STOCHASTIC MODELING FOR RIVER POLLUTION OF SUNGAI PERLIS

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Very special dedication for:

My beloved father: *Hj. Mohd Yunus Bin Hj. Abdullah*.

My dearest mother: *Hjh. Zawiaton Binti Hj. Zainal Ariffin*.

My precious brothers and sisters: *Redzuan, Shaliza, Nizamuddin & Siti Abidah.*

And

My adorable nieces: *Ryfaa Nur Safiya & Siti Nur Zafirah*.

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ABSTRACT

River pollution has been recognized as a contributor to a wide range of health problems and disorders in human. It can pose health dangers to humans who come into contact with it, either directly or indirectly. Therefore, it is most important to measure the concentration of biochemical oxygen demand (BOD) as a water quality parameter since the parameter has long been the basic means for determining the degree of water pollution in rivers. In this study, BOD is used as a parameter to estimate the water quality at Sungai Perlis. It has been observed that Sungai Perlis is polluted due to lack of management and improper use of resources. Therefore, it is of importance to model the Sungai Perlis water quality in order to describe and predict the water quality systems. The BOD concentration secondary data set is used which was extracted from the Drainage and Irrigation Department Perlis State website. The first order differential equation from Streeter – Phelps model was utilized as a deterministic model. Then, the model was developed into a stochastic differential equation (SDE) model. Results from this study shows that the SDE model is more adequate to describe and predict the BOD concentration and the water quality systems in Sungai Perlis by having smaller value of mean squared error (MSE).

ABSTRAK

Pencemaran sungai telah diiktiraf sebagai penyumbang kepada pelbagai masalah kesihatan dan gangguan kepada manusia. Ia boleh menimbulkan bahaya kepada kesihatan manusia yang bersentuhan dengannya, sama ada secara langsung atau tidak langsung. Oleh itu, ia adalah yang paling penting untuk mengkaji kepekatan permintaan oksigen biokimia (BOD) sebagai parameter kualiti air kerana parameter ini telah lama menjadi asas untuk menentukan tahap pencemaran air di dalam sungai-sungai. Dalam kajian ini, BOD digunakan sebagai parameter untuk menganggarkan kualiti air di Sungai Perlis. Sungai Perlis diperhatikan telah tercemar kerana kekurangan pengurusan dan penyalahgunaan sumber. Oleh itu, adalah penting untuk memodelkan kualiti air di Sungai Perlis bagi menerangkan dan meramalkan system-sistem kualiti air. Set data sekunder kepekatan BOD yang digunakan telah diperolehi daripada laman web Jabatan Pengairan dan Saliran Negeri Perlis. Persamaan pembezaan peringkat pertama dari model Streeter – Phelps telah digunakan sebagai model berketentuan. Kemudian, model itu dikembangkan menjadi model persamaan pembezaan stokastik (SDE). Hasil daripada kajian ini menunjukkan bahawa model SDE adalah lebih mencukupi untuk menerangkan dan meramalkan kepekatan BOD dan sistem-sistem kualiti air di Sungai Perlis dengan mempunyai nilai ralat purata kuasa dua (MSE) yang kecil.

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LIST OF ABBREVIATIONS

INWQS	-	Interim National Water Quality Standards
WQI	-	Water Quality Index
DOE	-	Department of Environment
BOD	-	Biochemical Oxygen Demand
SDEs	-	Stochastic Differential Equations
SDE	-	Stochastic Differential Equation
DID	-	Drainage and Irrigation Department
MSE	-	Mean Squared Error
WHO	-	World Health Organization
NWRC	-	National Water Resources Council
DO	-	Dissolved Oxygen
USEPA	-	U.S. Environmental Protection Agency
APHA	-	American Public Health Association
ODEs	-	Ordinary Differential Equations
ODE	-	Ordinary Differential Equation
GWN	-	Gaussian White Noise
TSS	-	Total Suspended Solid
GBM	-	Geometric Brownian motion
1S1R	-	One State One River

