EXACT SOLUTIONS OF UNSTEADY FREE CONVECTION FLOW PAST AN OSCILLATING PLATE WITH NEWTONIAN HEATING

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ABSTRACT

The motion of a viscous fluid caused by the oscillations of a vertical plate is important in many applied problems such as acoustic streaming around an oscillating body and an unsteady boundary layer with fluctuations. In boundary layer, free convection flow is a motion that results from the interaction of gravity with density differences within a fluid. These differences occur due to temperature or concentration gradients or due to their composition. The situation where the heat be transported to the convective fluid via a bounding surface having finite heat capacity is known as Newtonian heating (or conjugate convective flows). This configuration occurs in convection flows set up when the bounding surfaces absorb heat by solar radiation. In this thesis, the unsteady free convection flow of an incompressible viscous fluid past an oscillating vertical plate with Newtonian heating is studied. The free convection flow with either heat or heat and mass transfer with radiation effect is considered. The problem of magnetohydrodynamic free convection flow in a porous medium is also studied. Appropriate non-dimensional variables are used to reduce the dimensional governing equations along with imposed initial and boundary conditions into dimensionless forms. The exact solutions for velocity, temperature and concentration are obtained using the Laplace transform technique. The corresponding expressions for skin friction, Nusselt number and Sherwood number are also calculated. The graphical results are displayed to illustrate the influence of various embedded parameters such as Newtonian heating parameter, radiation parameter, Grashof number and phase angle. The results obtained show that the effect of Newtonian heating parameter increases the Nusselt number but reduces the skin friction. However, the Nusselt number is decreased when the radiation parameter is increased. Also, the skin friction is decreased when the radiation parameter, phase angle and Grashof number are increased.

ABSTRAK

Gerakan bendalir likat yang disebabkan oleh plat menegak berayun penting dalam kebanyakan masalah kenaan contohnya penjurusan akustik yang mengelilingi jasad berayun dan lapisan sempadan tak mantap yang turun naik. Dalam lapisan sempadan, aliran olakan bebas merupakan gerakan yang disebabkan oleh interaksi graviti dengan perbezaan ketumpatan di dalam bendalir. Perbezaan ini berlaku disebabkan oleh kecerunan suhu atau kepekatan atau komposisi kedua-duanya. Situasi di mana haba dipindahkan ke bendalir berolak melalui permukaan tertutup, yang mempunyai muatan haba terhingga dikenali sebagai pemanasan Newtonan (atau aliran berolak jodoh). Dalam aliran olakan, konfigurasi ini berlaku apabila permukaan tertutup menyerap haba melalui sinaran suria. Dalam tesis ini, aliran olakan bebas bagi bendalir likat tak boleh mampat merentasi plat menegak berayun beserta pemanasan Newtonan dikaji. Aliran olakan bebas tersebut sama ada dengan haba atau haba dan jisim dengan kesan sinaran dipertimbangkan. Masalah aliran olakan bebas hidrodinamik magnet di dalam bahantara berliang juga dikaji. Pembolehubah tak matra yang bersesuaian digunakan untuk menurunkan persamaan menakluk beserta syarat awal dan syarat sempadan bermatra ke bentuk tak bermatra. Penyelesaian tepat bagi halaju, suhu dan kepekatan diperoleh menggunakan teknik penjelmaan Laplace. Ungkapan yang sepadan untuk geseran kulit, nombor Nusselt dan nombor Sherwood juga dihitung. Keputusan grafik dipaparkan untuk menggambarkan pengaruh pelbagai parameter yang ditetapkan seperti parameter pemanasan Newtonan, parameter sinaran, nombor Grashof dan sudut fasa. Keputusan yang diperoleh menunjukkan kesan parameter pemanasan Newtonan meningkatkan nombor Nusselt tetapi menurunkan geseran kulit. Namun, nombor Nusselt berkurang apabila parameter sinaran meningkat. Juga, geseran kulit berkurang apabila parameter sinaran, sudut fasa dan nombor Grashof meningkat.

