DISCRETE ADOMIAN DECOMPOSITION METHOD FOR SOLVING FREDHOLM INTEGRAL EQUATIONS OF THE SECOND KIND

SALAR HAMEED MOHAMMED

UNIVERSITI TEKNOLOGI MALAYSIA

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SALAR HAMEED MOHAMMED

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To my beloved mother, my wife and sister.

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ABSTRACT

The nonlinear Fredholm integral equation (FIE) represents a large amount of nonlinear phenomena that usually produces a considerable amount of difficulties. This dissertation will display some methods used for solving this problem, such as an Adomian Decomposition Method (ADM) which is based on decomposing the solution to infinite series and numerical implementation of ADM for the special case when the kernel is separable. In addition, it discusses the process of applying the Discrete Adomian Decomposition Method (DADM) which gives the numerical solution at the nodes using quadrature rules like Simpsons rule and trapezoidal rule. The comparison of DADM with of both rules with the exact solution also are given. Furthermore the results from DADM, Triangles orthogonal functions (Tfs) and Rationalized Haar function (RHf) for two dimensional linear and nonlinear FIE of the second kind respectively are compared with exact solution. Hence the results obtained show equivalent accuracy when linear FIE of the second kind for two dimension were solved by DADM with Simpson's rule and by Tfs. Whereas the results show of DADM with Simpson's rule is more accurate than RHf to solve nonlinear FIE of the second kind for 2-D.

ABSTRAK

Persamaan kamiran Fredholm bagi masalah tak linear mewakili sejumlah besar masalah persamaan tak linear yang biasanya menghasilkan kesukaran. Dalam dissertasi ini beberapa kaedah untuk menyelesaikan masalah persamaan tak linear akan dinyatakan. Kaedah Adomian Decomposition Method (ADM) yang berasaskan penyelesaian untuk siri tak terhingga digunakan. Kaedah berangka juga diaplikasikan bagi kes khas dimana kernel nya terpisah. Di samping itu, ia turut membincangkan kaedah Discrete Adomian Decomposition Method (DADM) yang memberikan penyelesaian berangka bagi titik-titik pada rantau menggunakan peraturan Simpson.dan peraturan trapezoidal Perbandingan kaedah Discrete Adomian Decomposition Method (DADM) menggunakan kedua-dua kaedah penyelesaian berangka dengan penyelesaian yang tepat juga diberikan. Seterusnya, hasil yang diperolehi menggunakan Discrete Adomian Decomposition Method (DADM) dan fungsi Rationalize Haar untuk (RHf) masalah persamaan tak linear dua dimensi jenis kamiran Fredholm kedua (FIE) di bandingkan dengan penyelesaian tepat.Oleh itu, keputusan yang diperolehi menunjukkan ketepatan setara apabila FIE linear jenis kedua untuk dua dimensi telah diselesaikan oleh DADM dengan peraturan Simpson dan dengan TFS. Manakala keputusan menunjukkan daripada DADM dengan peraturan Simpson adalah lebih tepat daripada RHF untuk menyelesaikan FIE linear jenis kedua untuk 2-D.

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LIST OF ABBREVIATIONS

FIE	–	Fredholm integral equation
FIE 2^{nd}	–	Fredholm integral equation of the second kind
ADM	–	Adomian Decomposition Method
DADM	–	Discrete Adomian Decomposition Method
1-D	–	One-dimensional
2-D	–	Two-dimensional
TFs	–	triangular orthogonal functions
RHf	–	Rationalized Haar functions

LIST OF SYMBOLS

λ	–	Parameter
h, k, s, t	–	Variables
u, z, c, S	–	Functions
A_n	–	Adomian polynomial
x, y	–	Independent variable
F, N	–	Nonlinear operator
i, j, n, m	–	Integer
$K(x, t)$	–	Kernel
$\|u(x)\|$	–	Norm of $u(x)$
α	–	Constant

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CHAPTER 1

INTRODUCTION

1.1 Background Of The problem

An integral equation is an equation in which the unknown function u(x) appears under an integral sign. According to Bocher [1914], the name integral equation was suggested in 1888 by du Bois-Raymond.

The integral equation can be classified into two classes. First, it called Volterra integral equation (VIE) where the Volterras important work in this area was done in 1884-1896 and the second, it called Fredholm integral equation (FIE) where the Fredholms important contribution was made in 1900-1903. Fredholm developed the theory of these integral equations as a limit to the linear system of equations.

