

MONITORING THE VARIABILITY OF PETROCHEMICAL ZA FERTILIZER
PRODUCTION PROCESS BASED ON SUBGROUP OBSERVATIONS

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ABSTRACT

The development in technology and requirement to control the process quality characteristics simultaneously have leads to the use of multivariate control chart. This chart considered the correlations between quality characteristics, hence improve the performance of that statistical chart. This study deals with the Shewhart-type control chart of multivariate process of ZA fertilizer production in petrochemical industry. The main objective of this study is to monitor the process variability. In order to perform the monitoring process, Phase I and Phase II need to be done. In Phase I, the historical data set (HDS) with $m = 36$ observations were used to construct the control limits in order to detect any outlier observations, and then removed them to calculate the new control limits with the remaining observations. This step was repeated eight times before the in-control process was obtained together with the estimated parameter, which is the average of sample covariance matrices. Thus, a clean HDS with $m = 20$ observations were used to calculate the control limits for monitoring multivariate process variability of new observation in Phase II operation. Here, the study concluded whether the process is stable or not. In purpose of the study, generalized variance (GV) chart is presented as well as vector variance (VV) chart to perform the operations in both phases. Based on GV chart, no outlier is detected and the process is in-control. However, VV chart had detected outliers and concluded that the process is out-of-control. This illustrated that the VV chart is more effective in detecting process variability rather than GV chart.

ABSTRAK

Perkembangan teknologi dan permintaan untuk mengawal ciri-ciri kualiti proses secara serentak telah membawa kepada penggunaan carta kawalan pelbagai pembolehubah. Carta ini mempertimbangkan hubungan di antara ciri-ciri kualiti proses, seterusnya meningkatkan prestasi carta statistik tersebut. Penyelidikan ini berkaitan dengan carta kawalan jenis *Shewhart* terhadap proses pelbagai pembolehubah bagi pembuatan baja ZA dalam industri pertokimia. Objektif utama bagi penyelidikan ini ialah untuk memantau *variability* proses. Untuk melaksanakan proses pemantauan, Fasa I dan Fasa II perlu dilakukan. Dalam Fasa I, set data asal (SDA) dengan $m = 36$ pemerhatian telah digunakan untuk membina had-had kawalan bagi mengesan sebarang pemerhatian nilai-nilai terencil, dan kemudian menyingkirkannya bagi mengira had-had kawalan baru dengan baki pemerhatian yang ada. Langkah ini diulang sebanyak lapan kali sebelum process dalam-kawalan diperolehi bersama-sama dengan parameter anggaran, iaitu purata bagi sampel matrik kovarians. Jadi, SDA yang bersih dengan $m = 20$ pemerhatian telah digunakan untuk mengira had-had kawalan bagi memantau *variability* proses pelbagai pembolehubah terhadap pemerhatian baru di operasi Fasa II. Di sini, penyelidikan menyimpulkan sama ada proses adalah stabil atau tidak. Bagi tujuan penyelidikan, carta *generalized variance* (GV) dipaparkan sebagaimana carta *vector variance* (VV) untuk melaksanakan operasi di kedua-dua fasa. Berdasarkan carta GV, tiada nilai-nilai terencil dikesan dan proses adalah di dalam-kawalan. Walaubagaimanapun, carta VV telah mengesan nilai-nilai terencil dan menyimpulkan bahawa process adalah di luar-kawalan. Ini menjelaskan bahawa carta VV adalah lebih berkesan dalam mengesan perubahan proses berbanding carta GV.

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LIST OF ABBREVIATIONS

GV	-	Generalized Variance
HDS	-	Historical data set
LCL	-	Lower control limit
UCL	-	Upper control limit
VV	-	Vector Variance

LIST OF NOTATIONS AND SYMBOLS

$^{\circ}\text{C}$	-	Degree Celcius
CO_2	-	Carbon dioxide
NH_3	-	Ammonia
\mathbf{S}_k	-	Covariance matrix for sample k
\mathbf{S}_k^2	-	Squared covariance matrix for sample k
$\bar{\mathbf{S}}$	-	Average of sample covariance matrices
$\bar{\mathbf{S}}_{GV}$	-	Average of sample covariance matrices for GV
$\bar{\mathbf{S}}_{VV}$	-	Average of sample covariance matrices for VV
$\bar{\mathbf{S}}^2$	-	Squared average of sample covariance matrices
$ \mathbf{S} $	-	Determinant of covariance matrix (GV) for sample
$ \Sigma $	-	Determinant of covariance matrix (GV) for population
$ \mathbf{S}_k $	-	Determinant of sample covariance matrix
Tr	-	Trace
$Tr(\mathbf{S}_k^2)$	-	Sum of all diagonal elements of \mathbf{S}_k^2 (VV)

