MATHEMATICAL MODELLING OF RIVER POLLUTION BY DETERMINING THE LEVEL OF DISSOLVED OXYGEN AND POLLUTANT CONCENTRATION.

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To my beloved parents,

Mahadi bin Sulaiman and Jamaliah binti Aris

My sisters and my brother,

Azlina, Siti Nurlinda, Herryezman, Siti Syarifah Nor, Siti Rohaidah, Siti Noradiah, Siti Nur Najwa,

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ABSTRACT

A mathematical model of a coupled advection-diffusion equation of river pollution problem is investigated. The concentration of pollutants and dissolved oxygen were obtained from the model with and without dispersion coefficient for steady and unsteady states. The unsteady models were solved analytically using the method of Laplace transform for linear case and numerically by finite difference method for nonlinear case. The calculations also involved the absence and presence of half saturated oxygen with Michaelis-Menten term. A parameter analysis was carried out in order to observe the effects of each parameter involved in this model with respect to pollutant and dissolved oxygen concentration. The values of certain parameters such as velocity, cross section area, dispersion of pollutant and dissolved oxygen were obtained from the observational data in other relevant studies. The research methodology which is being implemented in this work was generic and can be used with little modification for other rivers.The mathematical model and its validated solution for Skudai River act as a decision support on the restriction and regulation to be imposed for monitoring the river pollution.

ABSTRAK

Satu model matematik bagi pasangan persamaan alir lintang-penyerapan untuk masalah sungai diselidiki. Kepekatan bahan pencemar dan oksigen terlarut diperoleh daripada model ini dengan dan tanpa pekali penyerakan untuk keadaan tetap dan tidak tetap. Model keadaan tidak tetap diselesaikan secara analitik dengan menggunakan kaedah penjelmaan Laplace bagi kes linear, manakala kes tidak linear diselesaikan secara berangka iaitu menggunakan kaedah perbezaan terhingga. Hitungan juga melibatkan ketidakhadiran dan kehadiran oksigen separuh tepu dengan ungkapan Michelis-Menten. Analisis parameter dijalankan untuk melihat kesan bagi setiap parameter yang terlibat dalam model ini terhadap kepekatan bahan pencemar dan oksigen terlarut. Nilai parameter seperti kelajuan, luas keratan rentas, penyerakan bahan pencemar dan oksigen terlarut diperoleh daripada data pemerhatian kajian lain yang berkaitan. Metodologi kajian yang dilaksanakan bersifat generik dan boleh digunakan untuk sungai lain dengan sedikit pengubahsuaian. Model matematik bagi Sungai Skudai dan penyelesaiannya yang disahkan dalam kajian ini berperanan membantu membuat keputusan terhadap sekatan dan peraturan yang dikenakan bagi memantau pencemaran sungai.

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LIST OF SYMBOLS AND PARAMETERS

The following symbols are used in this thesis:

t	=	Time (day)
x	=	The position (m)
P	=	The pollutant concentration (kg m^{-3})
X	=	The dissolved oxygen concentration (kg m^{-3})
α	=	mass transfer of oxygen from air to water or re-aeration ($m^2 day^{-1}$)
v	=	Water velocity in the x-direction (m day^{-1})
A	=	Cross section area (m^2)
D_p	=	Dispersion coefficient of pollutant in the *x*-direction ($m^2 day^{-1}$)
D_x	=	Dispersion coefficient of dissolved oxygen in the *x*-direction ($m^2 day^{-1}$)
S	=	Saturated oxygen concentration (kg m^{-3})
k_1	=	Degradation rate coefficient at 20^0C for pollutant (day^{-1})
k_2	=	De-aeration rate coefficient at 20^0C for dissolved oxygen (day^{-1})
q	=	Added pollutant rate along the river (kg $m^{-1} day^{-1}$)
k	=	Half saturated oxygen demand concentration for pollutant decay (kgm^{-3})
erfc	=	Complement error function
s	=	Laplace parameter
B	=	Biochemical oxygen demand
b_1	=	BOD decay
k_d	=	Deoxygenation rate constant of BOD
k_a	=	First order reaeration rate constant of DO
k_r	=	First order degradation reaction with decay rate constant of BOD
C	=	Reactant concentration

D = Mixing and dispersion coefficient

R = Reactivity of chemical and biological processes

Q = External loading to aquatic system from the point and nonpoint sources.

