UNSTEADY MIXED CONVECTION FLOW OVER A CYLINDER OF ELLIPTIC CROSS SECTION NEAR FORWARD AND REAR STAGNATION POINTS

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To my beloved family,
Dr. Sharidan Shafie
Dr Anati Ali
and friends……

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ABSTRACT

In this thesis, the unsteady mixed convection flow over a cylinder of elliptic cross section when the major axis are horizontal (blunt orientation) and vertical (slender orientation) have been studied. The study focused on the solution near the forward and rear stagnation points subjected to constant temperature placed in an incompressible viscous fluid. The unsteadiness is due to an impulsive motion of the free stream. The governing boundary layer equations are first reduced into a non-dimensional form, and then, transformed into a set of non similarity boundary layer equations, which are solved numerically using an efficient implicit finite-difference method known as Keller-box method. The numerical results are obtained for various values of the Prandtl numbers, Pr, the mixed convection parameter, α and parameter for blunt and slender orientation, ω. The effects of these parameter on velocity profiles, temperature profiles as well as Nusselt number are presented through graphs and tables. It is found that the increasing value of ω leads to a decreases in the velocity profiles for both cases, blunt and slender orientation near the forward and rear stagnation point, respectively. An increased value of temperature profiles is found near the forward stagnation point while the value is decreased near the rear stagnation point for the case of slender orientation. The temperature profiles are fixed for the case of blunt orientation. Further, near both the forward and rear stagnation points, it is also found that the Nusselt number is fixed with increasing ω for the case of blunt orientation. However, for the case of slender orientation, the Nusselt number near the forward stagnation point is decreased whereas near the rear stagnation point it is increased.

ABSTRAK

Dalam tesis ini, masalah aliran lapisan sempadan tidak mantap berserta olakan campuran yang melepasi silinder berbentuk elip bagi kedua-dua paksi utama adalah menegak (orientasi tumpul) dan melintang (orientasi langsing) di kaji. Fokus kajian ini adalah kepada penyelesaian di sekitar titik genangan hadapan dan belakang pada suhu malar di dalam bendalir pekat yang tidak termampat. Ketidakmantapan aliran adalah disebabkan oleh gerakan dedenyut arus bebas. Persamaan-persamaan lapisan sempadan menakluk, pada mulanya diubah bentuk kepada bentuk tak bermatra, kemudian diubah kepada set persamaan lapisan sempadan tak serupa, seterusnya diselesaikan secara berangka dengan menggunakan kaedah beza terhingga tersirat yang efektif dikenali sebagai kaedah kotak-Keller. Keputusan-keputusan berangka yang diperolehi meliputi pelbagai nilai nombor Prandtl, Pr, parameter olakkan campuran, α dan parameter untuk orientasi tumpul dan langsing, ω. Kesan parameter - parameter ini terhadap profil halaju, profil suhu dan juga nombor Nusselt dipaparkan menerusi graf dan jadual. Didapati dengan meningkatnya nilai ω menyebabkan profil halaju menurun di sekitar titik genangan hadapan dan belakang bagi kes orientasi tumpul dan langsing. Peningkatan nilai profil suhu didapati berlaku di sekitar titik genangan hadapan tetapi menurun di sekitar titik genangan belakang bagi kes orientasi langsing. Namun, bagi kes orientasi tumpul, profil suhu adalah tetap. Seterusnya, hasil kajian juga menunjukkan nilai nombor Nusselt adalah tetap dengan peningkatan ω di sekitar titik genangan hadapan dan belakang bagi kes orientasi tumpul. Walaubagaimanapun, bagi kes orientasi langsing, nombor Nusselt menurun di sekitar titik genangan hadapan manakala meningkat di sekitar titik genangan belakang.

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LIST OF SYMBOLS

C_f - skin friction coefficient

C_p - specific heat at constant pressure

erf - error function

c - thermal conductivity

b - length of the semi-minor axis

a - length of the semi-major axis

b/a - the ratio of the major and minor axis of cylinder

g - acceleration due to gravity

Gr - Grashof number

Nu - Nusselt number

$\bar{p}$ - dimensional pressure

p - nondimensional pressure

Pr - Prandtl number

q_w - convection heat transfer coefficient

Re - Reynolds number

$\bar{t}$ - dimensional time

t - nondimensional time

t_s - separation time

$\bar{T}$ - dimensional fluid temperature

T - nondimensional fluid temperature

T_w - surface temperature

T_∞ - external temperature

$\bar{u}, \bar{v}$ - dimensional velocity components along $\bar{x}$ and $\bar{y}$ axes

u, v - nondimensional velocity components along x and y axes

U_∞ - stream velocity

$\bar{x}, \bar{y}$ - dimensional Cartesian coordinates measured along the surface of the cylinder and normal to it, respectively

x, y - nondimensional Cartesian coordinates measured along the surface of the cylinder and normal to it, respectively

Greek symbols

α - mixed convection parameter

β - thermal expansion coefficient

ϕ - angle between outward normal and downward vertical

γ - eccentric angle

κ - vortex viscosity

η - similarity variable

μ - dynamic viscosity

τ_w - wall shear stress

υ - kinematic viscosity

ρ - density

ψ - stream function

ω - constant

Superscripts symbols

‘ - differentiation with respect to η

Subscripts symbols

w - wall condition

∞ - far field condition

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CHAPTER 1

INTRODUCTION

1.1 Research Background

The transfer of heat through fluid (liquid or gas) caused by molecular motion is known as convection. Besides that, convection is also defined as a movement of fluids regardless of the cause in fluids mechanics field. In general, the convection type of heat transfer can be divided into two basic processes. The first process is named as free or natural convection and the second process is forced convection. Forced convection occurs when a fluids flow is induced by an external force, such as a pump, fan or a mixer, while free convection is caused by buoyancy forces due to density differences created by the temperature variations in fluids. However, when the effect in these both convections becomes significant, then the process is described combined forced and free convection, which is also known as mixed convection. The effect is particularly marked in situations where the forced fluid flow velocity is low and/or the temperature difference is large.

