# FREE AND MIXED CONVECTIVE BOUNDARY LAYER FLOW OF A VISCOELASTIC FLUID PAST A HORIZONTAL CIRCULAR CYLINDER

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*For my dear family ,*

*Dr. Sharidan Shafie*

*and friends ~*

*Thank you for everything…*

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# ABSTRACT

The study of viscoelastic fluid has become increasingly important in the last few years. This is mainly due to its many applications in petroleum drilling, manufacturing of food and paper, and many other similar activities. In this thesis, the steady free and mixed convective boundary layer flow of a viscoelastic fluid past a horizontal circular cylinder has been studied separately subject to their own constant surface temperature boundary conditions. For the problem of mixed convection, the study also considered the problem that subjected to constant heat flux boundary conditions. The constitutive equations of viscoelastic fluids usually generate a higher-order derivative term in the momentum equation than equations of Newtonian fluid. Thus, there are insufficient boundary conditions to solve the problems of viscoelastic fluid completely. Therefore, the augmentation of an extra boundary condition is needed at infinity (far from the wall). In each case, the governing boundary layer equations are first transformed into a non-dimensional form, and then into a set of non similar boundary layer equations which are solved numerically using an efficient implicit finite-difference method known as Keller-box scheme. Numerical result presented include velocity profiles, temperature profiles, heat transfer characteristics, namely the local heat transfer, local skin friction coefficient and local wall temperature distribution for a wide range of material paramater K (viscoelastic parameter), prandtl number Pr, and mixed convection parameter $\lambda$. In each problem, it is found that velocity distributions decrease when the value of viscoelastic parameter, K increases, whereas the opposite behaviour is observed for the temperature distribution. It is worth mentioning that the results obtained in viscoelastics fluids when the parameter K = 0 (Newtonian fluids) are in excellent agreement with those obtained in viscous fluids (Newtonian fluids).

# ABSTRAK

Penyelidikan tentang masalah bendalir likat kenyal menjadi semakin penting sejak beberapa tahun kebelakangan ini. Ini adalah disebabkan oleh aplikasinya dalam penggerudian minyak, pembuatan kertas dan makanan serta aktiviti seumpamanya. Dalam tesis ini, konveksi bebas dan campuran mantap pada lapisan aliran sempadan bendalir likat kenyal melewati silinder sirkular melintang dikaji secara berasingan terhadap suhu malar syarat sempadan masing-masing. Untuk masalah konveksi campuran, kajian turut meneliti masalah berkaitan syarat sempadan fluks pemalar panas. Persamaan-persamaan juzuk bendalir likat-kenyal ini terjana dengan sebutan terbitan peringkat tinggi di dalam persamaan momentumnya berbanding persamaan bendalir Newtonan. Oleh itu, permasalahan yang di hadapi ialah ketidakcukupan syarat-syarat sempadan untuk menyelesaikan masalah bendalir likat-kenyal ini. Oleh yang demikian, penambahan syarat sempadan di infiniti (jauh dari permukaan) diperlukan. Bagi setiap masalah, pertama sekali, persamaan menakluk di ubah menjadi bentuk tak bermatra dan kemudian menjadi satu set persamaan lapisan sempadan tak serupa yang mana diselesaikan secara berangka menggunakan skim beza terhingga tersirat yang efektif yang dikenali dengan kaedah kotak-Keller dengan menambah syarat sempadan di infiniti. Penyelesaian berangka dipaparkan meliputi profil kelajuan, profil suhu, ciri ciri pemindahan haba, antaranya pemindahan haba setempat, pekali geseran kulit setempat dan taburan suhu dinding setempat juga diperolehi bagi nilai-nilai parameter K (parameter likat kenyal), Nombor Prandtl, Pr, dan parameter olakan campuran, $\lambda$. Bagi setiap masalah, keputusan menunjukkan taburan kelajuan menurun apabila nilai parameter likat kenyal, K meningkat sedangkan keadaan berbeza di lihat pada taburan suhu. Keputusan yang diperolehi dalam kajian ini apabila parameter K = 0 (bendalir newtonan) menunjukkan hasil yang memuaskan setanding dengan keputusan yang telah diperolehi menerusi bendalir likat (bendalir newtonan).

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| | | |
|---|---|---|
| $a$ | - | Radius of the cylinder |
| $C$ | - | Dimensional constant |
| $C_f$ | - | Skin friction coefficient |
| $d$ | - | Dimensionless injection or suction parameter |
| $f$ | - | Dimensionless stream function |
| $m$ | - | Velocity exponent parameter |
| $n$ | - | Temperature exponent parameter |
| Gr | - | Grashof number |
| Pr | - | Prandtl number |
| Nu | - | Nusselt number |
| Re | - | Reynolds number |
| $T$ | - | Fluids temperature |
| K | - | Viscoelastic parameter |
| $u$ | - | Velocity component in $x$-direction |
| $U_0$ | - | Dimensional constant |
| $v$ | - | Velocity component in $y$-direction |
| $u_e(x)$ | - | Non-dimensional velocity outside the boundary layer |
| $x$ | - | Coordinate in direction of surface motion |
| $x_s$ | - | Boundary layer separation point |
| $y$ | - | Coordinate in direction normal to surface motion |

| | | |
|---|---|---|
| $U_\infty$ | - | Free stream velocity |
| $q_w$ | - | Surface heat flux |
| $Q_w$ | - | Local heat transfer coefficient |