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LIST OF SYMBOLS/ NOTATIONS

u'	-	dimensional velocity
t'	-	dimensional time
x'	-	coordinate axis parallel to the plate
y'	-	coordinate axis normal to the plate
g	-	acceleration due to gravity
M	-	magnetic parameter
k	-	thermal conductivity of the fluid
k_1	-	permeability
k^*	-	mean absorption coefficient
K	-	porosity parameter
R	-	radiation parameter
D	-	mass diffusivity
t	-	dimensionless time
U_0	-	amplitude of the plate oscillations
C'	-	Species concentration in the fluid
C_ω	-	species concentration near the plate
C_∞	-	concentration in the fluid far away from the plate
C	-	dimensionless concentration
C_p	-	specific heat at constant pressure
T	-	temperature of the fluid near the plate
T_∞	-	ambient temperature
B_0	-	applied magnetic field
Gr	-	Grashof number
Gm	-	modified Grashof number
Sc	-	Schmidt number
Pr	-	Prandtl number
q_r	-	radiative heat flux along the y'-axis

h_s	-	heat transfer parameter for Newtonian heating
u	-	dimensionless velocity
y	-	dimensionless coordinate axis normal to the plate
q	-	Laplace transform parameter

Greek Symbols

ω	-	frequency of the plate oscillation
τ	-	dimensionless skin friction
τ'	-	dimensional skin friction
θ	-	dimensionless temperature
ϕ	-	porosity of the medium
ρ	-	density of the fluid
σ	-	electrical conductivity of the fluid
σ^*	-	Stefan-Boltzman constant
ν	-	kinematic viscosity
μ	-	viscosity
γ	-	Newtonian heating parameter
β	-	volumetric coefficient of thermal expansion
β^*	-	volumetric coefficient of mass expansion

Subscripts

ω	-	condition at the wall
∞	-	ambient condition

Superscript

$'$	-	dimensional variables

CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter will discuss about the research background, problem statement, objectives and scope of research, research methodology, significance of the study and finally thesis outlines.

1.2 Research Background

A fluid is a substance that continuously deforms under an applied shear stress. Fluids which obey Newton's law of viscosity and for which dynamic viscosity has a constant value are known as Newtonian fluids.

Mathematically,

$$\tau \propto \frac{du}{dy}\,, \tag{1.1}$$

$$\tau = \mu \frac{du}{dy}\,. \tag{1.2}$$

In above expression τ is the shear stress exerted by the fluid, μ is the dynamic viscosity of fluid and du/dy is the shear strain or rate of deformation perpendicular to the direction of shear. Equation (1.2) is known as Newton's law of viscosity. Simply, this means that the fluid continues to flow regardless of the forces acting on it. For example, water is Newtonian, because it continues to exemplify fluid properties no matter how fast it is stirred or mixed.

Heat transfer (or heat) is thermal energy in transit due to a spatial temperature difference. Whenever there exists a temperature difference in a medium or between media, heat transfer must occur. Likewise if we have a multicomponent system with a concentration gradient, one constituent of the mixture gets transported from the region of higher concentration to the region of lower concentration till the concentration gradient reduces to zero. This phenomenon of the transport of mass as a result of concentration gradient is called mass transfer (Cengel *et al*. 1998 and Incropera *et al.* 2011).

There are three types of heat transfer: conduction, convection and radiation. Conduction is heat transfer by means of molecular agitation within a material without any motion of the material as a whole. Convection is the transfer of thermal energy from one place to another by the movement of fluids or gases. Whereas radiation is heat transfer by the emission of electromagnetic waves which carry energy away from the emitting object. Further in convection, if the fluid motion is induced by some external resources such as fluid machinery or vehicle motion, the process is generally called forced convection flow. While if the motion in the fluid is induced by body forces such as gravitational or centrifugal forces, this kind of flow is said to be free or natural convection. Mixed convection flow occurs when free (natural) and forced convection mechanisms simultaneously and significantly contribute to the heat transfer.

