ENHANCED PRIM'S ALGORITHM FOR FINDING THE HAMILTONIAN CYCLE IN A GRAPH

NUR ATIQAH BINTI DINON

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ABSTRACT

The Travelling Salesman Problem (TSP) is known as one of the oldest combinatorial optimisation problem which solves the path problem in weighted graph. With the main objective to visiting all places (nodes) in a round trip that start and end in one specific place, TSP shared the same problem with a lot of applications in the world nowadays. In short, the goal of TSP is to find a Hamiltonian cycle. Hamiltonian cycle was introduced in 1800's which is as old as the moment TSP captured the mind of the thinker. Lots of great and major discussions have been made till now and TSP has seen applications in the areas of logistics, genetics, manufacturing, telecommunications, neuroscience and many more. TSP has been intervening in many of the everyday experience by most people and not only for a salesman. Be it a usual errand around the house or the major project by the company or government, TSP has an innate connection in tour finding which lead the attention from various personnel. In another related graph problem, Prim's Algorithm (PA) is widely used to compute the Minimum Spanning Tree (MST) of a graph. In this research, the two algorithms are being related by modifying the PA in order to work out the TSP which will find the Hamiltonian cycle of the graph. This new approach is called Enhanced Prim's Algorithm (EPA) which solves the TSP for fast result.

ABSTRAK

Masalah Perjalanan Jurujual (TSP) dikenali sebagai salah satu daripada masalah pengoptimuman gabungan tertua yang mana ia menyelesaikan masalah perjalanan dalam graf berwajaran. Dengan objektif utama untuk melawat semua tempat (nod) dalam perjalanan di dalam satu pusingan yang bermula dan berakhir di satu tempat tertentu, TSP berkongsi masalah yang sama dengan banyak aplikasi di dunia pada masa kini. Secara ringkasnya, matlamat TSP adalah untuk mencari kitaran Hamilton. Kitaran Hamilton telah diperkenalkan pada tahun 1800-an. Ianya adalah seusia dengan masa TSP mendapat perhatian penyelidik. Banyak perbincangan yang mendalam telah dibuat sehingga kini dan TSP telah muncul dalam banyak aplikasi seperti dalam bidang logistik, genetik, pembuatan, telekomunikasi, neurosains dan banyak lagi. TSP telah terlibat dalam kebanyakan amalan sehari-harian oleh kebanyakan orang dan bukan tertakluk hanya kepada permasalahan jurujual. Tidak kira samada permasalahan di dalam rumah atau di projek-projek utama oleh syarikat atau kerajaan, TSP mempunyai sambungan semula jadi dalam mencari aturan perjalanan yang mendapat perhatian dari pelbagai pihak. Dalam menyelesaikan masalah graf, salah satu algoritma yang digunakan untuk menyelesaikan masalah mencari pohon rentangan yang minimum adalah Algoritma Prim (PA). Dalam kajian ini, kami akan mengaitkan dua algoritma dengan mengubah PA yang sedia ada untuk menyelesaikan masalah TSP yang berfungsi untuk mencari kitaran graf Hamiltonian. Pendekatan baru ini dipanggil Peningkatan Algoritma Prim (EPA) yang menyelesaikan TSP bagi mendapat hasil yang cepat.

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LIST OF SYMBOLS AND ABBREVIATIONS

$V(G), v_i$	-	Vertex, node
$D(G), d_{i,j}$	-	Edge, coordinates
N	-	Total number of nodes
$W(C)$	-	Weight of the cycle
C	-	Cycle
n	-	Number of nodes
FCTP	-	Fixed-Charge Transportation Problem
LC-MST	-	Least-Cost Minimum Spanning Tree
DC-MST	-	Delay-Constrained Minimum Spanning Tree
MRCT	-	Minimum Routing Cost Tree
CMST	-	Capacitated Minimum Spanning Tree
HCP	-	Hamiltonian Cycle Problem
HA	-	Heuristic Algorithm
EPA	-	Enhanced Prim’s Algorithm

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CHAPTER 1

INTRODUCTION

1.1 Introduction

In our daily lives, there are a lot of situations that require us to make decisions that we presume could maximise or minimise our objectives such as taking the shortest route to our destination or looking for the cheapest price for certain items. It is agreed that with the concept of optimisation the best selection of component is made from a set of presented options. This has been done in almost everything in our lives by solving such problems based on our experience and knowledge about the system without resorting to any mathematical formulation. However, this old-fashioned method is limited to a certain degree of problem whereas the system becomes large and complicated, therefore a specific mathematical model is needed and a formulation and solution by the help of computer may be necessary. By using the computer, the optimisation theories can be exploited to their maximum level. The existence of numerous computer programming we have today is expected to facilitate researchers to expand the mathematical optimisation field.

