ANALYTICAL SOLUTIONS OF MAGNETOHYDRODYNAMIC (MHD) NEWTONIAN AND SECOND GRADE FLUIDS IN A POROUS MEDIUM

FARHAD ALI

A thesis submitted in fulfillment of the
requirements for the award of the degree of
Master of Science (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

DECEMBER 2010

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TO MY BELOVED FAMILY

ACKNOWLEDGEMENT

Incalculable thanks to ***Allah Almighty,*** Creator of all of us, Worthy of all persons, Who always guides in glooms & obscurities and subsists of assistance in difficulties, when all other sourced channels' upper frontier ends. **P**rincipal compliments and honors to ***His last Holy Prophet (SAW)- Prophet of Revolution and Mercy for all the worlds,*** who enabled the man to recognize his Originator. (من لم يشكر الناس لايشكر الله) "He who does not thank to people is not thankful to Allah", ***Holy Prophet (SAW)***

It is extremely easier said than done to incorporate the names of all those persons who did involve directly or indirectly in the completion of this work. Nonetheless, I am extraordinarily grateful to my entire team of teachers ever since my school days.

I put forward myself to an unfoiled honor to express my gratitude to my esteemed, learned, ingenious, and reverend Research Supervisor- ***Dr. NORZIEHA MUSTAPHA*** and co- Supervisor ***DR. SHARIDAN SHAFIE*** for their invaluable guidance, sympathetic attitude and nimble-fingered, competent -cum-prime supervision, during this research period, without that all that was fallacy. My special thanks to Research Management Centre (RMC) – Universiti Teknologi Malaysia (UTM) for the financial support through vote number 78338.

I wish to acknowledge my vital thanks to my brother ***Mr. ILYAS KHAN*** for his beyond price services, valuable suggestions and guidance throughout my project made me able in completing my project successfully.

I would like to express my heartiest gratitude to my family who provided the initial impetus in understanding the world of knowledge, their love, prayers, constant care and invaluable cooperation were the assets for me during the span of my studies from start till end. I also would like to thank all those who have assisted me directly or indirectly towards the completion of this thesis especially ***MacKichan Software***, *Inc. Scientific Word/WorkPlace* other than else.

ABSTRACT

This study is concentrate on developing the exact solutions of Newtonian and non-Newtonian fluid (second grade) problems. The fluid is considered electrically conducting (MHD) and passing through the porous medium. The modified Darcy's law is also incorporated in the mathematical formulation of the second grade problem. In addition to no-slip conditions, the influence of slip condition is also taken into account. Since, in many polymeric liquids when the weight of the molecule is high, the molecules near to the boundary show slip. Then the no-slip boundary condition is not appropriate. Therefore, this study incorporates the slip effects to handle this problem. The linear partial differential equations are reduced to the ordinary differential equations by using Laplace transform method. Then the final solutions for velocity and shear stress fields are obtained by taking the Laplace inverse, even though, to get the inverse Laplace is not always a trivial matter. Graphical results for the velocity profile and shear stress are presented and discussed. Comparison with earlier results for a limiting cases show a very good agreement. These solutions facilitate the verification of numerical solvers and also aid in the stability analysis of the solutions.

ABSTRAK

Pembangunan penyelesaian tepat untuk masalah bendalir Newtonian dan bukan Newtonian (gred kedua) merupakan tumpuan utama dalam kajian ini. Pengaliran bendalir berelektrik (MHD) yang mengalir melalui bahantara berliang telah dipertimbangkan. Pengubahsuaian hukum Darcy juga telah digunakan ketika formulasi matematik untuk masalah gred kedua. Selain daripada keadaan tanpa gelincir, masalah juga melibatkan pengaruh keadaan gelincir. Keadaan gelincir ini berlaku kerana dalam kebanyakkan cecair polimer, apabila berat molekul semakin bertambah, molekul berdekatan dengan sempadan akan menyebabkan berlakunya gelincir. Maka keadaan tanpa gelincir tidak lagi bersesuaian. Oleh kerana itu, kajian bagi menyelesaikan masalah kesan gelinciran ini juga dibincangkan. Kaedah penjelmaan Laplace digunakan bagi menurunkan persamaan terbitan biasa linear kepada persamaan terbitan separa. Seterusnya, penyelesaian akhir bagi halaju dan medan tegasan ricih diperoleh dengan menggunakan penjelmaan Laplace songsang, walaupun penyelesaian dengan penjelmaan Laplace songsang ini bukanlah sesuatu yang mudah. Keputusan grafik bagi profil halaju dan profil tegasan ricih dipersembahkan dan diperbincangkan. Perbandingan dengan keputusan terdahulu telah menunjukkan persetujuan.

