

NUMERICAL SOLUTION OF NON-ISOTHERMAL COMPRESSIBLE
NATURAL GASES MIXED WITH HYDROGEN IN PIPELINE USING METHOD OF
LINES

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For my beloved family

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ABSTRACT

The effect of hydrogen injection into natural gas pipelines is investigated in particular the pressure and temperature conditions, real gas effects, Joule–Thomson effect, linepack and energy consumption of the compressor station. Real gas effects are a form of compressibility factor calculated using a particular predictive group contribution method. It is being incorporated with the conventional mixing rule, which in turn being used as a parameter in the non-isothermal transient flow model. A non-isothermal gas flow is a two-way interaction between gas flow and heat transfer. Although many other engineering cases isothermal model is usually adopted in situations over a long pipeline system, the non-isothermal case is considered here. When a gas is subjected to a temperature change, properties such as density and viscosity, change accordingly. In some situations, these changes are large enough to have a substantial influence on the flow characteristics. Because the gas transports heat, the temperature is in turn affected by changes in the flow characteristics.

While it is convenient to treat that flow in pipeline is steady because it is easier to solve and under many conditions produces adequate results, gas pipeline system does not usually operate under steady condition. As such, an assumption that the gas flow in pipeline is in an unsteady condition is made. The gas flow is described by a set of partial differential equations (PDE) resulting from the conservation of mass, momentum and energy. The PDE are solved using the Method of Lines (MOL) and which in turn resulted in ordinary differential equations (ODE). The ODE is solved using the fourth order Runge Kutta Method.

The Yamal-Europe gas pipeline on Polish territory is selected as case study, where results for the thermodynamic properties of mixed hydrogen-natural gas

significantly differed from those of natural gas. The presence of hydrogen in natural gas mixture impacted the pressure and temperature gradient in the pipeline as hydrogen injection into the pipeline reduced the molecular weight of the gas mixture. Besides that, Joule-Thomson effect dominates the temperature profile and even causes a temperature drop below the soil temperature.

ABSTRAK

Kesan suntikan hidrogen ke dalam saluran paip yang mengandungi gas asli telah dikaji. Tumpuan diberi terhadap tekanan dan keadaan suhu, kesan 'Joule-Thomson', 'linepack' dan penggunaan tenaga di stesen pemampat. Kesan gas tulen merupakan salah satu faktor pemampatan yang dihitung menggunakan kaedah 'predictive group contribution'. Kesan gas tulen ini digabungkan bersama peraturan campuran lazim, di mana kemudian ianya diguna pakai sebagai parameter kepada model aliran bukan isoterma. Aliran gas bukan isoterma merupakan hubungan dua hala di antara aliran gas dan pemindahan haba. Walaupun terdapat pelbagai aplikasi kejuruteraan yang melibatkan model aliran isoterma di dalam kes sistem paip yang panjang, namun di dalam kes ini aliran bukan isoterma di ambil kira. Apabila gas mengalami perubahan suhu, ciri-ciri lain seperti ketumpatan dan kelikatan turut berubah. Pada masa tertentu, perubahan ini terlalu besar sehingga mempengaruhi bentuk aliran. Memandangkan gas memindahkan haba, secara tidak langsung suhu dipengaruhi oleh perubahan bentuk aliran tersebut.

Dalam pelbagai keadaan, adalah mudah untuk menganggap bahawa aliran gas di dalam saluran paip adalah sekata. Ini memandangkan sebarang permasalahan melibatkan aliran di dalam paip mudah untuk diselesaikan dan menghasilkan keputusan yang memuaskan. Hakikatnya, sistem saluran paip gas tidak semestinya beroperasi dalam keadaan sekata. Oleh yang demikian, adalah wajar untuk membuat andaian bahawa aliran gas adalah tidak sekata. Aliran gas dihuraikan dengan lebih jelas menggunakan persamaan pembezaan separa. Kesemua persamaan ini adalah hasil daripada prinsip pemuliharaan jisim, momentum dan tenaga. Persamaan pembezaan separa ini diselesaikan menggunakan 'Method of Lines' (MOL), di mana hasilnya merupakan persamaan pembezaan biasa (ODE). ODE ini diselesaikan menggunakan 'Fourth Order Runge Kutta Method'.

