

**FORCED CONVECTIVE BOUNDARY LAYER FLOW OF A NANOFUID
PAST A CONTINUOUS STRETCHING SURFACE**

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PAST A CONTINUOUS STRETCHING SURFACE

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To my beloved mother, father, family, lecturers and friends.

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ABSTRACT

Nanofluid is an advanced kind of fluid containing small quantities of nanoparticles that are uniformly and stably suspended in a base liquid. It has a higher thermal conductivity and enhanced heat transfer coefficient than its base fluid. In this study, the steady forced convective boundary layer flow of a nanofluid past a continuous stretching surface with suction or injection and magnetic effect is investigated. This mathematical model describes the flow of nanofluid that incorporates the effects of Brownian motion and thermophoresis. The governing nonlinear partial differential equations are transformed into a system of nonlinear ordinary differential equations using similarity transformation which is then solved numerically using a finite difference scheme known as the Keller-box method. The numerical results obtained show that the value of the reduced Nusselt number decreases with the increasing value of the Brownian motion parameter, Nb and thermophoresis parameter, Nt . Whereas, the value of the reduced Sherwood number increases with Nt but decreases with Nb . It is also observed that the reduced Nusselt and Sherwood numbers decrease with an increase of the velocity stretching parameter, m , the magnetic parameter, Mn and the injection parameter, $d > 0$. The suction parameter, $d < 0$, on the other hand, tends to increase the skin friction coefficient, reduced Nusselt and Sherwood numbers. The results show that the Brownian motion and thermophoresis influence the flow characteristics of nanofluids by either warming the boundary layer or aggravating the collisions between the nanoparticles and molecules of the base fluid.

ABSTRAK

Bendalir nano merupakan sejenis bendalir yang mengandungi sejumlah kecil zarah nano yang terampai secara seragam dan stabil dalam bendalir asasnya. Ia mempunyai konduktiviti terma yang tinggi dan dapat meningkatkan pekali pemindahan haba berbanding bendalir asasnya. Dalam kajian ini, aliran mantap untuk olakan paksaan lapisan sempadan bagi bendalir nano merentasi permukaan regangan yang berterusan dengan kesan sedutan atau suntikan dan juga kesan magnet telah dikaji. Model matematik yang digunakan untuk aliran bendalir nano ini menggabungkan kesan gerakan Brownian dan juga kesan termoforesis. Persamaan terbitan separa tak linear dijelmakan kepada sistem persamaan terbitan biasa tak linear dengan menggunakan penjelmaan keserupaan dan seterusnya diselesaikan secara berangka dengan menggunakan skema beza terhingga yang dikenali sebagai kaedah kotak Keller. Keputusan-keputusan berangka yang diperoleh menunjukkan bahawa nombor Nusselt berkurangan dengan peningkatan parameter gerakan Brownian, Nb dan termoforesis, Nt . Manakala nombor Sherwood meningkat dengan Nt tetapi berkurangan terhadap Nb . Pemerhatian juga menunjukkan bahawa nombor Nusselt dan nombor Sherwood berkurangan dengan peningkatan parameter halaju regangan, m , parameter magnet, Mn , dan parameter suntikan, $d > 0$. Parameter sedutan, $d < 0$ pula meningkatkan pekali geseran kulit, nombor Nusselt dan nombor Sherwood. Keputusan menunjukkan bahawa gerakan Brownian dan termoforesis mempengaruhi ciri-ciri aliran bagi bendalir nano dengan memanaskan lapisan sempadan atau meningkatkan pelanggaran di antara zarah-zarah nano dan bendalir asasnya.