LIST OF SYMBOLS

X_t	-	Random variables for stochastic process
t	-	Time of reaction
mg/L	-	Milligram per liter
$^{\circ}$C	-	Degree Celsius
%	-	Percent
$\frac{dx}{dt}$	-	Ordinary differential equation
$f(t,x)$	-	Deterministic function
$F(t,X,W)$	-	Random function
W	-	Random coefficient
$\frac{dX(t)}{dt}$	-	Stochastic differential equation
$\frac{dW(t)}{dt}$	-	Derivative of the Wiener process
aX_t	-	Drift term
bX_t	-	Diffusion term
°	-	Degree
kg/day	-	Kilogram per day
B	-	BOD concentration level, mg/L
k	-	Reaction rate coefficient, day $^{-1}$
B_0	-	Ultimate BOD, mg/L
ξ	-	White noise process

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CHAPTER 1

INTRODUCTION

1.1 Introduction

River is a natural stream flowing towards an ocean, a lake, a sea, or another river. In few cases, a river simply flows into the ground or dries up completely before reaching another water body. Small rivers may also be called by several other names, including stream, creek, brook, rivulet, run tributary and rill. There are no official definitions for generic terms, such as river, is applied to geographic features, although in some countries or communities a stream may be defined by its size (*Source*: water.ualberta.ca/en/About/WhyAlberta/Rivers.aspx). When the rivers are classified using Roman numerals I to V according to the Interim National Water Quality Standards (INWQS), it is known as water quality index (WQI). WQI is a measurement used as a basis for assessing the level of river water pollution.

Nowadays, it has become a priority to model the water quality for simulating the movement of pollutants. Many researchers from all over the world get involved in the water quality modeling research and they had created a large number of water quality model. The key elements for suitable water quality model creation are related with the good knowledge about a change in the river system. Most of these models are necessary to tackle the specific water quality problems encountered in specific environmental and socioeconomic conditions (Tsakiris and Alexakis, 2012). Water quality models can be classified according to the type of approach, pollutant items, area of applications, nature and so on. However, this study focuses more on how to

model the water quality due to the necessity to describe and predict the water quality conditions. There are many different types of models used in the water quality modeling for many different problems and purposes. Thus, the appropriate model and the required data depend on the purpose of the specific study.

Recent studies show that most of the rivers in Malaysia are polluted. In 2004, according to the Department of Environment (DOE), based on Biochemical Oxygen Demand (BOD), 18 river basins were classified polluted, 37 river basins were slightly polluted and 65 river basins were in clean condition. Based on the Malaysian River Classification and River Water Quality Monitoring Project Reports, the general trend of the overall river water quality in Malaysia is deteriorating. The deterioration of river water quality is normally due to several factors such as discharge from domestic sewage, industry, livestock and agriculture (Abu Bakar and Dalilah, 2007). If the water quality problems are being ignored, it will pose serious adverse environmental problems. Therefore, the level of water quality must be analyzed as the first step to solve the water quality problems. Figure 1.1 shows the water quality status for river basins of Peninsular Malaysia.

