MATHEMATICAL MODEL OF A KIDNEY DIALYSER

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GOD

For giving inner strength

My beloved parents

For inspiration and motivation

My dearest hubby and daughter

For being very supportive

My sister and brother

Love you always

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ABSTRACT

Hemodialysis is a technique of removing waste materials and extra fluids (creatinine and urea) from the blood of kidney patients. A mathematical model of an artificial kidney dialysis machine (dialyser) is used to analyse the clearances of waste materials against the flow rate of the blood. The mathematical model is formulated using the theory differential equations. The clearance of creatinine and urea versus the flow rate of the blood are calculated as solution to the model. The graphs of the solution are plotted using Maple 12 software and compared with experimental data. We find that the calculated clearances of creatinine and urea against the rate of blood flow with experimental data are much closed.

ABSTRAK

Hemodialisis merupakan satu teknik untuk membuang bahan-bahan buangan dan cecair (creatinine and urea) yang berlebihan yang terdapat dalam darah pesakit buah pinggang. Satu model matematik bagi mesin dialisis buah pinggang digunakan untuk menganalisis ‘clearance’ bahan buangan di dalam aliran darah. Model matematik ini diterbitkan dengan mengguna teori persamaan pembezaan. ‘Clearance’ creatinine dan urea berbanding kadar aliran darah di kira sebagai penyelesaian kepada model matematik tersebut. Graf-graf penyelesaian tersebut dilakarkan dengan menggunakan perisian Maple 12 dan dibandingkan dengan data yang didapati dari eksperimen. Didapati ‘clearance’ bagi creatinine dan urea berbanding dengan kadar aliran darah dari kiraan menghampiri dengan data eksperimen.

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LIST OF SYMBOLS

AHL	-	ascending limb of the loop of Henle
CD	-	connecting duct
CI	-	cortical interstitium
Cl	-	clearance of waste material
CST	-	continuously stirred tank
DHL	-	descending limb of the loop of Henle
DT	-	the distal tubule
ECV	-	extracellular fluid volumes
f	-	unbound solute fraction
GU	-	glomerular ultrafiltrate
ICV	-	intracellular fluid volumes
k	-	diffusive mass transport coefficient
MI	-	medullary interstitium
PT	-	proximal tubule
$u(x)$	-	concentration of waste product in the blood
$v(x)$	-	concentration of waste product in dialysate
x	-	distance along the dialyser

α	-	convective transport coefficient for water
δx	-	small length
ΔP	-	osmotic pressure difference
ΔCl	-	difference of theoretical and experimental clearances
Cl^C	-	clearance of creatinine
Cl^U	-	clearance of urea
ΔCl^C	-	difference of theoretical and experimental clearances of creatinine
ΔCl^U	-	difference of theoretical and experimental clearances of urea
σ^C	-	standard deviation of creatinine
σ^U	-	standard deviation of urea
A_M	-	area of membrane
Cl_{PR}	-	clearance of phenol red
K_C	-	mass transfer coefficient of creatinine
$K_D A$	-	dialyser mass transfer area coefficient
Q_B	-	flow rate of blood
Q_D	-	dialysate flow rate
Q_P	-	plasma flow rate

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CHAPTER 1

INTRODUCTION

1.1 Background of the Research

The kidney is a major organ in human body which excretes waste products and excessive fluid. Kidney's main function is to regulate fluid and electrolyte balance to maintain fluid volumes and ion compositions (Stephen Baigent et. al., 2000). Without the kidney, toxic will be accumulated in our body and may lead to death.

Nowadays, many people are facing chronic kidney failure problems which can be life threatening. When the kidney fails to function, an artificial kidney is needed to perform the essential tasks that have been done by the kidney (see Figure 1.1). Figure 1.1 depicts hemodialysis where it is a process of removing excessive waste products and water from blood. The machine in Figure 1.1 which acts as a substitute for kidney is known as dialyser. For healthy individuals with healthy kidneys, the removal of fluid and waste products is a continuous process. When the kidney fails to perform, excessive fluid and toxic chemicals are retained in the body. This situation can be very dangerous.

Therefore, an artificial kidney known as dialyser has been used to remove excessive fluid and waste products in the blood (D.N Burghes, and M.S. Borrie, 1981).

The process of removing excessive water and toxic from our blood using dialyser is called dialysis. There are two main types of dialysis, hemodialysis and peritoneal dialysis. The former uses external and artificial membrane to filter the waste products in the blood whereas the latter uses the patient's peritoneal membrane as the filter (Stephen Baigent et. al., 2000). Peritoneal dialysis is a continuous process. On the other hand, hemodialysis is routinely done three times per week where each session can last from 4 to 6 hours (Stephen Baigent et. al., 2000).

During hemodialysis, blood with concentrated toxic chemicals is taken from the body of the patient and passed into the dialyser. In the adjacent compartment of the dialyser, a cleaning fluid which is known as dialysate is being flowed in the opposite direction. These two compartments are being separated by a semipermeable membrane with minute pores which are too small for the blood cells to go through. However, these pores are large enough for the molecules of waste products to pass through (D.N. Burghes and M.S. Borrie ,1981). The waste products will flow from high to low concentration through the membrane, i.e. from blood to dialysate. Hence, a cleaner blood will flow out of the dialyser back to the patient's body.