B_0 = First stage BOD concentration at time *t=0*

X_0 = First stage DO concentration at time *t=0*

B_t = BOD concentration at time, *t*.

X_t = DO deficit at time, *t*.

k_b = BOD sendimentation removal rate

B_a = Additional of BOD from bottom deposits and sideinflow.

D_B = Net rate of DO removal by berthal demand, plant respiration and photosynthesis.

P_m = Maximal rate where oxygen is released by photosynthesis

R_1 = Rate at which oxygen is depleted by respiration.

D_r = BOD removal rate

L_u = BOD distributed sources

L_d = BOD point sources/ sink

C_1 = Concentration of COD

D_u = DOD distributed sources

U = Advection Velocity in *x*-direction

L_0 = Initial of BOD

Q_f = River flow

a, b = Constant number

LIST OF ABBREVIATION

1D	-	One- dimensional
ADE	-	Advection Diffusion Equation
BOD	-	Biochemical Oxygen Demand
DO	-	Dissolved Oxygen
PDE	-	Partial Differential Equation
ODE	-	Ordinary Differential Equations
QUAL2E	-	Enhanced Stream Water Quality
QUASAR	-	Simulation Along River
COD	-	Chemical Oxygen demand
DOE	-	Malaysia's Department of Environment
USEPA	-	United States Environment Protection Agency
DOD	-	Dissolved Oxygen deficit
CBOD	-	Carbonaceous biochemical oxygen demand
NH^-_4	-	Ammonium
NO_2^-	-	Nitrite
NO_3^-	-	Nitrate
SOD	-	Sediment Oxygen Demand
EM	-	Effective Microorganism
Sg.	-	Sungai
C#2008	-	C programming 2008

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study.

River is the main water supplier for human life, besides rainfall and lake. However, river pollution problem being more critical, and becomes a common environmental problem in third world countries nowadays. The problem arises especially in the developing country because of human such as industrial and agricultural activities. From these activities, it can confer serious impact to the environment especially for aquatic ecosystem and the human that using water every day. Moreover, the current increasing trends in population growth and socio-economic development, the quality and quantity of water are gaining widespread attention worldwide. This increasing concern about water quality and quantity necessitates the interventions in water systems to meet the objective of sustainable

water supply and prevent potential environmental deterioration. Any method that integrates water quantity and quality in water resource allocation would have the potential to add value to decision makers who face these challenges (Zhang et al., 2010). Zacharias et al. (2005) emphasized that sustainable water management which incorporates both socio-economic and environmental perspectives is a difficult but essential task in order to prevent potential environmental deterioration. In recent years the large amounts of polluted water was discharge into the rivers and causing serious future uncertainty in the water quality.

A model is a means to represent a portion of a reality. The model is valid if the points of the model predict accurately the corresponding points of the system being modeled. There are two general categories in a study of surface water which includes river pollution problems, which are experimental model and mathematical model. An experimental model is built to a particular range and uses water to produce a scaled flow that can be measured and related back to the real water system. In Malaysia, there are many researches using a physical or experimental model but rarely via mathematical model. The research uses variety of treatment method to resolve river pollution problem, such as effective microorganism (EM) technology. EM is studied for the degradation of pollutants in a river. The most favourable amount of EM is determined to achieve the optimum decontamination level of pollutants. For a mathematical model, it represents the flow of the water and other processes with a set of mathematical equations that need to be solved numerically by using computer. A mathematical model can be modified so that the model can realistically represent a certain characteristics of a river pollution problem.

Streeter and Phelps is the first researcher that develop water quality modeling in 1925 by focusing on the urban waste load allocation problem where the impact on dissolved oxygen conditions that studied using biochemical oxygen demand (BOD) to describe the bacterial decomposition of organic carbon. This approach has led to the discovery of two-state variable model that contain settling rate of the particular matter and the sediment oxygen demand as parameter. In 1970's, the concern of the society towards the environment has extend the horizon of quality issues to include in the mechanistic representation of biological process into the model.

1.2 Problem of Statement.

Water is a vital resource that is essential for all human and ecosystem survival and health. In addition, water is used in the production, industrial and manufacturing processes, such like hydroelectric power generation, waste assimilation, recreation, navigation, enhancement of fish and wildlife. There are variety purposes of using the water. Water pollution can be defined as changes in the level of constituents or parameters to the extent that it degrades the quality of water for an intended use.