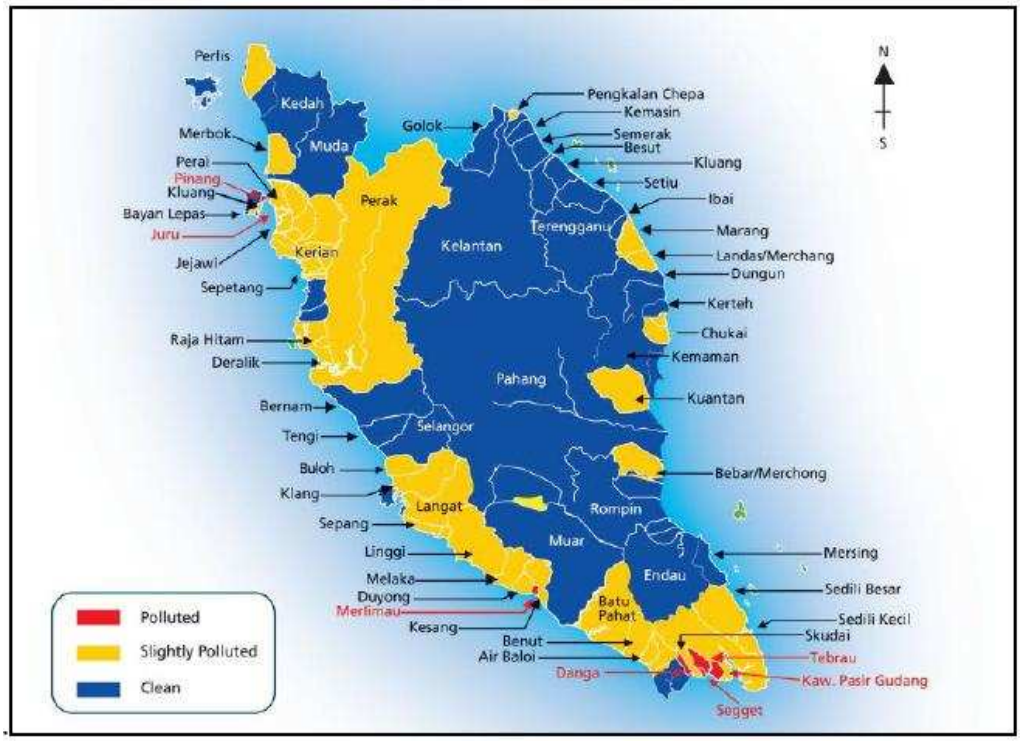

Figure 1.1 Water quality status for river basins of Peninsular Malaysia (Izamudin, 2011)

According to the National Report Malaysia in 2003, the main factors such as sediment run-off, industrial waste, domestic waste, agricultural, livestock and heavy metal that influenced the water quality in Malaysia will lead to deterioration of river water. In order to solve the deterioration of river water, it is very important to model the water quality of the rivers. There are many types of water quality models due to wide variety of river systems. Thus, each type of river water body needs the appropriate type of model.

In some studies, there are researchers that comparing a mathematical model with another mathematical model such as stochastic model and deterministic model. The studies show the result from the stochastic model is more dynamic if compared to the result from the deterministic model. For example, in a study of Water Pollution Models based on Stochastic Differential Equations (Huang and Morimoto, 2004) it is observed that from the beginning of the material input, mixing and reaction, to the end of material input, the system is dynamic on inner components such as turbulent influx in feed-rates, random perturbation in concentration and the system changes with changing factor. The study shows the three-step model has the advantage of prediction and control the water pollution by using the numerical solution of stochastic differential equations (SDEs) of the three-step model.

In this study, it is focusing on how to model the water quality of Sungai Perlis. The stochastic model will be derived from the deterministic equation using the perturbation of white noise. In applied science, white noise is a mathematical idealization of phenomena involving sudden and extremely large uncertainties. It is one of the stochastic processes. Stochastic processes are processes that proceed randomly in time. Specifically, a stochastic process is a collection of time indexed random variables, X_t where t belongs to an index set. Stochastic processes have been applied in many fields such as engineering, physics, biology, electronics, management science, economics, and psychology, and in operations research, they have spread widely over queuing, finance, and insurance. Especially, in reliability theory, stochastic processes are the most powerful mathematical tools for analyzing reliability models (Nakagawa, 2011).

1.2 Background of the Study

Perlis is the smallest state in Malaysia. It measures approximately 810 km^2 and has a population of 217,480. Sungai Perlis has a catchment area of 350 km^2 and the length of 9.6 km. The river is located at the north of Peninsular Malaysia with latitude 6.40° and longitude 100.13°; and it is flowing in the middle of Perlis from Bandar Kangar to Kuala Perlis. The upstream area of Sungai Perlis is located in Malaysia – Thailand border and flows to Kuala Perlis. Nowadays, Sungai Perlis is the center stage for the public to socialize and to perform recreational activities especially at Denai Larian, Perlis. The estimated number of visitors who use the river neighborhood as a rest and recreation area is about 10,000 a month. The map of Sungai Perlis is shown in Figure 1.2 and the plan of Sungai Perlis and its surrounding area is shown in Figure 1.3.

However, due to lack of management and improper use of resources have revealed Sungai Perlis is exposed to various pressures and the major issues from this problem are listed below. All of the issues have resulted in contamination and deterioration of Sungai Perlis water quality.

i. Low river water quality which currently ranked at Class IV.
ii. Erosion occurred on the banks of the river.
iii. At low-water conditions, the river water levels become too shallow and reveal the impurities in the river causing transportation such as boats and dugouts cannot be used.
iv. The river reserve area is occupied by too many squatters.

