MATHEMATICAL MODELLING OF UNSTEADY FREE CONVECTION BOUNDARY LAYER FLOW OVER A THREE-DIMENSIONAL STAGNATION POINT

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To my beloved Mak, Abah, Dely, Haili, Ali,

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ABSTRACT

The three-dimensional axisymmetric stagnation point flow have applications in many manufacturing processes in industry such as the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheet, and the cooling of an infinite metallic plate in cooling bath. In this thesis, mathematical models to study the heat and mass transfer of an unsteady three-dimensional body near the stagnation point are developed. Problems considered involve the flow in viscous fluid and micropolar fluid. In addition, the effect of heat generation is also considered for viscous fluid problem. The governing equations which consist of coupled nonlinear partial differential equations are solved numerically through an implicit finite difference scheme known as the Keller-box method. The results presented include the velocity, temperature and microrotation profiles as well as the fluid flow and heat transfer characteristics for various parametric physical conditions such as the absorption parameter, Q and the material or micropolar parameter, K. The results obtained show that the effect of heat generation, Q gives rises to the skin friction and heat transfer coefficients. However, the skin friction and heat transfer coefficients are decreased when the material parameter, K is increased.

ABSTRAK

Aliran titik genangan tiga matra simetri sepaksi mempunyai pelbagai kegunaan di dalam proses pembuatan di industri seperti lapisan sempadan di sepanjang pengelolaan bahan-bahan penghantaran, penyemperitan aerodinamik bagi kepingan plastik dan penyejukan plat logam tak terhingga di dalam penyejuk mandian. Dalam tesis ini, model-model matematik dibina untuk mengkaji pemindahan haba dan jisim bagi jasad tiga matra tak mantap berhampiran dengan titik genangan. Masalah-masalah aliran yang dipertimbangkan adalah aliran dalam bendalir likat dan bendalir mikrokutub. Bagi aliran bendalir likat, kesan penjanaan haba turut dipertimbangkan. Persamaan menakluk yang terdiri daripada persamaan terbitan separa yang tak linear diselesaikan secara berangka menggunakan skema beza terhingga tersirat yang dikenali sebagai kaedah kotak Keller. Keputusan-keputusan yang merangkumi profil-profil halaju, suhu dan mikroputaran, serta ciri-ciri aliran bendalir dan pemindahan haba dipaparkan secara grafik bagi beberapa parameter penyerapan, Q dan parameter bahan atau mikrokutub, K. Keputusan yang diperoleh menunjukkan bahawa kesan penjanaan haba, Q meningkatkan pekali-pekali geseran kulit dan pemindahan haba. Walau bagaimanapun, pekali-pekali ini semakin berkurangan apabila berlakunya peningkatan nilai-nilai parameter bahan, K.

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LIST OF SYMBOLS

a	-	acceleration
$\mathbf{a}_1$, $\mathbf{a}_2$	-	unit vectors
a_x, a_y, a_z	-	scalar acceleration in *x*-, *y*- and *z*-components
a, b	-	principles curvature in the *y*- and *x*-planes
c	-	curvature parameter
c_p	-	heat at constant pressure
e	-	internal energy
f	-	body force
f_x, f_y, f_z	-	body force in *x*-, *y*- and *z*-components
F	-	force
F_x, F_y, F_z	-	scalar force in *x*-, *y*- and *z*-components
g	-	gravity acceleration
g_0	-	gravity acceleration at initial time
H	-	arbitrary vector
H	-	microrotation vector
H_1, H_2	-	microrotation components along *x*- and *y*-axes
j	-	microinertia density
	-	index point on η plane
J_σ	-	surface curvature
k	-	fluid conductivity
K	-	material parameter
m	-	mass
n	-	unit normal
n	-	index point on ξ plane
N	-	nodal stagnation point

p	-	pressure
p_D	-	dynamic pressure
Q	-	heat generation/ absorption parameter
$\mathbf{R}$	-	vector position
$\mathbf{r}$	-	surface of the body S
S	-	body surface
t	-	time
T	-	fluid temperature
T_w	-	wall temperature
T_∞	-	ambient temperature
u, v, w	-	velocity components along x-, y-, z-axes
$\mathbf{V}$	-	velocity vector
x, y, z	-	cartesian coordinates
y_s	-	typical variable