'Yamal-Europe gas pipeline on Polish territory' dipilih sebagai rujukan kes di mana keputusan ciri-ciri termodinamik bagi campuran hidrogen dan gas asli bebeza dengan saluran paip yang mengandungi gas asli sahaja. Kewujudan hidrogen di dalam aliran gas asli mempengaruhi kadar suhu dan tekanan di dalam saluran paip, memandangkan hidrogen mengurangkan berat molekul keseluruhan campuran gas tersebut. Selain daripada itu, kesan 'Joule-Thomson' memberi kesan kepada profil suhu sehingga tahap di mana suhu tersebut jatuh di bawah paras suhu tanah.

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LIST OF SYMBOLS

ROMAN SYMBOLS	DESCRIPTION
A	Cross sectional area, m^2
a	Equation of state mixture parameter
a_s	Isentropic wave speed, ms^{-1}
c_p	Isobaric heat capacity, $Jkg^{-1} K$
f	Friction factor
g	Gravitational acceleration, ms^2
K	Pipe roughness, m
d	Diameter of pipe, m
L	Pipe length, m
\dot{m}	Mass flow rate, $kg s^{-1}$
p	Gas pressure, Pa
q	Heat flow into the pipe, $Jm^{-1}s^{-1}$
R	Gas constant, $Jmol^{-1} K$
Re	Reynolds number
t	Time, s
T	Gas temperature, K
v	Velocity, ms^{-1}
ω	Frictional force per unit length and time, Nm^{-1}
Z	Compressibility factor

ρ	Density, kgm^{-3}
θ	Inclination angle of the pipe, <i>radian</i>

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CHAPTER 1

INTRODUCTION

1.1 Research Background

Natural gas is a naturally occurring hydrocarbon gas mixture consisting primarily of methane, with up to 20% of other hydrocarbons as well as impurities in varying amounts such as carbon dioxide. Natural gas is widely used as an important energy source in many applications including heating buildings, generating electricity, providing heat and power to industry, as fuel for vehicles and as a chemical feedstock in the manufacture of products such as plastics and other commercially important organic chemicals.

Natural gas is found in deep underground natural rock formations or associated with other hydrocarbon reservoirs, in coal beds, and as methane. Petroleum is also another resource found near and with natural gas. Most natural gas was created over time by two mechanisms: biogenic and thermogenic. Biogenic gas is created by methanogenic organisms in marshes, bogs, landfills, and shallow sediments. Deeper in the earth, at greater temperature and pressure, thermogenic gas is created from buried organic material. Before natural gas can be used as a fuel, it must undergo processing to clean the gas and remove impurities including water in order to meet the specifications of marketable natural gas. The by-products of processing include ethane, propane, butanes, pentanes, and higher molecular weight hydrocarbons, hydrogen sulphide, carbon dioxide, water vapor, and sometimes helium and nitrogen.

Because natural gas is not a pure product, as the reservoir pressure drops when non associated gas is extracted from a field under supercritical (pressure or temperature)

conditions, the higher molecular weight components may partially condense upon isothermic depressurizing—an effect called retrograde condensation. The liquid thus formed may get trapped as the pores of the gas reservoir get deposited. One method to deal with this problem is to re-inject dried gas free of condensate to maintain the underground pressure and to allow re-evaporation and extraction of condensates. More frequently, the liquid condenses at the surface, and one of the tasks of the gas plant is to collect this condensate. The resulting liquid is called natural gas liquid (NGL) and has commercial value.