Optimisation consists of many branches and one of it is combinatorial optimisation. This type of optimisation arises in many areas and certainly in applied mathematics, operational research, computer science, artificial intelligence and many more. Combinatorial optimisation is widely studied as it can solve many problems such as transporting, scheduling, routing and installing. Some examples of combinatorial optimisations are Travelling Salesman Problem (TSP), Minimum Spanning Tree (MST) and Bin Packing problem. Combinatorial optimisation solves the problem of weighted graph. According to Wilson (1979), weighted graph is a connected graph in which a non-negative real number has been assigned to each edge. To solve a weighted graph, several approaches have been proposed to achieve its objective function. There are heuristic and meta-heuristic approaches to solve the optimisation problem. These approaches are created as they were expected to find the optimal result of the function.

A great fundamental knowledge on graph theory will help a better understanding on the general idea of the mathematical optimisation. The details on application of graph theory have been discussed by Marcus (2008) in his book entitled “Graph Theory: A Problem Oriented Approach”. In his book, one of the problems of graph theory discussed by Daniel is on solving path problems which include Euler and Hamiltonian Path. A further discussion on cycle including Hamiltonian Cycle was one of the subtopics discussed in the book. A discussion on Hamiltonian Path and Hamiltonian Cycle will be further elaborated in Chapter 2 since the main objective of the research is to find a Hamiltonian cycle. In this study, we are interested in constructing an algorithm modified from a prominent algorithm named Prim’s Algorithm (PA) to solve the TSP in finding the Hamiltonian cycle.

In this chapter, the flow of the research is presented where it begins with the background of the problem and problem statement of the research. Later on, the objectives, scope and the significant of the study are discussed and elaborated for further understanding. In Chapter 2, the basic terminology, theory and the literature review on the previous study are written out in details to understand the elementary knowledge of the research and the development of the topic in research field. In this chapter, a brief introduction on algorithms that motivate the research is discussed. In the next chapter, the research methodology which includes the details of algorithm and the proposed algorithm is focused. A deep explanation of the new proposed algorithm is made to facilitate the understanding. Later on, in Chapter 4, two case studies are provided to solve the problem using the proposed algorithm and a brief discussion on the result will be made in the end of the chapter. Finally, the analysis of the study is provided and the discussion on some further research directions into this problem that could take place based upon the works provided by this study.

1.2 Background of the Problem

The Travelling Salesman Problem (TSP) is known as one of the oldest combinatorial optimisation problem which solves the path problem in weighted graph. This problem receives a lot of attention as it had been the most cited problems in the operations research (Von Poser and Awad, 2006). With the main objective to visit all places (nodes) in a round trip that starts and ends in one specific place, TSP shared the same problem with a lot of applications in the world nowadays. In other words, according to Behzad and Modarres (2002) they claimed that the goal of TSP is to find a Hamiltonian cycle. They also define Hamiltonian cycle as a closed path that visits each node once and only once. In this way, the problem of finding the shortest route is identical to finding the minimum weight of Hamiltonian cycle of a weighted completed graph (Agnarsson and Greenlaw, 2007).

According to Xu (2003), if there exists a Hamiltonian cycle, it is called an optimal cycle. In his book, he also claimed that there is no specific efficient algorithm to solve TSP problem. One of the best known algorithms for approximately solving the TSP is called Christofides' approximation algorithm. It was first discovered by Nicos Christofides in 1976. This algorithm is called approximation algorithm because an approximation to the required result is within the known bound. The goal of this algorithm is to find a solution to the TSP where the edge weights satisfy the triangle inequality.

Other than TSP, another marvel problem in combinatorial optimisation problem is MST problem. This problem is to find a spanning tree having minimum weight among all spanning trees in the graph. There are two main procedures to constructs the tree which is by building up and by reducing down. There are a number of known efficient algorithms to find a minimum tree in a weighted connected graph. The best known algorithm is Prim's Algorithm (PA). PA is known as one of the greedy algorithms to solve MST problem. Greedy heuristic approach had been choosen to solve the problem as the feature is to find the best element in every stage to get a solution set that will give the optimal solution.