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LIST OF SYMBOLS\NOTATIONS

B	—	induced magnetic field
J	—	density of the current
B	—	total magnetic field
B_0	—	applied magnetic field
M	—	Hartmann number
K	—	permeability of the medium
K	—	dimensionless permeability of the medium
R	—	Darcy's resistance
(d/(dt))	—	material time derivative
T	—	time
P	—	scalar pressure
I	—	identity tensor
A_1	—	first Rivilin-Ericksen tensor
A_2	—	second Rivilin-Ericksen tensor
U	—	characteristic velocity
$C_{i(i=1-6)}$	—	arbitrary constants
V	—	velocity vector
v	—	dimensional velocity of the fluid in *x*-direction
U	—	dimensionless velocity of the fluid
Y	—	coordinate axis normal to the plate

Greek Symbols

ω	—	frequency of the plate
σ	—	electrical conductivity of the fluid
τ	—	dimensionless time

ξ	—	dimensionless coordinate axis normal to the plate
α	—	second grade parameter
γ ,λ	—	dimensionless slip parameters
β	—	dimensional slip parameter
β_o	—	applied magnetic field
φ	—	porosity of the medium
μ_c	—	magnetic field
ρ	—	density of fluid
σ	—	conductivity of the fluid
ν	—	kinematic viscosity
α	—	cross viscosity
α_2	—	viscoelasticity
μ	—	dynamic viscosity

Subscripts

c	—	cosine
s	—	sine \steady
t	—	transient

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CHAPTER 1

INTRODUCTION

1.1 Introduction

In this chapter, review of the Newtonian and non-Newtonian fluids is given. A background of the research and problem statement is presented. We have also highlighted the objectives and scope of the study, research methodology and finally, significance of the study.

1.2 Research Background

To most fluid dynamists, fluid dynamics is concerned with the study of Navier-Stokes' fluids. Probably no other fluid model has been as carefully scrutinized as the Navier-Stokes' model. While Newton (1687) formulated a one dimensional form as the model. Navier modified the Newton's model by adding the molecular interaction force. We attribute Navier (1827) and Stokes' (1845) for the derivation of equations of motion for Newtonian fluids. Poisson (1831) removed the Navier force, derived the Newtonian viscous flow model. Saint-Venant (1843), published the correct derivation of the Navier-Stokes equations for a viscous fluid and was the first to properly identify the coefficient of viscosity and its role as a multiplying factor for the velocity gradients in the flow. Finally the work of Stokes' (1851) led to the model now referred to as the Navier-Stokes' fluid model (Newtonian fluid model). The Navier-Stokes' fluid model

has occupied a central place in fluid mechanics for over a century and a half. Basic questions related to the mathematical properties of equations governing the flow of viscous fluids remain open. To predict the response of such fluids in case of turbulent conditions has evaded the physicist and engineers. However great efforts have been made to gain an understanding of the laminar flow of such fluids (for example see Ladyzhenskaya (1970)). Fluid mechanics is a base of advanced research, even for those working in different areas. In the beginning of the 20^{th} century Lunding Prandtl has given a new dimension to fluid mechanics. He introduced viscosity in the fluid and thus unifying hydraulics and theoretical hydrodynamics. The Navier Stokes' equations were used for complete description of Newtonian fluid flows. Due to the non-linearity effect, it is not easy to solve the Navier Stokes' equations and only in very few cases exact analytical solutions are exist in literature. The situations to find an exact solution for the Navier Stokes' equations having importance particularly in investigations to describe the viscous fluid motion. Same Navier-Stokes' solutions can be obtained for other engineering applications, which also yield considerable physical insight. However, the applicability of Navier-Stokes' equations is limited to simple fluids like air and water, known as Newtonian fluids. A large number of practical Newtonian applications deal with important problems, for instance, external past air planes, internal flows within jet engine and free surface flows about ships, submarines etc.

These fluids obeying the Newton's law of viscosity which has the following mathematical expression

$$\tau_{yx} \propto \frac{du}{dy}, \tag{1.1}$$

$$\tau_{yx} = \mu \frac{du}{dy}. \tag{1.2}$$

In above expression τ_{yx} is shear stress, μ is dynamic viscosity and du/dy is the rate of strain for unidirectional and one-dimensional flow. Simply, this means that the fluid continues to flow regardless of the forces acting on it. For example, water is Newtonian because it continues to exemplify fluid properties no matter how fast it is stirred or mixed.