Greek Symbols

α	-	thermal diffusivity
β	-	thermal expansion
∇	-	gradient operator
∇_S	-	surface gradient operator
$\delta\xi_i$	-	steplength to i^{th} interval
$\delta\eta_m$	-	steplength to m^{th} interval
η	-	plane along y-axis
γ	-	spin gradient viscosity
$\dot{h}$	-	volumetric heat addition
$\dot{h}_x, \dot{h}_y, \dot{h}_z$	-	heat transferred in x-, y- and z-directions
κ	-	vortex viscosity
μ	-	dynamic viscosity
ν	-	kinematic viscosity
ϕ	-	viscous dissipation
ρ	-	density

σ	-	surface
τ	-	dimensionless parameter
	-	viscous stress
τ_{yx}, τ_{yz}	-	shear stresses
τ_{yy}	-	normal stresses
θ	-	dimensionless parameter
ξ	-	plane along x-axis

Superscripts

$'$	-	differentiation with respect to η

Subscripts

s	-	steady-state flow
w	-	wall condition
∞	-	far field condition

Nondimensional numbers

C_{fx}	-	skin friction coefficient in x-direction
C_{fy}	-	skin friction coefficient in y-direction
Gr	-	Grashof number
Nu	-	Nusselt number
Pr	-	Prandtl number

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The fluid immediately adjacent to the surface to stick to the surface happened because of the effect of friction. In a simple word, there is no slip condition at the surface and that frictional effects were experienced only in a thin region near the surface. In other word, this region is known as the boundary layer. This theory was first proposed by Ludwig Prandtl on 8 August 1904. On that day, a few mathematicians and scientists assembled including Prandtl. Just in 10 minutes of presentation, Prandtl was able to bring the new dimension in fluid dynamics by introducing the theory of boundary layer. Boundary layer equation was reduced from Navier-Stokes equation that was first derived by Claude Louis Navier in 1822, while George Stokes independently derived it in 1845. Boundary layer equation has the parabolic behavior, which gives simplicity to analytical and numerical solutions. By marching downstream from where the flow encounters a body, these equations can be solved step by step. In 1905, Prandtl proposed the boundary layer equations for steady two-dimensional flow. He also proposed some solution approaches to solve the equations.

Later, in 1908, Prandtl's student, Heinrich Blasius published a paper entitled "Boundary Layers in Fluids with Little Friction". He solved both cases of flat plate and circular cylinder. He performed more accurate solution for the skin friction drag compared to the original paper of Prandtl for the case of flat plate. On the other hand, the solution gave the separation points on the back of the cylinder. Although the important finding by Blasius and Prandtl's research group have paid slight attention by scientists and researchers, former student of Prandtl, Theodore von Karman in 1921 successfully obtained a momentum integral equation that is proved to be directly applicable to a huge number of practical engineering problems especially in the field of fluid dynamics. Since then, the theory of boundary layer equations has attracted much attention among the technical society.

Skin friction drag on airships and airplanes was the first serious application of the boundary layer theory in late 1920s. After that, this theory become standardizes among the airplane designers and researchers have written many books such as "Boundary Layer Theory" by Hermann Schichting. In 2000, this book is already in its 8^{th} edition since it has been published in early 1930s, thus this shows the importance of Prandtl's theory throughout the last decades until today. Nowadays, problems involving boundary layer flow near the stagnation point can be found in many high technology products including engineering applications such as geothermal energy recovery, food processing and glass fibre. Various aspects of the flow and heat transfer problems for boundary layer flow near the stagnation point have been explored in many investigations.

Poots (1964) derived the boundary layer equations for steady three-dimensional stagnation point flow. From his study, a similar its solution of the boundary layer equations can be found if the parametric lines of the curvilinear coordinates on the surface are chosen to be lines of curvature. These solutions depend on several parameters such as the Prandtl number, Grashof number and ratio of two principal radius of curvature of the surface at the stagnation point. Afterward, several researchers considered the cases of boundary layer on three-dimensional stagnation point flow such as Banks (1974), Ingham *et al.* (1984), Kumari and Nath (1986), Slaouti *et al.* (1998) and Sharidan *et al.* (2007) and various effects have been taken into the problems investigated.

