinear Fredholm integral equation, the evaluation of integrals analytically may become impossible or too complicated. Thus another method was introduced namely ‘discrete Homotopy analysis method (DHAM)’, it is a discretized version of HAM.

In fact, DHAM is used when the quadrature rules are used to approximate the definite integrals which cannot be computed analytically. There are some researches done in using DHAM in order to find the approximation solutions of linear or nonlinear partial differential equations (PDEs). The convergence of DHAM was proved that the theoretical basic of the DHAM can be provided under suitable and rational hypotheses. There are many advantages similar among DHAM and continuous HAM. For instance, an auxiliary parameter, $\hbar$, was introduced to control and adjust the convergence region.

There were many approaches for determining the numerical solution for the integral equations. The aim in this research is to extend the application of discrete Homotopy analysis method to find the numerical solution for Fredholm integral equations. The method of integral equation is proposed for some one-dimensional thermal problems in real-world engineering and science problem. Presented models or problems then lead to integral equations to be solved. In order to get the approximation numerical solution, a computational program will be studied by using

C programming. In addition, a comparison is made with other numerical methods in this research. The pictures of one-dimensional heat conduction show as below:

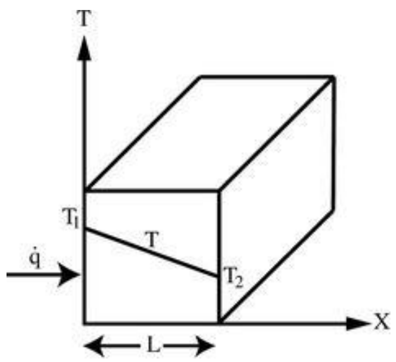


Figure 1.1: One-dimensional heat transfer

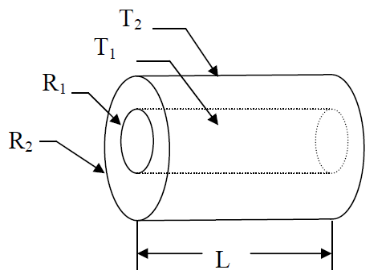


Figure 1.2: Conduction through a cylinder

## 1.3 Statement of the Problem

Modelling of natural phenomena and problems mostly leads to solving nonlinear equations. In engineering and science world, there are many problems such as heat problems which is difficult to be solved but if these problems modelled into the integral equations, the final equation will become simple and easier to be work out. Mathematical model in the form of integral equations can describe more well about the current engineering problems. In the heat conduction problems and several physical, technological and biological problems such as epidemiology problems, the integral equations work as an important role. There are various methods such as Trapezium rule, Simpson rule, Gaussian quadrature and many others methods are exist for solving the integral equations. But these techniques are suitable and valid only for simple and weakly nonlinear problems. DHAM has been discovered that the highly accuracy numerical solution can be obtained if applied to the problems model. In short, DHAM is a powerful method and higher accuracy in solving nonlinear one-dimensional heat conduction problems.

## 1.4 Objectives of the Study

The objectives of the study are as follow:

1. To apply discrete Homotopy analysis method for the one-dimensional thermal integral equation.

2. To analyze the numerical results of engineering problems in the Fredholm integral equations.

3. To make comparison of numerical result between the discrete Homotopy analysis method and trapezoidal rule.

## 1.5 Scope of the Study

This study discuss about the application of discrete Homotopy analysis method to solve one-dimensional of heat integral equation in science and engineering problem. The problems especially heat problem would be modeled into nonlinear integral equation. Integral equation is easier to be solved if compare with other equation such as differential equation. Therefore, discrete Homotopy analysis method was used to solve the Fredholm integral equation. In addition, we present a numerical method to solve the problem that is trapezoidal rule. So that we can make comparison between the numerical solution get from the methods and show which of method is more accurate in solving nonlinear problem. This proves that engineering required mathematics in emphasizing concepts and solving problems. Furthermore, the use of mathematical software or tool is necessary. The availability of fast and inexpensive computer allows problems which are difficult and intractable can be solved mathematically. In this research, we will use C programming to solve the nonlinear Fredholm integral equation in order to get the numerical solution.

## 1.6 Significance of the Study

The result of this research will give benefits to mathematics and engineering fields. Many engineering problems are nonlinear problems. As we known nonlinear equation is difficult to be solved analytically if compare with linear equation. Therefore, the mathematical concept, analytic method or mathematic tools such as discrete Homotopy analysis method, integral equations and C programming are used to simplify the complicated problems. Liao (2009) has proved that many types of nonlinear equations are successfully be solved by applying HAM into the equations. Therefore, this research will show that discrete version Homotopy analysis method is a useful mathematical tool for solving nonlinear equations.