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LIST OF SYMBOLS

Roman Letters

B	-	magnetic field vector
$B(x)$	-	magnetic field in the y -direction
B_0	-	constant
c_p	-	nanoparticle specific heat
C	-	nanoparticle volume fraction
C_f	-	skin-friction coefficient
C_w	-	nanoparticle volume fraction at the surface
C_∞	-	ambient nanoparticle volume fraction
d	-	suction/injection parameter
D_B	-	Brownian diffusion coefficient
D_T	-	thermophoretic diffusion coefficient
f	-	dimensionless stream function
F	-	body forces vector
g	-	gravity acceleration vector
h_p	-	nanoparticle specific enthalpy
I	-	identity vector
j _{p}	-	nanoparticle mass flux
j _{p,B}	-	mass flux due to Brownian diffusion
j _{p,T}	-	mass flux due to thermophoresis
J	-	electrical current vector
k	-	thermal conductivity
L	-	characteristics length of plate

Le	-	Lewis number
m	-	velocity stretching parameter
M	-	mass
\dot{M}	-	mass generation rate
Mn	-	magnetic parameter
\mathbf{n}	-	unit normal
Nb	-	Brownian motion parameter
Nr	-	bouyancy-ratio parameter
Nt	-	thermophoresis parameter
Nu	-	Nusselt number
Pr	-	Prandtl number
p	-	pressure
\mathbf{q}	-	energy flux
q_m	-	wall mass flux
q_w	-	wall heat flux
Ra_x	-	local Rayleigh number
Re_x	-	local Reynolds number
S	-	surface
Sh	-	local Sherwood number
t	-	time
T	-	fluid temperature
T_w	-	temperature at the surface
T_∞	-	ambient temperature
U_0	-	constant
u, v	-	velocity components along x - and y -axes
u_w	-	velocity of the stretching sheet
v_w	-	wall mass suction velocity
V	-	arbitrary control volume
x, y	-	Cartesian coordinates measured along the wall and normal to it, respectively

Greek Letters

α	-	thermal diffusivity
β	-	volumetric expansion coefficient of the nanofluid
δ	-	boundary layer thickness
η	-	similarity variable
θ	-	dimensionless temperature
μ	-	dynamic viscosity
ν	-	kinematic viscosity
ρ_f	-	fluid density
ρ_p	-	nanoparticle mass density
$(\rho c)_f$	-	heat capacity of the fluid
$(\rho c)_p$	-	effective heat capacity of the nanoparticle material
τ	-	stress vector
τ	-	heat capacity ratio
τ_w	-	wall shear stress
ϕ	-	rescaled nanoparticle volume fraction
ψ	-	stream function
σ	-	electric conductivity of the fluid

Subscripts

w	-	condition at the surface
∞	-	condition at ambient medium

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CHAPTER 1

INTRODUCTION

1.1 Introduction

During the past decade, technology to make particles in nanometer dimensions was improved and a new kind of solid–liquid mixture that is called nanofluid appeared. Nanofluids are described as suspended nano-sized particles in conventional heat transfer basic fluids. This term was first utilized by Choi (1995) to illustrate the suspension of the nanoparticles in a base fluid. Nanoparticles are particles that are between 1 and 100 nm in diameter. Moreover, modern technology can produce metallic or nonmetallic particles of nanometer dimensions. Nanomaterials have unique mechanical, optical, electrical, magnetic, and thermal properties. Nanofluids have higher thermal conductivity and single-phase heat transfer coefficients than their base fluids (Buongiorno, 2006). A very small amount of guest nanoparticles, when dispersed uniformly and suspended stably in base fluids, can provide dramatic improvements in the thermal properties of base fluids.

Nanoparticles used in nanofluids can be fabricated within physical processes and chemical processes. The physical techniques include mechanical grinding and the inert gas condensation technique. The chemical processes include chemical precipitation, spray pyrolysis, and thermal spraying. In addition, there are two ways to produce nanofluids which are by a single-step technique or a two-step technique. The single-step technique simultaneously makes and disperses the nanoparticles directly into the base fluid. Meanwhile, for the two-step technique the nanoparticles are made and then dispersed in the fluid (Das *et al.*, 2008).

Nanofluids typically employ metal or metal oxide nanoparticles, such as copper and alumina, and the base fluid is usually a conductive fluid, such as water or ethylene glycol. Nanofluids commonly contain up to a 5% volume fraction of nanoparticles to see effective heat transfer enhancements. Nanofluids are studied because of their heat transfer properties: they enhance the thermal conductivity and convective properties over the properties of the base fluid. Typical thermal conductivity enhancements are in the range of 15-40% over the base fluid and heat transfer coefficient enhancements have been found up to 40% (Yu *et al.* 2008).