Greek symbols

| | | |
|---|---|---|
| $\alpha$ | - | Thermal diffusivity |
| $\eta$ | - | Dimensionless similarity variable |
| $\theta$ | - | Dimensionless temperature |
| $\lambda$ | - | Mixed convection parameters |
| $\mu$ | - | Dynamics viscosity |
| $\rho$ | - | Fluid density |
| $\rho_\infty$ | - | Constant local density |
| $\nu$ | - | Kinematic viscosity |
| $\psi$ | - | Stream function |

Subscripts

| | | |
|---|---|---|
| $w$ | - | Condition at the surface |
| $\infty$ | - | Condition at ambient medium |

Superscripts

| | | |
|---|---|---|
| $'$ | - | Differentiation with respect to $y$ |
| - | - | Dimensional variables |

# LIST OF APPENDICES

# CHAPTER 1

# INTRODUCTION

## 1.1 General Introduction

The convective mode of heat transfer is generally divided into two basic processes, which are natural or free convection, and forced convection. Natural convection is caused by buoyancy forces due to density differences of temperature variations in the fluid. When heated, the density change in the boundary layer causes the fluid to rise and be replaced by cooler fluid, which also heats and rises. This continues as a phenomenon called free or natural convection. In any forced convection situation, natural convection effects are also present under the presence of gravitational body forces. In addition, when the effect of force flow in free convection becomes significant, the process is then called mixed convection flows which are a combination of natural and force convection flows. Recently, The study on free and mixed convection has received much attention to many researchers due to the numerous engineering applications. An exact analytical solution is still out of reach due to the nonlinearities in the Navier-Stokes and energy equations. The earliest attempts to compute this problem is involved the solving of simplified boundary layer equations.

Boundary layer is a narrow region of thin layer that exists adjacent to the surface of a solid body when a real fluid flows past the body. In this region, the effect of viscosity is obvious on the flow of the fluid that results in large velocity gradient and the presence of shear stress. The various transfer processes which take place in fluids and between solids and fluids are momentum, mass, and heat transfer. When formulating the conservation laws of mass, momentum, and energy, the laws of thermodynamics and gas dynamics have to be observed. This means that along with the boundary layer flow, there are also the thermal boundary layer and the mutual influence of these boundary layers upon one another to be accounted for. The concept of boundary layer plays an important role in many branches of engineering sciences, especially in hydrodynamics, aerodynamics, automobile and marine engineering (Kundu and Cohen, 2004).

Although extensive research work has been devoted to heat transfer in viscous (Newtonian) fluids, more recently, research in non-Newtonian fluids has gained momentum as well. Therefore, in this thesis, we considers a few problems that apply the boundary layer concept into the viscoelastics fluid. viscoelasticity is the nature of a second-grade fluid which is also the type of non-Newtonian fluids, and it is found in polymer fluids where these fluids exhibit both the viscous and elastic characteristics. Viscous materials, like honey, resist shear flow and strain linearly with time when a stress is applied, while the elastic materials strain instantaneously when stretched and just as quickly return to their original state once the stress is removed. The viscous property is due to the transport phenomenon of the fluid molecules while the elastic property is due to the chemical structure and configuration of the polymer molecules.

Viscoelastic fluids, which is also known as second-grade fluids, are more accurate than the first-order fluids with exponential dependence of viscosity and temperature. A detailed discussion on second and third-order fluids can be found in the study of Dun and Rajagopal (1995). The simplest model for viscoelastic fluids was formerly proposed by Rivlin and Erickson (1955) when they considered the stress deformation relation for isotropic materials. The idea of viscoelastic fluid was also very well documented by Byron (1977), who described the fluid as never moving very far or very rapidly from its initial configuration. In addition, an investigation into the flow of elastic-viscous fluids past a circular cylinder was done by Harnoy (1987).

Damseh, *et al.* (2008) studied the problem of transient mixed convection flow of a second-grade visco-elastic fluid over a vertical surface and they found that the velocity decreases inside the boundary layer as the viscoelastic parameter is increased and consequently, the local Nusselt number decreases. This is due to higher tensile stresses between viscoelsatic fluid layers which has a retardation effects on the motion of the layers. Hsia, and Hsu, (2009) has investigated a conjugate heat transfer of mixed convection for visco-elastic fluid past a triangular fin. The Results indicated that the elastic effect in the flow can increase the local heat-transfer coefficient and enhance the heat transfer of a triangular fin. Very recently, the problem on mixed convection flow of a viscoelastic fluid through a porous medium in a vertical channel with permeable walls has been studied by Reddy and Raju (2010).

The problems that considered the viscoelastic fluids, which is a type of non-Newtonion fluids, has gained considerable importance because of its applications in various branches of science, engineering, and technology, particularly in material processing, chemical and nuclear industries, geophysics, and bio-engineering. The study of non-Newtonian fluid flow is also of significant interest in oil reservoir engineering. For a variety of reasons, non-Newtonian fluids are classified on the basis of their shear properties.