Therefore, it is of importance to model the Sungai Perlis water quality in order to describe and predict the water quality systems. The model is expected to play a very important role for acquiring sufficient water quality data and for better understanding in describing and predicting the water quality systems. Also, it may be used for specific pollution prevention or remediation programs. Thus, the modeling

of water quality is very useful for the purpose of describing and predicting the environmental state of a river system.

Figure 1.2 Maps of Sungai Perlis

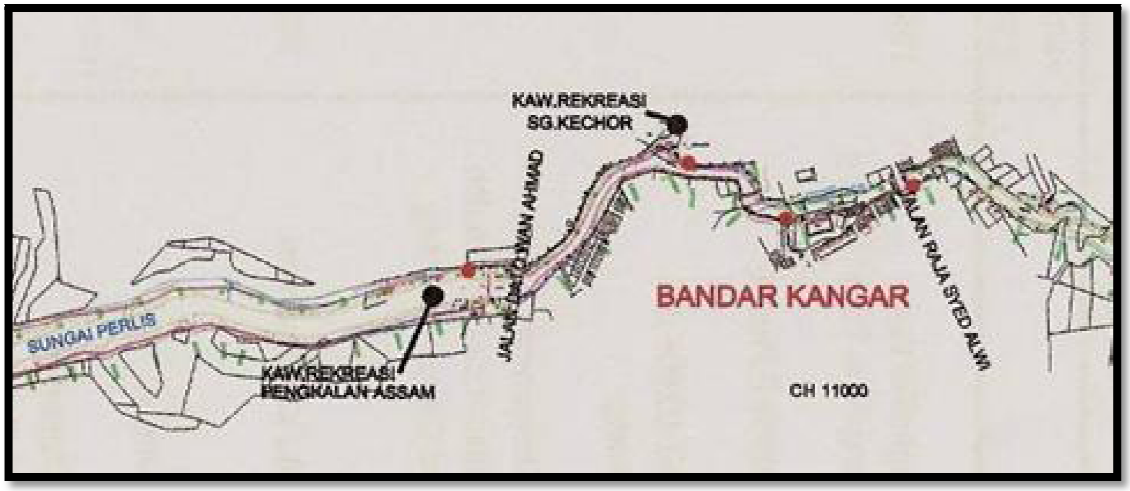

Figure 1.3 Plans of Sungai Perlis

There are many types of water quality models that have been used in many different problems and purposes. Many of today's water quality models are adequate for the simulation of simple chemical and biological reactions. They are good tools for helping people to understand and quantify water quality in river systems. It is widely known that hundred of scientific papers have been published on the field of water quality modeling. The end result of water quality modeling is to give information which can provide a basis and technique to support the environmental management agencies to make right decisions in pollution control measures.

1.3 Statement of the Problem

River pollution has been recognized as a contributor to a wide range of health problems and disorders in human. It can pose health dangers to mankind who come into contact with it, either directly or indirectly. River pollution also had shown drastically negative impacts on wild animals and the environment as a whole.

Simulation is a tool to measure the effects of river pollution on the water quality. Simulation models that contain no random variables are classified as deterministic. A deterministic model is used in that situation where in the result is established straightforwardly from a series of conditions. While, a stochastic simulation model is used in a situation where in the cause and effect relationship is stochastically or randomly determined as random inputs and will lead to random outputs. Since the outputs are random, they can be considered only as estimates of the true characteristics of a model. Now the problem is to determine the water quality model that will represent a good model which can provides more adequate information in describing and predicting the water quality systems that is between deterministic model and stochastic model.

1.4 Objectives of the Study

The main objectives of the study are listed as follows:

1. To model the water quality with deterministic and stochastic models.
2. To estimate the parameters in the deterministic and stochastic models.
3. To solve the stochastic differential equation (SDE) with the analytical and numerical solutions.
4. To evaluate the performance of the deterministic and stochastic models.