1.2 Problem Statement

Hydrogen is considered a promising fuel alternative in our future energy demand. The transport of a mixture of natural gas and hydrogen is possible through the existing natural gas networks without pipeline modification as long as the mass fraction of hydrogen remains sufficiently low; that is about less than 17% of hydrogen. Mixing higher percentages of hydrogen requires special attention regarding durability and integrity of the pipeline and functioning of end user appliances like flame stability and increased nitrogen oxide emissions. What is the effect of hydrogen injection into natural gas pipelines? What are the differences in the flow characteristics between natural gas and natural gas-hydrogen mixture?

Several authors have investigated the effect of hydrogen injection into existing natural gas pipelines. However, these studies assume a steady state gas flow with a constant temperature. When modeling pipeline systems, it is convenient to assume that the flow is steady because it is easier to solve and under many conditions produces adequate results. Gas pipeline systems seldom operate under steady flow conditions. As such, an assumption that the gas flow in pipeline is in an unsteady condition is made.

Although, Elaoud and Hadj-Tarib (2008) studied fast transient behavior in gas

pipelines with hydrogen and natural gas mixtures, they assumed an isothermal flow and instead of using an equation of state, an expression for the average density of the mixture related to the hydrogen mass ratio is deployed. Isothermal flow is a model of compressible fluid flow whereby the flow remains at the same temperature while flowing in a pipeline. Despite many other engineering cases isothermal model is usually adopted in situations over a long pipeline system, the non-isothermal case is considered here. When a gas is subjected to a temperature change, properties such as density and viscosity, change accordingly. In some situations, these changes are large enough to have a substantial influence on the flow characteristics. Because the gas transports heat, the temperature is in turn affected by changes in the flow characteristics.

Chapman and Abbaspour (2008) also concluded that in case of fast transients it is extremely necessary to assume that the flow is non-isothermal. This dissertation addresses the gas flow dynamics of mixed hydrogen and natural gas transport via high pressure pipelines in more detail including Joule Thomson effect, linepack and energy consumption of the compressor station. The assumption that the flow is isothermal affects the running cost of the compressors, linepack and pressure loss in gas pipelines. The non-isothermal one dimensional compressible flow is described by the time dependent continuity, momentum and energy equations and an equation of state.

It is important to mention that the transport of hydrogen and natural gas mixtures via high pressure gas networks is not self-evident. Albeit beyond the scope of this dissertation, material issues like hydrogen embrittlement, leakages and the effect of hydrogen on compressor and pressure reduction stations might complicate the utilization of high pressure networks for the transport of hydrogen and natural gas mixtures.

1.3 Objectives & Scope

The purpose of this dissertation of numerical solution of non-isothermal

compressible natural gas mixed with hydrogen in pipelines is

- To investigate the effect of hydrogen injection into natural gas pipelines; in particular the pressure and temperature conditions, Joule-Thomson effect, linepack and energy consumption of the compressor station. A compressor station is a facility that helps the transportation of natural gas through gas pipelines by providing energy (thru constant pressure by turbines) to move the gas through the pipelines. An explanation of linepack is as per in the **Appendix: Linepack**
- To determine the non-isothermal flow characteristics for mixed hydrogen-natural gas in pipelines

This research is scoped to the following

- Reproduction of International Journal of Hydrogen Energy 34 by F. E. Uilhoorn (2009) entitled ‘Dynamic Behavior of Non-Isothermal Compressible Natural Gases Mixed with Hydrogen in Pipelines’
- Non-isothermal unsteady one dimensional compressible flow described by time dependent continuity, momentum and energy equations and equation of state

1.4 Significance of Research

Single phase fluid flow is encountered in many industrial processes; including that in the oil and gas industry. It has significant economic and scientific importance to these related industries. Industries usually rely on scientific models and simulations to predict the behavior of the fluid flow, to determine the flow dynamics and to observe the contact between similar phases but different chemical compositions or the contact

between different phases. These are useful information to assist related industry professionals to optimize their available infrastructure and resources in order to improve product delivery, productivity and cost efficiency.