According to the Malaysia's Department of Environment (DOE), 2006, many rivers experienced a loss of quality that affects people's health, the nation's economy, and the environment. The deterioration of the water quality is highly attributed by an increasing number of pollutant sources such as sewage treatment plants, agro-based factories, and animal farms that contributed in the increasing of pollutant loaded. According to DOE annual report, the rivers in Johor, can be considered to be slightly polluted due to the various developments along the rivers. The data from the DOE stations were examined to give general improvement ideas for water quality condition. In the interpretation of water quality data, it is important to identify the spatial trend of parameters such as dissolved oxygen (DO), biochemical oxygen demand (BOD) and chemical oxygen demand (COD). These spatial trends are particularly significant for water quality modeling, as it roughly show the assimilative size of the river.

News Straits Times published an article on October 2008, Unloved Rivers that dedicated especially for Malaysians on their disappointing attitude towards rivers. The article reported that even the waterway has improved, the spending on the rehabilitation of the polluted rivers still arises and yet the public awareness still decreased. This problem has been recognized and getting worst time by time. It is also reported that the dead river has become a fact of life, like what happened in Johor.

Initially this study is based on a mathematical model from Pimpunchatet et. al (2009). From the model, we have considered two cases which are with half saturated oxygen and without half saturated oxygen coefficient which involves Michaelis-Menten formulation. We have solved the steady case via an ordinary differential equation (ODE) and the unsteady case via partial differential equation (PDE). From both cases, the dispersion coefficient and half saturated oxygen are considered.

A transient spatial river pollution problem is constructed and solve analytically and numerically.

1.3 Objectives of the Research.

The objectives of this research are given as follows:

i) To solve a specific mathematical model from Pimpunchat et. al (2009) model of a coupled advection-diffusion equation for river pollution problems.

ii) To construct a transient spatial river pollution problems based on above mentioned model.

iii) To validate the mathematical model and its solution using observational data from Sg. Skudai.

1.4 Scope of the Research.

In this research, we focus only on river problems, not the lakes or estuaries. River is the most distinct characteristic and it is a natural downstream flow. The physical condition of river is linked to the physical condition of the surrounding watershed. The mathematical model that we used involved reaction between pollutant, DO and also involving added pollutant rate. Spatial distribution of pollutant sources along the river and the duration of the release can give effect to the river pollution problem. With these, we assumed that the pollutant sources remain the same from upstream to downstream and we consider for the added pollutant rate. We also considered the time duration that is used for each level of dissolving oxygen and pollutant concentration along the river.

We used unsteady case for this research in order to construct a more realistic model. Furthermore, we consider a one dimensional model that is most commonly used in river. Besides, it also can be used in special cases such as in estuaries and lakes with large length- to-width ratios. We presumed that the temperature is always the same.

1.5 Significance of the Research.

River pollution problem is important to solve since this is a critical problem for every country. Research about river pollution always upgraded and published every year to construct the best solution, in order to use it widely and effectively. To identify the pollution's level of waters, we need to measure through water quality model. The quality of the water will be low when the pollution happened and it

contains elements such like BOD, DO, pH and temperature. If the pollutant in the river is under control, the quality of the water will increase. It means that the qualities of the water are good and clean to be used by human and other organism life to continue living. From this research, we will help to improve or control water quality, to be less contaminated or polluted and useful to others. Such a mathematical model with solutions can help people to manage water quality to solve river pollution problems with slight modifications for other rivers.

1.6 Thesis Outline.

There are six chapters presented in this study, including the present introductory chapter. Chapter 1 comprises the background of study, statement of problem, objectives, significance and scope of the research. The literature review has been extensively studied in Chapter 2. In Chapter 2, there are five sections presented specially for each problem considered in this study. Chapter 2 reviewed the introduction of river pollution, sources of river pollution, relationship between dissolved oxygen (DO) and BOD and previous study about mathematical modeling for river pollution problem. Chapter 3 presented the solution for steady state condition in solving advection-diffusion equation. Chapter 4 provides the solution for unsteady state condition and using Laplace transform technique in this study. Chapter 5 discusses the result and discussion for this study. Finally, chapter 6 summarizes the research results and the conclusion. In this chapter, we also recommended suggestion for future study.

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