1.5 Scope of the Study

The study focuses on the modeling of Sungai Perlis water quality where the first order differential equation from Streeter – Phelps model is specifically used as a deterministic model. Then, the simple deterministic equation is extended into an SDE model using the perturbation of white noise.

In the study, the secondary data of BOD concentration were used to estimate the water quality model. The data used was extracted from the website of Drainage and Irrigation Department (DID) Perlis State. This study used daily observation data from January 2012 up to April 2013.

In order to achieve the objectives of the study, four hundred and thirty-four data were analyzed. Only one sampling station namely Sungai Perlis was used for the purpose of the study. The river water sample was collected every 15 minutes daily and the average for each day has been obtained.

1.6 Significance of the study

Sungai Perlis is the center stage for the public to socialize and to perform recreational activities. However, due to lack of management and improper use of resources resulted in a serious deterioration of water quality in Sungai Perlis. The major issues from this problem are Sungai Perlis has low river water quality which currently ranked at Class IV, the erosion occurred on the banks of the river, the water level of Sungai Perlis becomes too shallow and reveal the impurities causing boats and dugouts cannot be used as transport; and Sungai Perlis reserve area is occupied by too many squatters. Thus, it is crucial to model the Sungai Perlis water quality.

The study is necessary to determine the water quality model that will represent a good model which provides more adequate information in describing and

predicting the water quality conditions. Therefore, a good model for Sungai Perlis water quality can be discovered. The results from the study can be used in specific pollution prevention or remediation programs for better water resource management plan. Also, the water quality model that has been discovered from the study can be applied to other rivers that may have the same problem as Sungai Perlis. This study is the best measure to ensure good water quality in rivers as the model is more effective in describing and predicting the water quality systems by having a small value of mean squared error (MSE).

REFERENCES

Abu Bakar Yang and Dalilah Haji Dali. (2007). *Alam Sekitar Anugerah Tuhan*. (3rd ed.) Malaysia: Jabatan Alam Sekitar.

Allen, E. (2007). *Modeling with Ito Stochastic Differential Equation*. The Netherlands: Springer.

Amneera, W. A., Nor Wahidatul Azura Zainon Najib, Siti Rawdhoh Mohd Yusof and Santiagoo Ragunathan. (2013). Water Quality Index of Perlis River, Malaysia. *International Journal of Civil & Environmental Engineering*. 13(2), 1 – 6.

Blanchard, E., Sandu, A. and Sandu, C. (2007). Parameter Estimation Method using an Extended Kalman Filter. *Proceedings of Joint North America, Asia-Pacific ISTVS*. 23-36 June. Alaska, USA, 1 – 14.

Boano, F., Revelli, R. and Ridolfi, L. (2006). Stochastic Modelling of DO and BOD Components in a Stream with Random Inputs. *Advances in Water Resources*. 29, 1341 – 1350.

Custer, S. W. and Krutchkoff, R. G. (1969). *Stochastic Models for Biochemical Oxygen Demand and Dissolved Oxygen in Estuaries*. Virginia: Water Resources Research Center.

Dodson, C. S., Prinzmetal, W., and Shimamura, A. P. (1998). Using Excel to Estimate Parameters from Observed Data: An Example from Source Memory Data. *Behavior Research Methods, Instruments, & Computers*. 30(3), 517 – 526.

Gard, T. C. (1988). *Introduction to Stochastic Differential Equations*. New York: Marcel Dekker, Inc.

Ghangrekar, M. M. (2012). *Water Quality and Estimation of Organic Content*. Unpublished note, Indian Institute of Technology Kharagpur.

Hach, C. C., Klein, R. L. and Gibbs, C. R. (1997). *Introduction to Biochemical Oxygen Demand*. USA: Hach Company.

Haider, H., Al, W. and Haydar, S. (2013). A Review of Dissolved Oxygen and Biochemical Oxygen Demand Models for Large Rivers. *Pakistan Journal of Engineering and Applied Sciences*. 12, 127 – 142.

Haliza Abd. Rahman, Arifah Bahar, Norhayati Rosli, Mohd. Khairul Bazli Mohd.Aziz and Madihah Md. Salleh. (2010). Parameter Estimation of Stochastic Differential Equation: Bayesian Regression Spline. *Jurnal Karya Asli Lorekan Ahli Matematik*. 3(1), 17 – 28.