There are two kinds of Fredholm integral equations: and they are Fredholm integral equation where the unknown function is linear and when it is nonlinear.

$$u(x) = f(x) + \lambda \int_a^b K(x,t)\, F[u(t)]\, dt \tag{1.1}$$

The scaler $\lambda \neq 0$, $a \leq x \leq b$, where $f(x)$ is known continuous function on [a,b], $F[u(x)]$ is known nonlinear function ,$K(x,t)$ is the kernel function which is known, continuous and bounded on the square $D = \{(x,y) \mid a \leq x, y \leq b, c \leq s, t \leq d\}$ and $u(x)$ is unknown function which must be determined. (Wazwaz, 1997).

Up to now, There are several analytical and numerical methods that are used to solve nonlinear FIE's. Such as the variational iteration method (VIM), the successive approximation method, the direct computation method, the successive substitution method, the series solution method, the conversion to equivalent differential equations and the Adomian decomposition method (ADM).

For cases where the evaluation of integrals analysis is impossible or complicated the previously mentioned methods cannot be applied. there are many mathematicians have tried to find a way to overcome this obstacle. In previous research, a Modified ADM for solving the Volterra or Fredholm integral equation of the second kind. has been used, and recently Behiry et al have introduced a discrertized version of ADM, namely the Discrete Adomian dicomposion method (DADM) (Behiry, 2010). T. Allhaviranllo and M. Ghanbari overcome the same obstacle as Behiry et al. and they used an effective and reliable method homotopy analysis method (HAM) which was proposed by Liao. They introduce a discretized version of the HAM namely discrete homotopy analysis method (DHAM) for solving linear and nonlinear Fredholm integral equations (T.allahiranllo and M. Ghanbari ,2011).

1.2 Statement of the Problem

There are several analytical and numerical methods use to solve nonlinear FIE as mentioned in the previous section but these analytical solution methods are not easy to use and require tedious calculation. Also when applying these methods to solve linear and nonlinear Fredholm integral equations many definite integrals need to be computed.

For cases that evaluation of integrals analysis is impossible or complicated the previously mentioned methods cannot be applied. So many mathematicians have tried to find a way to overcome this obstacle. The recent Behiry et al have been introduced a discrertized version of ADM, namely discrete Adomian dicomposion method (DADM) (Behiry, 2010).

This research solves linear and nonlinear FIE. by using ADM and Behiry's DADM presenting some examples obtained from his paper (Bihery, 2010) and others employing Simpson's rule as he has done as well as trapezoidal rule.

1.3 Objective of the study

The objectives of this study are:

1- To reconstruct DADM from ADM .

2- To apply DADM to linear and nonlinear FIE problem .

3- To perform numerical experiment using DADM with Simpson's rule and trapezoidal rule.

1.4 Scope of the study

The research focus on the DADM to solve one and two dimensional linear and nonlinear FIE of the second kind. This study is limited to a comparison of the DADM with the Simpson's rule and Trapezoidal rule. And MATHEMATICA 7.0 software will use to compute the solution of several examples using DADM.

1.5 Significant of study

Nonlinear Fredhom integral equation of the second kind is important and appear in many applications in scientific fields, such as fluid dynamics, solid state physics, nonlinear phenomena, geophysics, plasma physics, electricity and magnetism, biology and chemical kinetics kinetic theory of gases, hereditary phenomena in biology, quantum mechanics, mathematical economics and queuing theory. These problems and phenomena may be modeled by integral equations.(T.allahiranllo and M. Ghanbari ,2011). Integral equation method is worthwhile to study since some of these problems when solved via integral equations gives high accuracy results.

ADM is a popular analytical method among the researchers who are using integral equations. DADM can be described as a new kind of ADM that solve an integral equation of the second kind numerically. This is a clear advantage over ADM for problems that cannot be solved analytically. It is hope that this work can be used as a reference for the future study.

1.6 thesis’s outline

This dissertation consists of five chapters. Including introductory and literature review chapters. The introductory Chapter 1 contains background of the problem, Statement of the Problem, objectives of the study, scope of the study, Significant of study and dissertation’s out line. The literature review is given in Chapter 2 of previous studies and states some application on Fredholm integral equation, theorems and definitions on concepts of ADM.

Chapter 3 provides a detailed study of the analysis of the methods for solving the problem. And chapter 4 presents the numerical result obtained from this study. The conclusion and recommendation for farther research is given in chapter 5.

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