1.2 Research Background

Several researchers have studied the free convection on boundary layer flow near the stagnation point. These studies have been considered as the important finding due to their applications in the technology solutions. Poots (1964) started to derive the steady three-dimensional boundary layer equations. Then, Banks (1974) took this advantage and presented the special results for the negative region of curvature parameter. Kumari and Nath (1986) presented the results for the case of hydromagnetic flow in the presence of an applied magnetic field. Besides, Slaouti *et al.* (1998) considered the case where there is an initial steady state that is perturbed by a step change in the wall temperature.

On the other hand, by taking the effect of large injection rates, Eswara and Nath (1999) studied the unsteady laminar incompressible mixed convection boundary layer flow at a three-dimensional stagnation point. The effect of a small fluctuating gravitational field characteristic of g-jitter on heat and mass transfer has been investigated by Sharidan *et al.* (2007). Hayat *et al.* (2010) obtained the analytical solution of three-dimensional magnetohydrodynamic (MHD) flow in a porous space by using the homotopy analysis method (HAM). Since all studies have presented comparison between the results of steady state and the results obtained by Poots (1964), we are inspired to investigate the behavior of heat and mass transfer on viscous and incompressible fluid as our first problem and to do the same comparison to check the validity of our results as compared with the previous studies.

Moreover, we have to extend our first problem by taking the effect of internal heat generation or absorption. The literature review on heat generation issues have been revealed by many researchers and published in various journals. Foraboschi and Federico (1964) were the first to introduce the volumetric rate of heat generation. In 1993, heat transfer characteristics in boundary layer of a viscous fluid over a stretching surface with the effect introduced by Foraboschi and Ferderico (1964) have been studied by Vajravelu and Hadjinicolaou. Chamkha and Camille (2000) considered the effect of thermophoresis together with heat generation of hydromagnetic flow over a flat plate.

The most recent would be the study on natural convection along a wavy surface of body that was investigated by Molla *et al.* (2004).

In the third problem, we consider the case that relates to the micropolar fluids. The theory of micropolar fluids, which was first proposed by Eringen (1966) has gained the most attention since the traditional Newtonian fluids cannot precisely describe the characteristics of the fluid flow with suspended particles. The model for unsteady boundary layer flow of a micropolar fluid near the forward stagnation point of two-dimensional infinite plane wall was reported by Lok *et al.* (2003a). Then, Lok *et al.* (2003b) changed the case of forward stagnation point to rear stagnation point (Lok *et al.*, 2003a). On the other hand, Sharidan *et al.* (2005b) demonstrated the behavior of g-jitter induced free convection in micropolar fluid near the stagnation point of three-dimensional body. Furthermore, Xu *et al.* (2006) presented the series solutions from Lok *et al.* (2003a) by means of HAM (Liao, 2003) that have produced more accurate results over the whole range of material parameter. Recently, Cheng (2010) examined the nonsimilar boundary layer solutions for double diffusion by natural convection along a sphere.

Since the unsteady three-dimensional body near the stagnation point gives much impact on the heat transfer process, it is necessary to extend the previous works to obtain much better results. These results will be used for validation purposes in related industries, for example in the manufacturing industry. Thus, it is the purpose of this study to investigate the unsteady three-dimensional body near the stagnation point on the free convection for viscous and micropolar fluids.

1.3 Problem Statements

This study will explore the following questions. How can we compare the unsteady three-dimensional stagnation point flow and the steady three-dimensional stagnation point flow? What are the transformations that can be used to reduce the number of variables on the governing equations that satisfy the numerical approach? What are the effects of heat generation and micropolar or material parameters to the flow characteristics on the wall shear stress (skin friction) and the wall heat flux (heat transfer from the surface)? How do the velocity, microrotation and temperature profiles affected due to the presence of heat generation effect and micropolar fluid?

1.4 Objective and Scope

The objective of the study is to investigate theoretically the unsteady boundary layer flow near the three-dimensional stagnation point by solving mathematical models for the following incompressible and viscous fluid problems:

1. Unsteady free convection flow near the stagnation point of a three-dimensional body.
2. Unsteady free convection flow near the stagnation point of a three-dimensional body with internal heat generation or absorption.
3. Unsteady free convection flow near the stagnation point of a three-dimensional body in a micropolar fluid.