Increase in thermal conductivity of this magnitude cannot be solely attributed to the higher thermal conductivity of the added nanoparticles, and there must be other mechanisms attributed to the increase in performance. Buongiorno (2006) considered seven slip mechanisms that can produce a relative velocity between the nanoparticles and the base fluid which are inertia, Brownian motion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage and gravity. In his observations only Brownian motion and thermophoresis are important slip mechanisms in nanofluids. Brownian motion is defined as random motion of particles suspended within the base fluid due to continuous collisions between the nanoparticles and the molecules of the base fluid. Meanwhile, thermophoresis is the phenomenon where the particles diffused under the effect of a temperature gradient. In this phenomenon, particles travel in the opposite direction of the increasing temperature which is from warmer to cooler areas. From the finding, Buongiorno (2006) developed a two-component nonhomogeneous equilibrium model for transport phenomena in nanofluids.

1.2 Background of Research

Numerous researches have been carried out to study convection of nanofluids. This is due to the importance of nanofluids to the industry. Choi (1998) discussed the properties of nanofluids and future challenges. The author wrote that the development of nanofluids is still hindered by several factors such as the lack of

agreement between results, poor characterization of suspensions, and the lack of theoretical understanding of the mechanisms.

Roy *et al.* (2004) investigated numerically the problem of laminar flow and heat transfer in a radial flow cooling system with the use of nanofluids. Maiga *et al.* (2005) carried out the research on the heat transfer enhancement by using nanofluids in forced convection flows. Meanwhile, Koblinski *et al.* (2005) discussed the problem of nanofluids for thermal transport. Buongiorno, (2006) discussed the convective transport in nanofluids.

In addition, Behzadmehr *et al.* (2007) carried out the research on the prediction of turbulent forced convection of a nanofluid in a tube with uniform heat flux using a two phase approach. Analytical prediction of forced convective heat transfer of fluids embedded with nanostructured materials (nanofluids) have been performed by Vasu *et al.* (2007). Mansour *et al.* (2007) carried out the effect of uncertainties in physical properties on forced convection heat transfer with nanofluids. On the other hand, laminar flow forced convection heat transfer of a nanofluid that consists of water and aluminum dioxide, Al_2O_3 inside a circular tube was investigated experimentally by Heris *et al.* (2007). Forced convective heat transfer was experimentally investigated by Ding *et al.* (2007). Effect of reduced specific heats of nanofluids on single phase, laminar internal forced convection was investigated by Bergman (2008).

Apart from that, Jung *et al.* (2009) carried out the research on forced convective heat transfer of nanofluids in microchannels. Bianco *et al.* (2009) performed the numerical investigation of nanofluids forced convection in circular tubes. Numerical study of developing laminar forced convection of a nanofluid in an annulus has been investigated by Izadi *et al.* (2009). Besides that, Kakaç and Pramuanjaroenkij (2009) has made some review of convective heat transfer enhancement with nanofluids. He *et al.* (2009) studied forced convective heat transfer of non-Newtonian nanofluids. Experimental study on forced convective heat transfer with low volume fraction of CuO-water nanofluid had been performed by Asirvatham *et al.* (2009). In addition, numerical simulation of convective heat

transfer in nanofluid based on copper, Cu nanoparticles and ethylene glycol in a horizontal minichannel has been studied by Kurowski *et al.* (2009). Nield and Kuznetsov (2009) studied the Cheng-Minkowycz problem of natural convection past a vertical plate, in a porous medium saturated by a nanofluid. Ho *et al.* (2010) has carried out an experimental investigation of forced convective cooling performance of a microchannel heat sink with Al_2O_3 -water nanofluids. Forced convective of a nanofluid that consists of water and Al_2O_3 in horizontal tubes has been studied numerically by Lotfi *et al.* (2010).