There are lot of problems about free convection constantly arise in engineering service. An example of free convection is the cooling process in heat exchanger components. Therefore, by understanding the properties of the convection occurring in the process, the lifespan of the heat exchanger component can be monitored and predicted.

In these recent years, unsteady flows have become significant in both different category of fluid mechanics and an area of convection heat and mass transfer. The problem of unsteady convective heat transfer has long been a major subject in the heat transfer theory because of its great importance from both a theoretical and practical viewpoint. The extra independent variables time which has been considered in the unsteady problem can increases of the complexity of its solution procedure. The unsteady effects can arise in two situations. The first situation is due to self-induced motion of the body and the second situation is due to the fluctuations or nonuniformities in the surrounding fluid. Besides that, some devices are required to execute time-dependent motion in order to perform their basis functions (McCroskey, 1977).

In general, unsteady viscous phenomena play an important role in the reentry of space vehicles. Unsteady viscous flows have been studied quite widely and all the characteristic features of unsteady effects are now more or less familiar to fluids mechanicists. The problem of unsteady mixed convection boundary layer flow of Newtonian and Non-Newtonian fluid past a circular cylinder have been considered by Ingham and Merkin (1981) and Ali *et al* (2010), respectively.

In addition, the study of stagnation point flow has attract much attention because of its capability in providing the governing equations to be much simplified besides allowing the process of bringing out all the essential features. The stagnation point solution, though it may valid in a small region in the vicinity of stagnation point, may function as a starting solution for the solution over the entire body, as proven by Lok (2008).

Following the above studies, the present study aspires to obtain the unsteady mixed convection flow over a cylinder of elliptic cross section near forward and rear stagnations points for the case of constant wall temperature. This study considered the problem near both forward and rear stagnation point. In addition, this study also looks into the difference caused by blunt orientation and slender orientation. This boundary layer problem is solved by Keller's Box method described in Ali *et al* (2007). According to Keller (1978), we know that this method have been found to be efficient and flexible in dealing with many type of problem, especially for free and mixed convection boundary layer flows. In fact, it is easily adaptable for solving equation of any order (Cebeci and Bradshaw, 1988)

1.2 Problem Statement

The study will investigate the following questions; (i) How Prandtl number, mixed convection coefficient and time affect the results of skin friction, Nusselt number, velocity profile and temperature profile in unsteady mixed convection over a cylinder elliptic cross section near both forward and rear stagnation point? (ii) What are the effects of blunt orientation and slender orientation to the skin friction, Nusselt number, velocity profile and temperature profile in unsteady mixed convection over a cylinder elliptic cross section near both forward and rear stagnation point?

1.3 Objectives of the Study

The objectives of this study are:

i. To transform the non- dimensional governing equation of the problem into a system of non-similarity equation using stream function and similarity variables.

ii. To solve the governing equation numerically using Keller box method and develop numerical algorithm using Matlab.

iii. To determine the separation times of boundary layer near the forward and rear stagnation points.

iv. To investigate the effects of Prandlt number, mixed convection coefficient and axis ratio for blunt and slender orientation on the velocity profiles, temperature profiles, skin friction, and Nusselt number.

1.4 Scope of the study

The unsteady mixed convection boundary layer flow is considered in an incompressible viscous fluid problem. The problem will be narrowed down to boundary layer flow over horizontal cylinders of elliptic cross section subjected to constant temperature. The analysis of this study is only focusing on the forward and rear stagnation point. The numerical schemed used is Keller box method and the numerical results are obtained from various values of time, Prandlt number, mixed convection coefficient and axis ratio for blunt and slender orientation. The results are discussed based on the velocity profile, temperature profile, Nusselt number and skin friction coefficient.

1.5 Significant of study

Mixed convection (combined forced and free convection) flow with and without mass transfer occurs in many technologies and industrial applications. Its applications are namely solar central receiver exposed to wind currents, nuclear reactors cooled during emergency shutdown, heat exchangers placed in low-velocity environments, boundary-layer control on airfoil, lubrication of ceramic machine parts and food processing. Mixed convection occurs during the motion of fluid that results from variety in density and heat change.

In environment and engineering services, mixed convection over cylinder is a basic and vital problem. Moreover, in manufacture industry, most of the manufacturing machine has its own heat exchanger components which are made from tubes of elliptic cross section. The benefit of this design is it creates less resistance for cooling the fluid pass by. Thus, the study of heat transfer for an elliptic cross section cylinder is useful to create an effective and efficient heat exchanger component and design.

1.6 Outline of Dissertation

This dissertation consists of six chapters including this introductory chapter, in which discuss about the background of research, the problem statement, objectives, scope and significant of the study. The literature review for the research problem is given in Chapter 2.

Chapter 3 contains a discussion on the mathematical formulation of the equations that involved in our problem of unsteady mixed convection boundary layer over a cylinder of elliptic cross section near forward and rear stagnation point.

Full explanation of the numerical method, the Keller box method are given in the Chapter 4, which are presented and described particularly for the problem in this study. Stepwise development of the method was stated. The Keller box method used in this study is programmed in Matlab. The complete program of the specific problem discussed in Chapter 4 is given the Appendix B.

Further, Chapter 5 includes the result and discussion of the problem. The numerical computation of results are presented both in the form of tables and graphs. Finally, Chapter 6 contains a summary of the dissertation and several recommendations for future research.

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