While some fluids can be well presented by the theory of Navier-Stokes' equations but there are many fluids of practical importance whose response cannot be ade-

quately characterized by the Navier-Stokes' model. In industrial applications, there are numerous fluids and even more numerous process to which the fluids are subjected for which the Nevier-Stokes' model is inappropriate. Thus fluids which do not obey the Newton's law of viscosity (1.2) are called non-Newtonian fluids. In other words, non-Newtonian fluids are those whose flow properties are not described by a single constant value of viscosity. For such fluids shear stress is directly and non-linearly proportional to the deformation rate. Mathematically

$$\tau = k\left(\frac{du}{dy}\right)^n, \quad n \neq 1, \tag{1.3}$$

where constant k is the consistency index and n is the behavior index. Many fluids such as ketchup, starch suspensions, paint, blood, shampoo, polymer solutions and molten polymers are non-Newtonian.

Recently, considerable attention has been devoted to the problem of how to predict the behavior of non-Newtonian fluids. This is due to the fact that such fluids like molten plastics, pulps, slurries, emulsions, petroleum drilling etc. do not obey the Newtonian postulate that the stress tensor is directly proportional to the rate of deformation tensor, are produced industrially in increasing quantities. The flow characteristics of non-Newtonian fluids are quite different from Newtonian fluids. Therefore, the flows of non-Newtonian fluids attracted the researchers more than Newtonian fluids simply because of their several technological and industrial applications. Non-Newtonian fluids are also important in the fields of processing of foods, movement of biological fluids, plastic manufacture, performance of lubricants etc. The interest of the researchers to study non-Newtonian fluids have been increased in the last five decades due to many connections with applied sciences.

In this dissertation we will be concerned with both Newtonian and non-Newtonian (second grade) fluids. Firstly a brief history of the unsteady flow of viscous fluid has been presented. The combined effects of magnetohydrodynamic and porosity of the medium are also discussed. Secondly, a short review of the non-Newtonian fluids is presented related to the problems chosen for this dissertation. There are very few cases in which the exact solutions of Navier-Stokes' equations exist. These solution are even difficult to obtain for Newtonian fluid. This is because the non-linearity occurs due

the inertial term in the Navier-Stokes' equations. However, under certain restrictions one can easily obtain the exact solutions for Newtonian fluids. Stokes' in 1851 and Rayleigh in 1911 (for detail discussion see Schlichting (2000)) obtained the exact solutions for the first and second problem of Stokes' for a Newtonian fluid by using similarity transformation. Since then the flow over a flat plate with different boundary and initial conditions has been investigated by many authors and become the focus of their research. As early as Penton (1968), has presented a closed form solution for the flow of viscous fluid due to oscillating motion of the plat. Tokuda (1968) determined the exact solution for a Newtonian fluid when the flow in fluid is induced due to the sudden motion of the plate.

Makinde and Mhone (2005) investigated the combined effects of transverse magnetic field and radiative heat transfer for the unsteady flow of a Newtonian fluid passing through a channel filled with saturated porous medium and nonuniform wall temperature. The motion in the fluid is obtained by the external pressure gradient of the oscillatory form and the exact solutions are obtained for the velocity and temperature fields. Fetecau *et al*. (2008) obtained the starting solutions of Stokes' second problem for Newtonian fluid by using Laplace transform method.

The inadequacy of Classical Navier-Stokes theory to describe rheological complex fluids such as polymer solutions, blood, paint, certain oils and greases led to the development of several theories of non-Newtonian fluids. Because of the complexity of these fluids several constitutive equations have been proposed. These constitutive equations are complicated and contain as special cases some of the previous fluids. The simplest and widely studied fluid in the grade models is called the second grade fluid. The unsteady unidirectional flows of a second grade fluid have been studied by Ting (1963) who was the first author on this subject. He obtained the solution of second order fluid in a bounded region. The equation of motion of incompressible second grade fluid is of higher order than the Navier-Stokes' equations and some additional boundary conditions are required. In order to overcome the difficulty, work has been done on acceptable boundary conditions by Fosdick *et al*. (1969). A critical review on the boundary conditions has been given by Rajagopal (1995).