On the other hand, research on natural convective boundary-layer flow of a nanofluid past a vertical plate was studied analytically by Kuznetsov and Nield (2010). Besides, the problem of laminar fluid flow which results from the stretching flat surface has been investigated by Khan and Pop (2010). Bachok *et al.* (2010a) investigated the steady boundary-layer flow past a moving semi-infinite flat plate in a uniform free stream. Meanwhile, Bachok *et al.* (2010b) studied the flow and heat transfer in an incompressible viscous fluid near the three-dimensional stagnation point of a body that is placed in a water based nanofluid.

Recently, the steady flow of an incompressible viscous fluid due to a rotating disk in a nanofluid is studied by Bachok *et al.* (2011). The Falkner-Skan problem for a static or moving wedge immersed in nanofluids is investigated numerically by Yacob *et al.* (2011a). Meanwhile, Yacob *et al.* (2011b) carried out the problem of the steady flow and heat transfer over a static or moving wedge with a prescribed surface heat flux in a nanofluid.

Researchers have conducted many experimental investigations to determine the heat transfer enhancement by nanofluids which focused either on the effective thermal conductivity or on the heat transfer coefficient. Subsequently, limited computer simulation and numerical studies dealing with thermal properties and heat transfer characteristics of nanofluids have been carried out. Therefore, this thesis is focusing on the numerical computation and simulations of nanofluids.

1.3 Statement of Problem

This study will explore the following questions. What are the appropriate mathematical models describing the convective boundary layer flow of nanofluids? What are the effects of Brownian and thermophoresis parameters on the heat and mass transfer characteristics as well as fluid temperature and nanoparticle fraction profiles? How do the suction or injection and magnetic field affect the flow, heat and mass transfer characteristics?

1.4 Objectives and Scope of Research

The objective of this study is to develop mathematical models for the problem of convective boundary layer flow of a nanofluid, to carry out mathematical formulations and to develop numerical algorithm for the computation in order to solve the following problems:

1. To derive the equations of motion governing convective boundary layer flow of a nanofluid.
2. To validate the solution obtained using Keller-box method against analytical method.
3. To solve the problem of forced convective boundary layer flow of a nanofluid past a continuous stretching surface.
4. To solve the problem of forced convective boundary layer flow of a nanofluid past a continuous stretching surface with suction or injection in the presence of magnetic field.

The analytical method considered is referring to Kuznetsov and Nield (2010). This study takes into consideration the boundary layer flow past a continuous stretching surface. Meanwhile, the model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis based on Buongiorno (2006).

1.5 Significance of Research

Nanofluids can be used to improve heat transfer and energy efficiency in a variety of thermal systems. Much of the work in the field of nanofluids is being done in national laboratories and academia. Recently, the number of companies that see the potential of nanofluid technology and are in active development work for specific industrial applications is increasing. Some of the applications are in transportation, microelectronics, defense, and biomedical. The applications described below are obtained from Yu *et al.* (2008).

In transportation, the standard automotive coolant is a relatively poor heat transfer fluid compared to water alone. Engine oils perform even worse as a heat transfer medium. The addition of nanoparticles to the standard engine coolant has the potential to improve automotive and heavy-duty engine cooling rates. Such improvement can be used to remove engine heat with a reduced-size coolant system. Smaller coolant systems result in smaller and lighter radiators, which in turn benefit almost every aspect of car and truck performances and lead to increased fuel economy. Alternatively, improved cooling rates for automotive and truck engines can be used to remove more heat from higher horsepower engines with the same size of coolant system.

Furthermore, in the microelectronics application nanofluids can increase the power density of integrated circuits and microprocessors, a trend that should continue for the foreseeable future. Future processors for high-performance computers and servers have been projected to dissipate higher power in the range of 100-300 W/cm². Whether these values actually become reality is not as significant as the projection that the general trend to higher power density electronics will continue. Single-phase fluids, two-phase fluids, and nanofluids are candidate replacements for air. All have increased heat transfer capabilities over air systems, and all are being investigated. Nanofluids have been considered as the working fluid for heat pipes in electronic cooling applications. The results showed a significant reduction in thermal resistance of the heat pipe with the nanofluid as compared with

deionized water. The measured results also showed that the thermal resistance of a vertical meshed heat pipe varies with the size of nanoparticles.