Various parameters such as the Prandtl number, curvature parameter, heat generation parameter and material parameter are considered in order to investigate the behavior of flow and heat transfer characteristics for various values of each parameter.

1.5 Significance of the Study

Boundary layer and stagnation point flow have significant impact on the technology applications. Most of the man-made technologies that are produced in various industries, such as the design of packed bed reactors, food processing and spacecraft maneuvers can be explained using boundary layer flow near a stagnation point theory as well some suitable effects.

Specifically, boundary layer theory find its application in the calculation of the skin friction drag which acts on a body as it is moved through all type of fluids. This theory is very useful on the situation where the drag experienced by a flat plate at zero incidence, the drag of the ship, of an aeroplane wing, aircraft nacelle or turbine blade. Boundary layer also gives the answer to the very important question of the shape that should be for the body in order to avoid such detrimental separation. Furthermore, this theory can explains the phenomena that happen at the point of maximum lift of an aerofoil, which is associated with stalling.

Subsequently, the study of heat generation in all type of fluids is important in viewing several physical problems, for instance those dealing with chemical reactions and those concerned with dissociating fluids. The temperature distribution and particle deposition rate can be affected by the possible heat generation. This may occur in such applications related to nuclear reactor cores, fire and combustion modeling, electronic chips and semi conductor wafers.

In addition, boundary layer flow near a stagnation point embedded in micropolar fluid is also considered to be an important field of study in various engineering applications, such as extrusion of polymer fluids, solidification of liquid crystals, cooling of a metallic plate in a bath, animal bloods, exotic lubricants and colloidal and suspension solutions, for which the classical Navier-Stokes theory is insufficient.

Recently, micropolar fluid is used to model various biological flows. It is because the flow of a micropolar fluid is less prone to instability than that of a classical fluid as part of angular momentum is lost in the rotation of particles. Motion of an air bubble in blood flow, and an exchange of fluid between a circular capillary with a rigid wall and the surrounding tissue were modeled by Maurya (1985). Thus, the biological fluids with inner structure (body fluids) is the interesting topic to be derive the attention since there is more secret in our body and maybe it can help the scientist to cure the AIDS viruses.

Finally, this study is a medium to enhance understanding of fluid mechanics and heat transfer phenomena and also the generation of efficient algorithms to solve the related computational fluid dynamics problems.

1.6 Thesis Outline

This thesis consists of six chapters. Chapter 1 begins with the introduction, background of the research, statement of problem, the objectives and scope, and the significance of this study in numerous applications. The literature review has been extensively studied in Chapter 2. In Chapter 2, there are three sections presented specifically for each problem considered in this study. Chapter 3 and Chapter 4 discuss the derivation of equation of motion in three-dimensional and boundary layer equation for laminar free convection on isothermal surface, respectively. For boundary layer equation, we do not consider the viscous dissipation and the work done against compression.

In Chapter 5, we discuss the first problem on unsteady free convection flow near the stagnation point of a three-dimensional body in a viscous and incompressible fluid. This chapter will be divided into five main sections where the first section is the introduction of the problem and the second section will describe the details on the basic equations. The solution procedure for this problem is explain in the third section that including the finite-difference method, Newton's method, block-elimination method and the starting conditions for the programming setup. The finals section contains the conclusion of this problem.

On the other hand, in Chapter 6, we discuss free convection boundary layer problem of three-dimensional stagnation point as well, but we considered the effect of internal heat generation or absorption inside the problem. We begin this chapter with introduction of the problem. Then, we describe in details the basic equations of the problem and show the results and discussion in second and third sections. Finally, we provide the conclusion for this problem.

Chapter 7 discusses the unsteady free convection flow near the stagnation point of a three-dimensional body in a micropolar fluid. The division of sections is similar to those in Chapter 6. All the problems in chapters 5 to 7 are solved by using implicit finite-difference scheme known as Keller-box method. The details for this method are shown in the third section of Chapter 5. The obtained numerical results which include the velocity and temperature profiles as well as the skin friction coefficient and the heat transfer coefficient are presented in chapters 5 to 7. Besides, the microrotation profiles are explained in Chapter 7 for the case of micropolar fluid. Finally, the summary of this study is given in Chapter 8. In this chapter, we also include some suggestions for future research.

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