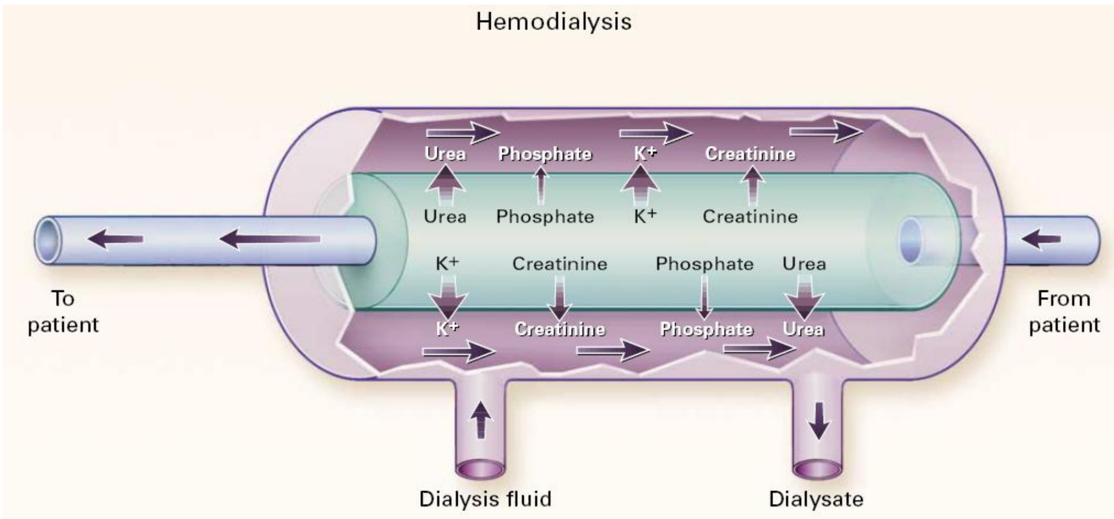

Figure 1.1 Hemodialysis : Combination of Diffusive and Convective Transport (Forni and Hilton, 1997)

1.2 Statement of the Problem

D.M. Burley (1975) mentioned that kidney dialysis machines are called artificial kidneys which are used to treat patients who have lost kidney function because of some disease or injury. The machine is essentially a mass transfer device that cleanses the patient's blood to remove elevated levels of salts, excess fluids, and metabolic waste products. This removal process is necessary to control blood pressure and maintain the proper balance of potassium and sodium in the body.

The dialyser is a large canister that contains thousands of small membrane pores. During the dialysis process, the patient's blood is passed a few ounces at a time through these membrane fibers, where it encounters a cleansing fluid (a chemical formulation called dialysate, whose composition is tailored for each patient) that helps to separate unwanted constituents from the blood. Once this highly specialized filtration process is complete, the clean blood is returned back to the body (see Figure 1.2).

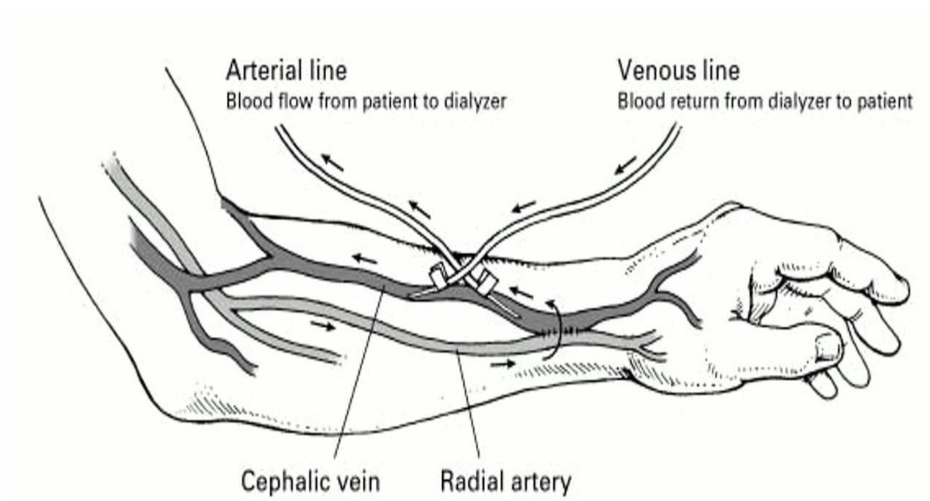

Figure 1.2 The Process of Dialysis (Ifudu, 1998)

To gain more understanding about the process of dialysis, we need to study the fundamental mechanism that operates in a dialyser by constructing a simple mathematical model.

Here are three related questions for my study:

1) How to derive mathematical modeling of a dialyser?

2) Is the governing equation can be solved theoretically?

3) How to calculate the amount of removed waste materials (creatinine and urea)?

1.3 Objectives of the Study

The main objectives of the study are

- To study the process of dialysis
- To formulate the mathematical model of the kidney machine
- To calculate the amount of removed waste material (creatinine and urea)
- To predict the amount of removed waste material (creatinine and urea) for any given flow rate of blood.

1.4 Scope of the Study

We only deal with a simple model of kidney machine where the mathematical model that is formulated can be solved using first order differential equations. We will only restrict our scope to a model with only one compartment which is divided by a membrane.

1.5 Significance of Study

This study is expected to help engineers to design an improvised version of kidney dialysis machine which are highly efficient and to bring down their costs. In order to design a more efficient model, a simple model is needed to be analysed and studied thoroughly. Therefore, the model I study which represents a simple model is expected to enhance the creation of a more detailed model in the future. The mathematical model of the dialyser is important to create better and more efficient dialysis process.

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