Huang, Z. and Morimoto, H. (2004). *Water Pollution Models Based on Stochastic Differential Equations*. Master Thesis. Nagoya University, Japan.

Hurn, A. S. and Lindsay, K. A. (1999). Estimating the Parameters of Stochastic Differential Equations. *Mathematics and Computers in Simulation*. 48, 373 – 384.

Iacus, S. M. (2008). *Simulation and Inferences for Stochastic Differential Equations with R Examples*. New York: Springer Science+Business Media, LLC.

Izamudin Mohamed. (2011). *Water Quality Assessment of Sungai Skudai*. Bachelor Thesis. Universiti Teknologi Malaysia, Skudai.

Kojiri, T and Teramura, T. (2003). Integrated River Basin Environment Assessment and Planning Through Hybrid Processes. XI IWRA World Water Congress.

Leduc, R., Unny, T. E. and McBean, E. A. (1988). Stochastic Models for First Order Kinetics of Biochemical Oxygen Demand with Random Initial Conditions, Inputs, and Coefficients. *Applied Mathematics Modelling*. 12, 565 – 572.

Li, Q. (2007). *Numerical Approximation for SDE*. Ph.D Thesis. University of Edinburgh, United Kingdom.

Maketab Mohamed. (2008). Water Quality Models in River Management. *Proceedings of the 1st Technical Meeting of Muslim Water Researchers Cooperation (MUWAREC)*. December. Johor, Malaysia, 14 – 26.

Nakagawa, T. (2011). *Stochastic Processes with Applications to Reliability Theory*. London: Springer-Verlag.

Nur Atikah Mohd Rohani. (2012). *Study on Water Quality at Manjung River and Its Tributaries*. Bachelor Thesis. Universiti Teknologi Petronas, Perak.

Nurul Hana Mohamed. (2010). *Stochastic Modelling of Water Quality*. Bachelor Thesis. Universiti Teknologi Malaysia, Skudai.

Ojha, P. K. (2012). *Numerical Approximation Methods for Solving Stochastic Differential Equations*. Master Thesis. National Institute of Technology, Rourkela.

Picchini, U. (2007). SDE Toolbox: Simulation and Estimation of Stochastic Differential Equations with Matlab. Retrieved 5th December from http://sdetoolbox.sourceforge.net.

Raja Zaharaton Raja Zainal Abidin. (2008). *Water Resources Management in Malaysia – The Way Forward.* Kuala Lumpur: The Board of Engineers Malaysia.

Sauer, T. (2012). *Numerical Solution of Stochastic Differential Equations in Finance*. Duan, J. C., Hardle, W. K. and Gentle, J. E. *Handbook of Computational Finance*. (529 – 550). Berlin: Springer-Verlag.

Singh, B. (2004). *Determination of BOD Kinetic Parameters and Evaluation of Alternate Methods*. Master Thesis. Thapar Institute of Engineering & Technology, Patiala.

Siwiec, T., Kiedryńska, L., Abramowicz, K., Rewicka, A. and Nowak, P. (2011). BOD Measuring and Modelling Methods – Review. *Annals of Warsaw University of Life Sciences, Land Reclamation*. 43(2), 143 – 153.

Streeter, H. W. and Phelps, E. P. (1925). *A Study of the Pollution and Natural Purification of the Ohio River*. United States: Public Health Service.

Tsakiris, G. and Alexakis, D. (2012). Water Quality Models: An Overview. *European Water*. 37, 33 – 46.

Varziri, M. S., McAuley, K. B. and McLellan, P. J. (2008). Parameter and State Estimation in Nonlinear Stochastic Continuous-Time Dynamic Models with Unknown Disturbance Intensity. *The Canadian Journal of Chemical Engineering*. 86(5), 828 – 837.

Young, P. and Beck, B. (1974). The Modelling and Control of Water Quality in a River System. *Automatica*. 10, 455 – 468.

Zaki Zainudin. (2010). Benchmarking River Water Quality in Malaysia. *Jurutera*. 12 – 15.