Being motivated by the method used in PA, this study has altered the existing PA in order to work out the TSP which will find the Hamiltonian cycle of the graph. In solving the combinatorial optimisation problem by greedy method, there are a lot of ways and algorithms that can be developed as long as it can produce reasonably optimal solution. With this fact, a modification to PA has been constructed to solve the TSP in order to obtain a Hamiltonian cycle in the graph.

1.3 Statement of the Problem

Travelling Salesman Problem (TSP) belongs to a class of NP-hard problem. Consequently, there are no known algorithms guaranteed to give an optimal solution. The existing Prim's Algorithm (PA) is one of the well known greedy methods to solve the optimization problems. The steps in PA were believed to be adapted in solving the TSP and further able to find the Hamiltonian cycle. The Enhanced Prim's Algorithm (EPA) has been constructed in order to answer the TSP to get the optimal result. In other words, the new proposed algorithm adds another possible method to find an optimal Hamiltonian path or cycle of TSP which can minimize the total weight of the cycle in the graph.

In this study, a Hamiltonian cycle of the weighted graph problem is studied to find the best route in minimizing the total objective function. We consider a graph G with a set of nodes $V = \{v_i\}$ and a set of coordinates D= $\{d_{ij}\}$ for $i, j = 1,2, \dots, N$ where d_{ij} represents the distance from i to j. A travelling salesman who wants to make a round trip through all the nodes and concluding his tour at the same point he starts the tour, where the optimal solution of the tour is expected to be obtained in the end of this study. In short, the problem is solved by arranging the nodes in a cyclic order in such a way that the sum of the d_{ij} between consecutive points is minimal.

1.4 Objectives of the Study

The objectives of this study are:

1. to have a better understanding of Prim's Algorithm and the steps used in solving the optimization problems.
2. to develop an algorithm which originated from Prim's Algorithm to solve the TSP and construct a Hamiltonian cycle in the graph.
3. to develop C++ language programme code to solve the problem computationally.
4. to solve some case study which cover both small and large scale problems.

1.5 Scope of the Study

This study focuses on solving the Travelling Salesman Problem (TSP) using greedy method. The present algorithm is studied to motivate a development of-or transformation and enhancement to develop-a new algorithm. In this study, Prim's Algorithm (PA) is used as the main model to form a new greedy algorithm to solve the TSP. The aim of this new algorithm is to construct a Hamiltonian cycle of the graph while minimizing the total weight. The new algorithm is coded in Microsoft Visual C++ programming to solve some case studies from TSP. The results obtained using the programming are analysed and compared with the manual solution.

1.6 Significance of the Study

In this study, the algorithm must find a cycle in the graph to solve the Travelling Salesman Problem (TSP). The closed path should go through every vertex in the graph which will create a Hamiltonian cycle. Since there is no exact algorithm to solve a TSP, an alternative algorithm is proposed to solve the problem which originated and stimulated by the steps in Prim's Algorithm (PA). Therefore, the enhanced algorithm can be used as one of the algorithm to solve TSP. The modified Prim's Algorithm is coded using C++ language programme with Microsoft Visual C++ to get the computational result in finding the optimal route of TSP that generates reasonably optimal result. This algorithm can be used as an alternative to existing algorithm in solving the optimization problem related to TSP since it arises in many areas and fields. From industrial practice to vehicle routing, there exist many problems involving minimising travelling cost. Therefore, by introducing the new enhanced algorithm to solve the real practice environment, it would give another solution to the problem especially in terms of optimising the cost.

1.7 Conclusion

In this chapter, the basic component of this research is discussed to get the overall picture of this research. This chapter begins with the introduction of the study and followed by the background of the problem. The study is commencing from two optimization problems which are Travelling Salesman Problem (TSP) and Minimum Spanning Tree (MST) Problem. These two problems are analyzed and emerge as the problem statement of this research. In this chapter, there is also a discussion on the objectives of the study. The main goal of this study is to find the Hamiltonian cycle of the weighted graph in solving the Travelling Salesman Problem (TSP) using modified Prim's Algorithm (PA). This chapter also discussed the scope and the significance of the study in the end of the chapter. In the next chapter, the reviews on previous research are made on TSP and MST. The further discussion on Hamiltonian cycle is included as the TSP is initially from it.

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