The motion of a fluid caused by the oscillations of a flat plate, also named as

Stokes' second problem is not only of fundamental theoretical interest but it also occurs in many applied problems. The starting solutions tend to the steady-state solutions. Such steady solutions, corresponding to the different oscillations of a rigid plate or to an oscillating pressure gradient in a second grade fluid, have been studied by many authors. Hayat *et al*. (1998) obtained exact analytic solution for the flow of non-Newtonian fluid of grade two generated by periodic oscillations of a plane. The velocity field and the moment of the frictional forces are calculated. Siddique *et al*. (1999) obtained the exact solutions for the flows of a non-Newtonian fluid between two infinite parallel plates by using the theory of Fourier transform. The flows discussed are generated by periodic oscillations of one of the plates. Some interesting flows caused by certain special oscillations are also studied. In an other paper Hayat *et al*. (2000) obtained the exact analytic solutions for a class of unsteady unidirectional flows and the frictional forces of an incompressible second grade fluid. The periodic Poiseuille flow and frictional force due to an oscillating pressure gradient are examined. Very recently Nazar *et al*. (2010) obtained the exact solutions for the flow of second grade fluid. He obtained the starting solutions by using Laplace transform.

Further the effect of an external magnetic field on flows through a porous medium has gained an increasing attention through the years. The interest in this field is due to the wide range of applications either in engineering or in geophysics. Thus an exact analytical solution for a Newtonian fluid between ecentric rotating disk with MHD was presented by Mohanty (1972). Erkman (1975) considered the steady flow of a conducting viscous incompressible fluid between two parallel non-conducting plates rotating about different axes with the same angular velocity in the presence of a uniform transverse magnetic field. He obtained an exact solution for velocity field and for the induced magnetic field. In addition, the analysis of hydromagnetic flows through a porous medium has also been studied by several authors. Hayat *et al*. $(2007\,(a))$ considered steady flow of a second grade fluid in a porous channel. The constitutive equations are those used for a second grade fluid. The fluid is electrically conducting in the presence of a uniform magnetic field applied in the transverse direction to the flow and passing through the porous medium. They obtained an analytical solution by employing a homotopy analysis method (HAM). Khan *et al*. (2007) obtained the exact solutions for electrically conducting Oldroyd-B fluid passing through a porous medium.

The flow is induced due to constantly accelerated and oscillating plate. Expressions for the corresponding velocity field and the adequate tangential stress are determined by means of the Fourier sine transform. In an other paper Khan *et al*. (2008) concentrated on the unsteady flows of a magnetohydrodynamic (MHD) second grade fluid filling a porous medium. The flow modeling involves modified Darcy's law. Three problems are considered. They are (i) starting flow due to an oscillating edge, (ii) starting flow in a duct of rectangular cross-section oscillating parallel to its length, and (iii) starting flow due to an oscillating pressure gradient. Analytical expressions of velocity field and corresponding tangential stresses are developed.

This project specifically considers the study of incompressible Newtonian and second grade fluids flows between two infinite parallel plates. The lower plate is taken at $y = 0$, while the upper plate is faraway from the lower plate such that there is no disturbance of fluid. The fluid is taken electrically conducting and passing through the porous medium.

In the present work, the flow in the fluid is induced due to oscillations and the accelerated motion of boundary. Moreover, the study on the flow of a viscous fluid over an oscillating and constantly accelerated plate is not only of fundamental theoretical interest but it also occurs in many applied problems such as acoustic streaming around an oscillating body. For the flow of an incompressible viscous fluid caused by the oscillation of the plane wall, when the fluid motion is set up from rest, the velocity field contains transient as well as steady parts. The transient parts gradually disappear in time.

An other important aspect in fluid mechanics is the consideration of slip condition. One of the corner stone on which the fluid mechanic is built is the no slip condition. But there are situations where the no slip condition does not work. For example in the case of many polymeric liquids when the weight of the molecules is high, the molecules near to the boundary show slip at the boundary. Then the no-slip boundary condition is not appropriate. In addition, in many problems like thin film problems, rarefied fluid problems, fluids containing concentrated suspensions and flow on multiple interfaces, the no-slip boundary condition fails to work. To tackle this problem, Navier (1823) for the first time suggested the general boundary condition which shows

