PREDATOR-PREY MODEL WITH CONSTANT RATE OF HARVESTING

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Special dedicate to my beloved family

Arifin Bin Abdullah

Rohani Binti Dali

Mohd Fakharuddin Bin Arifin

Nurulain Hanis Binti Shahruni

Mohamad Amirudin Bin Arifin

and

Nurul Atiqah Arifin

Thanks for all the supports, sacrifices, patients and willingness to share my dreams with me...

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ABSTRACT

Predator-prey model is the first model to illustrate the interaction between predators and prey. It is a topic of great interest for many ecologists and mathematicians. This model assumes that the predator populations have negative effects on the prey populations. The generalized equation of this model is the response of the populations would be proportional to the product of their population densities. Prey population grows with limited by carrying capacity, K and it is called the logistic equation. Thus, in this research, there are four different cases are analyzed which are predator-prey model, predator-prey model with harvesting in prey, predator-prey model with harvesting in predator and predator-prey model with harvesting in both populations. Systems of ordinary differential equation are used for all models. The objectives of this research are i) to study the concept of Lotka-Volterra predator-prey model, ii) to analyze the predator-prey model with constant rate of harvesting in prey, iii) to analyze the predator-prey model with constant rate of harvesting in predator, iv) to analyze the predator-prey model with constant rate of harvesting in both populations. In analyzing all four models, equilibrium points will be obtained and analyzed for the stability by using Routh-Hurwitz Criteria. Lastly, some numerical examples and graphical analysis are shown to illustrate the stability of the stable equilibrium points and the effects of harvesting to the systems.

ABSTRAK

Model Pemangsa-Mangsa merupakan model yang pertama untuk menggambarkan interaksi antara pemangsa dan mangsa. Ia adalah satu topik yang menarik minat besar bagi semua ahli ekologi dan ahli matematik. Model ini mengandaikan populasi pemangsa memberi kesan negatif kepada populasi mangsa. Persamaan umum model ini adalah tindak balas sesuatu populasi yang berkadar dengan hasil kepadatan sesuatu populasi. Populasi mangsa membesar dengan dihadkan oleh keupayaan membawa, K dan ia dipanggil sebagai persamaan logistik. Oleh itu, dalam kajian ini, terdapat empat kes berbeza yang dianalisis iaitu model pemangsa-mangsa, model pemangsa-mangsa dengan penangkapan dalam mangsa, model pemangsa-mangsa dengan penangkapan dalam pemangsa dan model pemangsa-mangsa dengan penangkapan dalam kedua-dua populasi. Sistem persamaan pembezaan biasa digunakan untuk keseluruhan model. Objektif kajian ini adalah i) untuk mengkaji konsep model Lotka-Volterra Pemangsa-mangsa, ii) untuk menganalisis pemangsa-mangsa model dengan kadar tetap penangkapan dalam mangsa, iii) untuk menganalisis pemangsa-mangsa model dengan kadar tetap penangkapan dalam pemangsa, iv) untuk menganalisis pemangsa-mangsa model dengan kadar tetap penangkapan dalam kedua-dua populasi. Dalam menganalisis keempat-empat model tersebut, titik keseimbangan akan diperolehi dan kestabilannya akan dianalisis dengan menggunakan Kriteria Routh-Hurwitz. Akhir sekali, beberapa contoh berangka dan analisis graf ditunjukkan untuk memperlihatkan kestabilan titik keseimbangan dan kesan penangkapan dalam sistem.

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LIST OF SYMBOLS

$x(t)$	-	Prey population
$y(t)$	-	Predator population
H_x	-	Prey harvesting
H_y	-	Predator harvesting
$\lambda_{1,2}$	-	Eigenvalues of equilibrium point
r	-	Growth rate of prey
K	-	Carrying capacity of the prey.
α	-	Rate of consumption of prey by predator.
β	-	Conversion of prey consumed into predator reproduction rate.
c	-	Death rate of predator

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CHAPTER 1

INTRODUCTION

1.1 Background of Research

In the ecology system, the predator-prey model is amongst the oldest studies and also the first model to illustrate the interaction between predators and prey. This model assume that the predator populations have negative effects on the prey populations and this system was formulated by Vito Volterra, an Italian mathematician and Alfred Lotka, an American mathematical biologist in 1925 (Boyce and DiPrima, 2010).

There are several known predator-prey models such as Lotka-Volterra model, Logistic equation, Holling Tanner Type 2 and Type 3 models, Yodzis model and so on. This study only focuses on the most well known Lotka-Volterra model. This model has been analyzed by various text books in dynamical systems, mathematical biology, ecology, differential equations etc. The generalized equation of this model is the response of the populations would be proportional to the product of their population densities. The prey population grows infinitely in the absence of predators. Therefore, the logistic equation or often called as carrying capacity of the environment was added to the prey equation given,

$$\frac{dx}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - \alpha x(t)y(t),$$
$$\frac{dy}{dt} = \beta x(t)y(t) - cy(t), \tag{1.1}$$

where

$x(t)$ denotes prey population.

$y(t)$ denotes predator population.

r denotes the growth rate of prey.

K denotes carrying capacity.

α denotes the rate of consumption of prey by predator.

β denotes the conversion of prey consumed into predator reproduction rate.

c denotes death rate of the predator.

Our study also continues by adding constant rate of harvesting to the model. In the previous research, Brauer (1977), Dai and Tang (1998), Martin and Ruan (2001), Kar and Pahari (2006), Syamsuddin and Malik (2008), Xia et al. (2009) and Agarwal and Pathak (2012) had analyzed the constant rate of harvesting either prey or predator population. According to Kar and Pahari (2006), many of interesting dynamical behaviors such as the stability of the equilibrium points, existence of Hopf bifurcation and limit cycles have been observed. Therefore, this study concern on analyze the stability of the model by adding the harvesting to both populations by using the method of differential equation.

Lotka-Volterra predator-prey model plays a crucial role in the population dynamics and also one of the major advanced theories introduced by Lotka and Volterra. They assumed the response of the population would be proportional to the product of their population densities without any delayed and constant rate of harvesting. The model formulation is in the equation (1.1). Some of the studies found that the rate of harvesting has been used to control the increasing of population and as a controller of the population density (Ouncharoen et al., 2010). This means that harvesting can make whether the population increases or decreases for a continuity yields but it will limited by a carrying capacity in equation (1.1).

In this project, the constant rate of harvesting will be added to either preys, predators or both populations and we will analyze the stability of the equilibrium points of the predator-prey model.

1.2 Problem Statement

This study is focused on how constant rate of harvesting affect the dynamics of the predator-prey system.

1.3 Objectives of Research

The objectives of this research are:

1. To study the concept of Lotka-Volterra predator-prey model.
2. To analyze the predator-prey model with constant rate of harvesting in prey.
3. To analyze the predator-prey model with constant rate of harvesting in predator.
4. To analyze the predator-prey model with constant rate of harvesting in both populations.

1.4 Scope of Research

The main scope of this research is to analyze the predator-prey model by with the constant rate of harvesting. In this research, we shall only focus on two populations which are prey and predator. We will formulate the model by adding the constant rate of harvesting prey, predator or both population and also find how this factor affects to the stability of equilibrium points in predator-prey model.

1.5 Significant of Research

The findings of this research is useful for the mathematicians who are interested in the ecology fields because this research will give us more understanding in predator-prey model with or without harvesting and the effects on the stability of the population. The result obtained can be a guide by applying to another predation model in population field.

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