Free convection has its applications such as those found in heat transfer from a heater to air, heat transfer from nuclear fuel rods to the surrounding coolant, heat transfer from pipes, cooling of the electronic devises, the spreading of pollutants from smoke stacks and atmospheric and oceanic circulation as explained by Ghoshdastidar (2004). Free convection flows occur not only due to temperature difference, but also due to concentration difference or the combination of these two.

The study of free convection flow with heat and mass transfer plays an important role in the design of chemical processing equipment, formation and dispersion of fog, crop damage due to freezing, nuclear reactors and environmental pollution.

In the advancement of space technology and in processes involving high temperatures thermal radiation effects play an important role. Recent developments in missile reentry, rocket combustion chambers, hypersonic flights, power plants for inter planetary flight, gas cooled nuclear reactors and power plants for inter planetary flight, have focused attention on thermal radiation as a mode of energy transfer and emphasize the need for improved understanding of radiative transfer in these process. Heat transfer by thermal radiation is basically very important in many aspects of practical engineering applications. Some well known examples are the composite structures applied in industry, chemical engineering, solar radiation in buildings, foundry engineering and solidification processes. The radiative heat transfer problems are also important in industrial textiles, textiles designed for use under hermetic protective barrier, multilayer clothing materials and needle heating in heavy industrial sewing as explained by Korycki (2006).

On the other hand, the motion caused by oscillations of the plate is known as Stokes's second problem in the literature. Such a motion is not only possess a theoretical appeal but it also occurs in many applied problems such as acoustic streaming around an oscillating body and in the study of unsteady boundary layer with fluctuations (Tanner, 1962). The transient solution for the flow of a viscous fluid due to an oscillating plate have been studied by Penton (1968). Erdogan (2000) considered the unsteady flow of a viscous incompressible fluid due to an oscillations of plane wall and obtained the exact solutions by mean of the Laplace transform technique. Extensive research work has been published on the flow of a viscous fluid due to oscillations of the plate for different constitutive models (see for instance Das *et al*. (2008), Fetecau *et al*. (2009) and the references therein). However, the motion induced by oscillations of the plate in free convection flow are rarely studied in the literature. Such investigations are further narrowed down when the exact solutions of free convection flow are desired particularly by using the Laplace transform technique. Perhaps, it is due to the difficulty in finding the inverse Laplace transforms.

In the mathematical modelling of convective boundary layer flow problems, the researchers usually use the boundary conditions of constant wall temperature or constant heat flux. However, in many practical situations where the heat transfer from the surface is taken to be proportional to the local surface temperature, the above assumptions fail to work. Such type of flows are termed as conjugate convective flows and the proportionally condition of the heat transfer to the local surface temperature is termed as Newtonian heating. This work was pioneered by Merkin (1994) for the free convection boundary layer flow over a vertical flat plate immersed in a viscous fluid. However, due to numerous practical applications in many important engineering devices, several other researchers are getting interested to consider the Newtonian heating condition in their problems. Few of these applications are found in heat exchanger, heat management in electrical appliances (such as computer power supplies or substation transformer) and engine cooling (such as thin fins in car radiator). Therefore, in view of such applications various authors have been used the Newtonian heating condition in their convective heat transfer problems and have obtained the solutions either numerically or in analytical forms.

Interestingly, so far no study has been reported in the literature to study the magnetohydrodynamics (MHD) free convection flow of viscous fluid over an oscillating plate with Newtonian heating. The MHD flow on the other hand has several applications in the field of agricultural engineering, geophysics and petroleum industries. Recently, considerable attention has been focused on applications of MHD and heat transfer such as metallurgical processing, MHD generators and geothermal energy extraction. Therefore, it is of great interest to study the effects of magnetic field and other participating parameters on the temperature distribution and heat transfer when the fluid is not only an electrically conducted but also when it is capable of absorbing-emitting radiation.