## 1.2 Statement of Problem

Interest in the viscoelastic fluids has increased substantially over the past decades due to the occurrence of these fluids in many technological applications. Therefore, this research is conducted to study one type of non-Newtonian fluid which is called the viscoelastic fluid. The study will explore the following questions. How do the viscoelastic mathematical models compare with the existing Navier-Stokes or Newtonian mathematical models in describing the nature of free and mixed convection boundary layer flow past a horizontal circular cylinder? What are the effects of viscoelastic fluids parameter on the skin friction, heat transfer, velocity profile and temperature profile?

## 1.3 Objectives of Research

The objectives of this research are to carry out mathematical formulations and develop numerical algorithm using FORTRAN 77 for the computation, in order to analyze and investigate the following problems:

1. Mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder with constant temperature
2. Mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder with constant heat flux
3. Free convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder with constant temperature

The analysis includes: (i) formulation of the mathematical models to obtain the governing boundary layer and heat transfer equation for the above mentioned new models, (ii) nonsimilar boundary layer transformation, and (iii) numerical computation to solve the problem using a finite difference scheme. The scheme employed is the Box method developed by Keller (1970,1971) and throughout the whole course of this research, the main reference for the Keller-box method are the books by Cabeci and Bradshow (1977, 1988) and Na (1979). The convergence criterion required that the maximum absolute error between two successive iterations was $10^{-6}$.

## 1.4 Scope of Research

This research takes into consideration the steady two-dimensional incompresssible viscoelastic fluid model. The problem is the limited boundary layer flow pass a horizontal circular cylinder. Since, for most engineering applications, the flow velocities are moderate; hence the viscous-energy-dissipation term becomes small and can be neglected.

## 1.5 Significance of the Research

The theory boundary layer problem of viscoelastic fluids has gained a lot of interest, and become important in recent years because of their applications in several industrial-manufacturing processes involving petroleum drilling, manufacturing of foods and paper. In engineering applications, it is possible to use viscoelastic fluids to reduce frictional drag on the hulls of ships and submarines. Some typical applications for viscoelastic boundary layer flow over a stretching sheet are polymer sheet extrusion from a dye, glass fiber and paper production, and drawing of plastic films. There are also many applications involving atomization of viscoelastic fluids such as paints, coating, inks, and jet fuels. The relationship between viscoelasticity and drop formation aimed at the production of mono-disperse colloidal sized droplets uses the same approach as ink jet printing and particle production.

Free convection has attracted a great deal of attention from researchers because of its presence both in nature and engineering applications. In nature, convection cells formed from air rising above sunlight and warming land or water are a major feature in all weather systems. Convection is also seen in the rising plume of hot air from fire, oceanic currents, and sea-wind formation (where upward convection is also modified by Coriolis forces). In engineering applications, convection is commonly visualized in the formation of microstructures during the cooling of molten metals, fluid flows around shrouded heat-dissipation fins, and solar ponds. A very common industrial application of free convection is free air cooling without the aid of fans; this can happen from on a small scale (computer chips) to large scale process equipment. Significant free convection is induced by the density stratification of air in the thermal boundary layer (Nazar, (2003)).

On the other hand, the mixed convection (combination of forced and free convection) flow with and without mass transfer occurs in many technological and industrial applications such as solar central receivers exposed to wind currents, nuclear reactors cooled during emergency shutdown, heat exchangers placed in low-velocity environments, boundary-layer control on airfoil, lubrication of ceramic machine parts and food processing. Mixed convection flows arise when the free stream, inertial and near wall buoyant forces have strong effects on the resulting convective heat transport.

Therefore, the study of free and mixed convection of viscoelastic boundary layer flow problems is important due to its imperative applications in real life. The result or output of this research enhances the understanding of the fluids flow phenomena and improves the development of related industries, for example the manufacturing industries. Besides that, the generation of efficient algorithm of the viscoelastics problem helps in solving the problem of computational fluid dynamics in future.

## 1.6 Outline of Thesis

This thesis consists of seven chapters, including this introductory chapter in which the statement of problem, objectives, scope and significance of research are presented. In Chapter 2, a literature review for the proposed problem is presented and discussed. All of the problems in this thesis are solved numerically using the Keller-box method.

Chapters 3, 4 and 5 contain the detailed solution for Problem 1 (mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder with constant temperature), Problem 2 (Mixed Convection Boundary Layer Flow of a Viscoelastic Fluid Over a Horizontal Circular Cylinder with Constant Heat Flux) and Problem 3 (Free Convection Boundary Layer Flow of a Viscoelastic Fluid Over a Horizontal Circular Cylinder with Constant Temperature) respectively. Each chapter consists of the basic equation, solution procedures, result and discussion and also conclusion of the proposed solution. In addition, in chapter 3, we also include further discussion and details about numerical algorithm using FORTRAN 77 for the first problem.

Finally the concluding chapter, Chapter 6 contains a summary of the main results of the research and several recommendations for future research. A complete program for this problem is given in appendix A to C.

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