$Tr(\bar{\mathbf{S}}^2)$	-	Sum of all diagonal elements of $\bar{\mathbf{S}}^2$
$Tr(\bar{\mathbf{S}}^4)$	-	Sum of the squares of all elements in $\bar{\mathbf{S}}^2$
m	-	Number of observations
n	-	Sample size
s^2	-	Variance for sample
\bar{x}	-	Mean for sample
p	-	Number of quality characteristics / variables
α	-	False alarm
μ	-	Mean for population
σ^2	-	Variance for population
χ^2	-	Chi-square

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study

The development of multivariate process control in order to monitor and control process quality characteristics simultaneously had allowed many proposed multivariate control chart technique. The control chart is the most useful process monitoring technique because, it easily can detect any assignable cause in the process which falls outside the control limits. It is necessary to identify the quality characteristics of the product in order to control a production process. The existence of these multiple quality characteristics for defining the quality of the process in industry, leads to the consideration of their correlations. In other words, the variability in the observed values is unavoidable. However, if practitioners monitor the process quality characteristics separately, it will causes incorrect decisions. Alternatively they can use a multivariate control chart for simultaneous monitoring of process quality characteristics. Sometimes, these quality characteristics also known as variables.

In the study, the Shewhart-type control charts are used in order to monitor the variables, and at the same time it will detect shifts in the process, which is the covariance between several related variables. Using the charts, there are two phases need to be considered, which is Phase I for parameter estimation and Phase II for future monitoring. Numerous studies focused on monitoring only the mean vector by using Hotelling T^2 , such as Mason and Young (1999). However there are little emphasis on

monitoring process variability. Due to this problem, a comparative study is conducted to monitor and control multivariate process variability based on generalized variance (GV), which is also known as the determinant of covariance matrix, and vector variance (VV) statistics.

1.2 Problem Statement

A lot of research has been carried out regarding the multivariate control charting methods that focused on detecting shifts in the mean vector. Most of them use Hotelling T^2 control charting. However, Alt and Smith (1988) and Montgomery (2005, p. 511) point out that monitoring process variability is also important as monitoring process mean vector. Motivated by this emphasis, a study on monitoring and controlling multivariate process variability is carried out. In order to illustrate the monitoring process focus on the sample covariance matrix, two charts based on GV and VV statistics are used. Based on these statistics, the advantage of VV is it does not have any limitation, in which GV needs the condition that the covariance matrix is non-singular. The study is implemented into real application of ZA fertilizer production process from petrochemical industry in order to verify the objectives of this study. From the results, the study compared the performance of both charts in detecting any shifts in variability.

1.3 Objectives of the Study

As stated by Woodall and Montgomery (1999), the multivariate process control is one of the most rapidly developing sections of statistical process control. Let us assume that there is only one process output variable measured and tested in univariate case, however in real production several related variables need to be considered for the purpose

of monitoring and controlling the process. So, the multivariate statistical process control methods that consider the variables jointly overcome this problem by monitoring the variables simultaneously. Since the research focused on monitoring the process variability based on Shewhart-type control chart, two phases are conducted to achieve the aims of the study. The objectives of the study are:

- (a) to determine a clean historical data set (HDS) by removing any outlier observation which can distort the parameter estimation in Phase I
- (b) to obtain parameter estimation based on clean HDS in Phase I control charting to be used in Phase II monitoring process
- (c) to determine whether the new observations in Phase II is in-control or out-of-control by monitoring the multivariate process variability using GV chart and VV chart
- (d) to compare the performance of GV and VV charts in monitoring multivariate process variability

1.4 Scope of the Study

The main focus of this study is to monitor the variability of multivariate process. In order to accomplish this purpose, two Shewhart-type control charts based on GV and VV are built for the purpose of comparison. From these methods, some related measures need to be used together with the data obtained after the process put in action. The related measures are process covariance matrix and the determinant of sample covariance matrix as well as the trace of the squared sample covariance matrix. All data analysis and control chart representations are constructed by using Microsoft Office Excel 2007, while the data of ZA fertilizer production process from petrochemical industry are studied to compare the performance of those two chart, thus give the clearer picture about the research.

1.5 Significance of the Study

The findings of this research is important to agree about the use of control charting which are GV and VV charts in monitoring the multivariate process variability. The results of the study can give an idea of a better control chart to be used in monitoring the multivariate process variability as well as quality control improvement in many field such as petrochemical, manufacturing, technology, health and care, and medication. Since the construction of both charts is simple, thus it is necessary to be implemented into the real industry.

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