A number of military devices and systems require high-heat-flux cooling to the level of tens of MW/m^2 . At this level, cooling with conventional fluids is challenging. Examples of military applications include cooling of power electronics and directed-energy weapons. Directed-energy weapons involve high heat fluxes and providing adequate cooling of them and the associated power electronics is a critical need. Nanofluids have the potential to provide the required cooling in such applications as well as in other military systems, including military vehicles, submarines, and high-power laser diodes. In some cases, nanofluid research for defense applications includes multifunctional nanofluids with added thermal energy storage or energy harvesting through chemical reactions. Meanwhile, in Navy the transformer cooling is important as well as the power generation industry with the objective of reducing transformer size and weight. The ever-growing demand for greater electricity production can lead to the necessity of replacing and/or upgrading transformers on a large scale and at a high cost. A potential alternative in many cases is the replacement of the conventional transformer oil with a nanofluid. Such retrofits can represent considerable cost savings. It has been demonstrated that the heat transfer properties of transformer oils can be significantly improved by using nanoparticle additives.

Nanofluids and nanoparticles have many applications in the biomedical industry. For example, to circumvent some side effects of traditional cancer treatment methods, iron-based nanoparticles could be used as delivery vehicles for drugs or radiation without damaging nearby healthy tissue. Such particles could be guided in the bloodstream to a tumor using magnets external to the body. Nanofluids could also be used for safer surgery by producing effective cooling around the surgical region, thereby enhancing the patient's chance of survival and reducing the risk of organ damage. In a contrasting application to cooling, nanofluids could be used to produce a higher temperature around tumors to kill cancerous cells without affecting nearby healthy cells.

1.6 Outlines of Thesis

This thesis is divided into seven chapters including this introductory chapter which contains the background of research, statement of problem, objectives and scope of research as well as significance of research. In Chapter 2, a literature review for the problem are presented and discussed.

The derivation of the governing equations for nanofluid is shown in Chapter 3. In this chapter, the governing equations consist of four equations which are continuity, nanoparticle fraction, momentum and energy equations are derived. Then, the boundary layer scale analysis is used to simplify the system of equations. These equations can be simplified to yield an approximate set of equations, namely the boundary layer equations.

In Chapter 4, we revisited the problem considered by Kuznetsov and Nield (2010) which is natural convective boundary layer flow of a nanofluid past a vertical flat plate. The solution obtained using Keller-box method is validated against analytical method. This chapter is divided into six sections where the first section is the introduction of the problem. The next section, we discuss about the governing equations of the problem. Then, the following sections discussed the similarity transformation in order to reduce the partial differential equations into the ordinary differential equations. The Keller-box method is used for the numerical formulation of the problem and then the results and discussion for the problem is also discussed. The conclusion for the problem is discussed in the last section.

In Chapter 5, the problem of forced convective boundary layer flow of a nanofluid past a continuous stretching surface is discussed. This chapter is divided into five sections which consist of the introduction of the problem, the governing equations of the problem, the similarity transformation, the results and discussion for the problem and lastly conclusion of the problem.

Meanwhile, Chapter 6 describes the problem of forced convective boundary layer flow of a nanofluid past a continuous stretching surface with suction or injection in the presence of magnetic field. This chapter is the extensions of the previous problem which is discussed in Chapter 5. There are four sections in this chapter, which consist of the introduction of the problem, the formulation of the problem as well as the results and discussion for the problem and lastly the conclusion of the problem.

Finally, Chapter 7 presents the summary of this thesis and recommendation for future research. All the references in this thesis are listed at the end of Chapter 7. Appendix A presents the mathematical formulation for the problem that considered in Chapter 4. Meanwhile, the mathematical formulation for the problem that is discussed in Chapter 5 and 6 is shown in Appendix B. Appendix C and Appendix D present the Keller-box numerical formulation for the problem considered in Chapter 4, 5 and 6. The Matlab programs for solving the specific problems discussed in Chapters 4, 5 and 6 are shown in Appendices E, F and G, respectively. Lastly, Appendix H shows the academic activities throughout my study.

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