Furthermore, the flow through porous media has received considerable attention in recent years because of its several important applications such as those involving heat removal from nuclear fuel debris, drug permeation through human skin, flow of oil through porous rock, filtration of solids from liquids, just to name a few. A porous medium is a material containing void spaces (pores), either connected or unconnected, dispersed within it in either a regular or random manner. These voids may contain a variety of fluids such as water, air and oil. If the voids represent a certain portion of the bulk volume, a complex network can be formed which is able to carry fluids. Porous media play an important role in applied science and engineering such as

- Soil Science: The porous medium (soil) contains and transports water and nutrients to plants.
- Hydrology: The porous medium is a water bearing and sealing layer.
- Chemical Engineering: Porous medium is applied as filter or catalyst bed.
- Petroleum Engineering: Porous medium (reservoir rock) stores, crude oil and natural gas.

In view of the above discussion and the immense need of the Newtonian heating in the free convection flow, the present study aims to investigate the free convection flow of an incompressible viscous fluid past an oscillating vertical plate with either heat or heat and mass transfer with Newtonian heating boundary condition. Moreover, the problem of MHD free convection flow in a porous medium is also studied. To the date, it is worth to mention that problem of MHD free convection flow in a porous medium with Newtonian heating is still not available in the literature. It is due to the complex nature of these problems and mostly the researchers could not obtain the exact solutions.

1.3 Problem Statement

The focus of this study is to analyze the effects of Newtonian heating and thermal radiation on the heat and mass transfer of an incompressible viscous fluid past an oscillating vertical plate. Heat transfer analysis is also considered in the presence of MHD and porosity effects. This study will explain the following questions. How does the Newtonian fluid model behave in the problem of unsteady free convection flow past an oscillating plate with Newtonian heating? How does the mathematical model behave in this problem involving heat and mass transfer? How does the presence of porosity, MHD and other fluid parameters affect the fluid motion? How the analytical solution for complicated free convection flow with Newtonian heating condition can be obtained? Specifically the problems investigated in this work are

Problem I. To find exact solution for free convection flow with heat transfer past an oscillating plate.

Problem II. To find exact solution for free convection flow with heat and mass transfer past an oscillating plate.

Problem III. To find exact solution for MHD free convection flow with heat transfer past an oscillating plate in a porous medium.

1.4 Research Objectives

Having obtained the solutions of each problem, the main interest of this study is to investigate the effect of

i. The frequency of oscillating plate on velocity in free convection flow with heat transfer.
ii. The frequency of oscillating plate on velocity in free convection flow with heat and mass transfer.

iii. The frequency of oscillating plate on velocity in MHD free convection flow through a porous medium with heat transfer.

1.5 Scope of Research

This study will focus on the unsteady free convection flow of an incompressible viscous fluid with either heat or heat and mass transfer together. The radiation effect on the free convection flows past an oscillating vertical plate with Newtonian heating will be investigated. Moreover, the radiation effect on MHD free convection flow past an oscillating plate in a porous medium will also be investigated in this project. The solutions for velocity, temperature and concentration fields are presented in simple forms in terms of the exponential function and complementary error function. In all the proposed problems, the exact solutions have been obtained by a Laplace transform technique (see appendix A) and plotted graphically using either Mathematica-5.2 or Mathcad-15.

1.6 Research Methodology

First of all, the physical problem in each case will be modeled in the form of coupled linear partial differential equations prescribed with a set of linear initial and boundary conditions. Then a set of non-dimensional variables will be introduced to transform the dimensional coupled partial differential equations into their dimensionless forms along with the imposed initial and boundary conditions. The non-dimensional partial differential equations along with imposed initial and boundary conditions are solved using a Laplace transform technique to get velocity, temperature and concentration fields. Moreover, the corresponding skin friction, Nusselt number and Sherwood number are also evaluated.