the fluid slip at the surface. According to Navier the difference of fluid velocity and the velocity of the boundary is proportional to the shear stress at that boundary. The proportionality constant is called the slip parameter having dimension of length. More specifically, by the slip conditions we mean that the velocity of the fluid particles in the neighborhood of the stationary plate is not the same as that of the plate. Some interesting investigations explaining the slip effects are given by many authors. Like Roux (1999) considered the slip effect. He obtained the exact solutions for the second grade fluid. Khalid *et al*. (2004) examined the consequences of the slip at the plates. Stokes' and Couette flows of viscous fluid produced by an oscillatory motion of a wall are analyzed under conditions where the no slip assumption between the wall and the fluid is no longer valid. The authors in this paper obtained the exact solutions by using the Laplace transform method. These solutions are expressed both in steady periodic and transient parts. Keeping in mind the importance of the slip condition, in this study we have considered the slip effects on the accelerated flows of Newtonian fluids.

Several analytical techniques are available in the literature for finding the exact solutions of fluid flow problems. In this project we will use Laplace transform method for finding the exact solutions of the proposed problems. Laplace transform method is a widely used integral transform method for finding the solutions of boundary and initial value problems. However, this technique is well suited for the initial value problems. It has several important applications in mathematics, physics, engineering, and probability theory. It has the ability to solve the differential equations which continuously arise in engineering problems. An important application of Laplace transform occurs in the solution of ordinary differential equations which are cast in the form of initial value problems. This method is widely used for finding the solutions for the free convection problems, thermal radiation problems and in most of the cases the free convections problems taking into account the combined effects of heat mass transfer. Few recent attempts in the field, tackled by Laplace transform method are made by Vieru *et al*. (2008) , Fetecau *et al*. (2008), Toki (2009), Narahari (2010) , Chandrakala (2010) , Chaudhary (2010) and Rajesh (2010). Properties of Laplace transform make the transform very appealing as means of finding solutions provided the inverse transform can be easily found. Here we briefly introduce this method for the solutions of partial differential equations and will discuss about its existence (see Appendix-E).

1.3 Significance of the Study

The significance of this study is vast and touches nearly every human endeavor. The science of meteorology, physical oceanography and hydrology are concerned with naturally occurring fluid flows, as are medical studies of breathing and blood circulation. All transportation problems involve fluid motion, with well-developed specialties in aerodynamics of air craft and rockets and in naval hydrodynamics of ships submarines. Almost our electric energy is developed either from water or from steam flow through turbine generators. All combustion problems involve fluid motion, as do the more classical problems of irrigation, flood control, water supply, sewage disposal, projectile motion, oil and gas pipelines. The motion of a viscous fluid caused by the oscillations of flat plate played a vital rule in many branches of science and engineering. Similarly certain materials such as polymers, molten plastic, lubricants, artificial fibres and many others do not come under the definition of the Newtonian fluid, such fluids are generally called non-Newtonain fluids. One can see the importance of these fluids in the field of technology. The study of non-Newtonian fluids have several applications that occur in the industry such as the extrusion of polymer fluids , solidification of liquid crystals, cooling of metallic plate in a bath. Due to the practical and fundamental association of these fluids to industrial problems several researchers have studied the flows of non-Newtonian fluids such as Tanner (1962) and Rajagopal (1984). The study of electrically conducting fluids play an important role in fluid flow problems. The most characteristic biological fluid is blood, which behaves as a magnetic fluid due to the complex interaction of the intermolecular protein, cell membrane hemoglobin as a form of iron oxides. Since blood is electrically conducting fluid, the MHD principles may be used to deccelerate the flow of blood in human arterial system and there by it is useful in the treatment cardiovascular disorders (see refs Katz *et al*.(1938) and Ramchandra Rao *et al*. (1986)). Keeping in mind the above facts, the results obtained from this project will be significant because of the following reasons.

These results can be used as the basis for fluid flow problems frequently occurring in engineering and applied sciences. The obtained results will help in checking the accuracy of the solutions produced via numerical schemes. High quality research will be promoted by producing papers in indexed journals.

1.4 Objectives and Scope of Research

The main objectives of this project is to develop the analytical solutions for the unsteady incompressible flow of Newtonian and second grade fluids for the following problems.

1. The MHD flow of a Newtonian fluid in a porous medium.

2. Slip effects on accelerated flows of a hydromagnetic fluid in a porous medium

3. Unsteady MHD flows of a second grade fluid in a porous medium.

The scope of this project is as follows:

This study takes into consideration the unsteady, incompressible, unidirectional and one-dimensional flows of a Newtonian and second grade fluid. The fluid is electrically conducting and passing through a porous medium. The no slip and slip conditions are considered for Newtonian fluid whereas only no slip condition for second grade fluid. The governing equations have been solved by using Laplace transform. The considered fluids in this project are electrically conducting and passing through the porous medium.