## 1.7 Outline of Report

This aim of this report is to apply the discrete Homotopy analysis method for solving one-dimensional thermal problem. This report consists of five chapters and it is organised as follows. In chapter 1, the introduction, describing background of the problem, statement of the problem, objectives of the study, scope and significance are demonstrated. In chapter 2, some examples that done by previous researcher related with the topic are discussed. The concept, theory and review of Homotopy analysis method, discrete Homotopy analysis method and Trapezoidal rule are introduced in chapter 2 as well. In chapter 3, DHAM and Trapezoidal rule are showed to be applied into nonlinear Fredholm integral equation. Chapter 4 discussed about the implementation of DHAM and trapezoidal rule into problem, calculations are carried out from C programming and the comparison of results calculated from the two methods. Some graphs may use to present the numerical result. In last chapter which is chapter 5, the study is be concluded and summarized. Last but not least, few useful recommendations are suggested for further research.

For the sake of clarity, the flow chart of the whole simulation is presented as below.

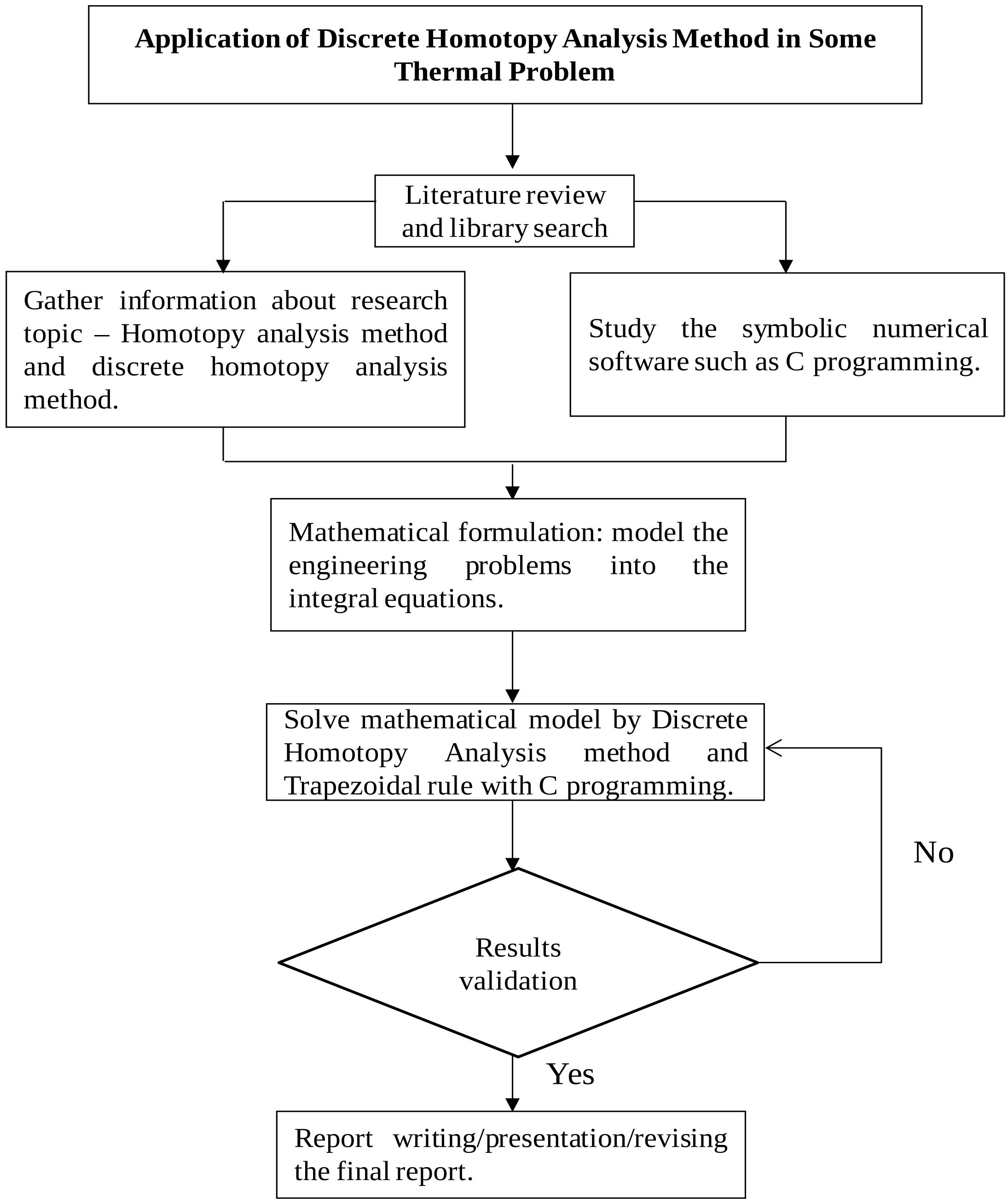


Figure 1.3: Research Methodology Chart

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