In order to understand an insight of the problem, the exact solutions will be plotted. The softwares used for plotting are Mathematica-5.2 and Mathcad-15. The choice of selection of one of these softwares or both simultaneously, depends on the nature of the problem. Graphical results will enhance the understanding of the physical phenomenon of the problems. Furthermore, these results will be used to ensure the correctness of the solutions by satisfying all imposed initial and boundary conditions. The effects of the pertinent flow parameters will be noticed on the velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number. The analytical results obtained through Laplace transform technique will be compared to the existing solutions available in the literature to verify the accuracy of the presented analysis.

1.7 Significance of the Study

The results obtained from this project will be significant because of the following reasons:

i. These results can be used as the basis for fluid flow problems frequently occur in engineering and applied sciences.
ii. The obtained results will be helpful in checking the accuracy of the solutions obtained through numerical schemes.
iii. The study of MHD free convection flow is important in view of its possible applications in astrophysics and geophysics.
iv. Convection in porous media has important applications in geothermal energy storage and flow through filtering devices.

1.8 Thesis Outlines

This thesis consists of six chapters including this chapter. Chapter 1 discuss some basic terminologies of fluid mechanics, research background, problem statement, research objectives, scope of research, research methodology and significance of the present study. Chapter 2 provides the required literature regarding the problems outlined in the objectives.

Chapter 3 discusses the unsteady free convection flow of an incompressible viscous fluid past an oscillating vertical plate with Newtonian heating. The effects of thermal radiation are considered. It is assumed that the fluid is electrically conducted and absorbing-emitting radiation but a non-scattering medium. The motion in the fluid is induced due to buoyancy force and oscillations of the plate. The Boussinesq approximation for a Newtonian fluid is employed to model the governing problem. The Rosseland approximation is used to described the radiative heat flux in the energy equation. By using non-dimensional variables, the resulting governing equations along with initial and boundary conditions are written in dimensionless form. These equations are solved for the exact solutions using Laplace transform technique. The expressions for velocity and temperature are obtained. They satisfy all imposed initial and boundary conditions. As a special case, these solutions can be reduced to the existing solutions in the literature. The skin friction and Nusselt number are evaluated analytically as well as numerically and presented in tabular forms. Numerical results for velocity and temperature are shown graphically for various parameters of interest and the physics of the problem is well explored.

In Chapter 4, an extension of Chapter 3, we also consider the effects of mass transfer on the free convection flow of a viscous incompressible fluid past an oscillating vertical plate with Newtonian heating. As in the previous chapter, the equations of the problem are first formulated using the Boussinesq approximation and transformed into their dimensionless forms. Then Laplace transform method is used to find the exact solutions for velocity, temperature and concentration. These solutions satisfy all imposed initial and boundary conditions. Moreover, expression for skin friction, Nusselt number and Sherwood number are obtained. The obtained numerical results for the pertinent flow parameters are plotted graphically and presented in tabular forms. The analytical results are compared to the existing solutions available in the literature to verify the accuracy of the presented analysis.

Chapter 5 is also an extension of Chapter 3. This chapter discusses the unsteady MHD flow of a viscous incompressible fluid passing through a porous medium with Newtonian heating. It is assumed that the fluid is electrically conducted and the magnetic field of uniform strength is applied perpendicular to the plate. The magnetic Reynolds number on the flow is considered to be small so that the induced magnetic field is negligible. The effect of viscous dissipation is also neglected in the energy equation. All the fluid properties are assumed to be constant except the influence of the density variation with temperature is considered only in the body force term. The motion in the fluid is induced due to buoyancy force and oscillations of the plate. The Boussinesq approximation is employed to model the governing flow problem and Laplace transform technique is used to obtain velocity and temperature fields. The corresponding skin friction and Nusselt number are also calculated. Numerical results for velocity are shown graphically for various parameters of interest. The solutions for hydrodynamic flow in a non porous medium are recovered from the presented analysis.

Finally, Chapter 6 presents the summary of this research and discuss several recommendations for future work. The references are given at the end of the thesis followed by some appendices.

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