1.5 Thesis Outlines

This thesis encounters six chapters including this introductory chapter, in which we have presented the research back ground significance and objectives of the research. In Chapter 2, a literature review of Newtonian and non-Newtonian (second grade) fluids is presented. The available work in the literature closely related to the problems considered in this project is discussed in details. The Stokes' problems are well described in this chapter. In additions several extensions of Stokes' problems made by different researchers in the field are also studied. The electrically conducting Newtonian and non-Newtonian fluids passing through a porous medium with slip and no slip conditions are also discussed. The problem in Chapter 3, is concerned to develop the analytical solutions for the unsteady incompressible flow of Newtonian fluid. The

fluid is electrically conducting and passing through a porous medium. The motion in the fluid is induced due to constantly acceleration and oscillations plate. The expressions for velocity and shear stress fields are determined by Laplace transform method. The derived steady-state and transient solutions satisfy the imposed initial and boundary conditions. Graphs are sketched and discussed for various emerging parameters for both sine and cosine oscillations of the plate. It is observed that the velocity and shear stress fields are strongly dependent on these parameters. Further, it is noted that both the velocity and shear stress fields decrease for both types of oscillations with an increase in the magnetic parameter M. This phenomenon is reversed for the permeability parameter K of a porous medium when compared with the magnetic parameter M. As expected that the strongest shear stress occurs near the plate in both cases and decreases away from the plate. The influence of time on the velocity and shear stress fields is also studied. It is found that the required time to reach the steady-state for cosine oscillations of the wall is smaller than that of the sine oscillations of the wall for both velocity and shear stress fields. However, the required time decreases if the frequency ω of the velocity increases. In order to check the accuracy of the results obtained in this chapter with the existing results in the literature, comparative diagrams have also been plotted.

Chapter 4 analyzes the unsteady magnetohydrodynamic (MHD) flow of a second grade fluid saturates a porous medium. The exact solutions for cosine and sine oscillations of the plate are obtained. The methodology used for the solutions of the problem is similar to the previous chapter. The obtained solutions are presented as a sum of steady-state and transient solutions. The known solutions for the second grade fluid in the absence of porous medium ($1/K \rightarrow 0$) and applied magnetic field ($M \rightarrow 0$) are also recovered. The analytical results have been displayed for dimensionless parameters through several graphs. It is found that the velocity increases with the increase in the values of second grade parameter α for cosine oscillations and decreases for sine oscillations of the plate. Further, it is noted that the velocity decreases with the increase in magnetic parameter M for both types of oscillations. It is because of the increasing values of the opposing force (Lorentz force) which cause the fluid to move slowly. If we compare the permeability parameter K of a porous medium with magnetic parameter M, it has an opposite effect on the velocity field. The comparison between our solutions with the existing solutions in the literature has also been made.

In Chapter 5, we investigate the influence of slip condition on the unsteady magnetohydrodynamic (MHD) flow of Newtonian fluid fills a porous space above the plate at $y = 0$. The magnetohydrodynamic (MHD) flow in the fluid is carried out due to the accelerated motion of the plate. Both constant and variables flow cases are considered. Darcy's law for Newtonian fluid is also incorporated. Using Laplace transform technique, the exact solutions for velocity and shear stress fields are obtained for constant and variable accelerated flows. The analytical results have been plotted for the indispensable dimensionless parameters to show the influences on velocity and shear stress fields. It is found that the velocity and shear stress for both types of accelerated flows decrease with the increasing values of the slip parameter γ near the wall. The effects of the magnetic parameter M on the velocity and shear stress fields are the same to that of slip parameter γ for both type of accelerated flows. It is due the fact that magnetic field produces resistance in the flows and results to decrease the velocity and shear stress by increasing the values of M. The behavior of K on the velocity and shear stress for constant and variable accelerated flows absolutely opposite to those of magnetic parameter M and slip parameter γ. It is worth noted that the permeability K of a porous medium reduces the resistance and hence cause the velocity and shear stress to increase with the increasing values of K. The dimensionless time τ is an increasing function of velocity and shear stress fields. The diagrams for the limiting cases have also been sketched. Finally this dissertation has been summarized in Chapter 6 and the recommendations for future research have been highlighted.

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