Natural gas is an extremely important source of energy for reducing pollution and maintaining a clean and healthy environment. In addition to being a domestically abundant and secure source of energy, the use of natural gas also offers a number of environmental benefits over other sources of energy, particularly other fossil fuels.

Confronting more and more of today's urgent environmental challenges, such as the control of the release of the gases with greenhouse effect, and facing with the ever increasing shortage in the fossil resources, the radical changes in energy policies seem now inevitable. Among the various domains, hydrogen is one of the energy alternatives full of promise. Preliminary studies have shown that the transport of a mixture of natural gas and hydrogen is possible through the existing natural gas networks without pipeline modification as long as the mass fraction of hydrogen remains sufficiently low. Although hydrogen substitution perturbs little the fluid mechanics constraints of the system, the limitations fall into the tolerance of the construction materials of the pipes, compressors and other elements of the natural gas infrastructures.

The transition towards the situation in which hydrogen becomes an important energy carrier, needs decades but worldwide great efforts are made in the field of hydrogen production, delivery, storage and utilization. In this view, an analysis of the potential of using the actual natural gas pipeline systems for the delivery of hydrogen is a valid argument. The chemical and physical properties of hydrogen and natural gas differ significantly, which have an effect on safety related to gas transport and its utilization as well as on the integrity of the network.

Among the recent works, the influence of hydrogen on the pressure drop in the pipelines has been calculated, the construction costs of natural gas transmission pipelines have been analyzed and the impact of hydrogen in the global cost has been

studied. From an economic viewpoint, the cost of natural gas pipelines is a function of pipe diameter and the cost of a hydrogen pipeline can be between 50% to 80% higher than that of a natural gas pipeline of the same size. Regional transportation costs could be as much as five times higher than natural gas, primarily because of the lower volumetric energy density of hydrogen. Besides, hydrogen embrittlement of the steel under the high pressure environment of hydrogen constitutes a major concern. Consequently, the transportation of a hydrogen rich gas requires great attention since hydrogen embrittlement is characterized by a loss of ductility of a steel.

In a world where energy demand is growing at unprecedented rates, pipelines will continue to play an important role in safety and efficiently transporting oil and gas from remote areas to their markets. Hydrogen is foreseen as an important and reliable energy carrier in the future sustainable energy society. This energy vector, which can be produced from different primary sources among which the renewable energies, is exploitable in different stationary or portable applications. Hydrogen deployment scenarios can be based on one of the two different fundamental assumptions concerning the level of decentralization in production. Regardless of the primary energy sources and technologies used, hydrogen can be produced both at large scale facilities and then distributed to individual customers over a range of few tens to some hundred kilometers (centralized production), or in the proximity of dispensing facilities or end user appliances (on site generation). Consequently, this yields principally to two separate families of production and distribution path ways made of neighboring stages allowing the adoption of different technologies.

1.5 Outline of Dissertation

In Chapter 2 of this dissertation, review is made to a number of journals pertaining to the area of fluid flow in pipelines. The review includes governing equations of fluid flow which are continuity equation, momentum equation and energy equation.

This chapter also discusses the different numerical methods used by the varying journals to solve their respective governing equations. From the reviews, the intended equations and method of solution are chosen. Chapter 3 then focuses on the mathematical model of the selected equations and then formulating the problem with appropriate boundary conditions.

Subsequently in Chapter 4, the solution procedure is detailed out. This chapter looks at the method of lines (MOL); its definition and how it is to be applied. It also looks at the Runge Kutta Method; the different types of Runge Kutta method and which of the method is adopted for the numerical solution to the subject matter of the dissertation.

Following the mathematical modeling and procedures detailed out in the previous two (2) chapters, analysis is made and results are obtained. These are mentioned in Chapter 5. In this chapter also, suggestions are made to develop into future work and potential research areas.

Last but not least in Chapter 6, conclusion is drawn from the elaborations made in the earlier chapters. A summary will be made on the numerical solution of the non-isothermal transient flow in mixed hydrogen-natural gas pipelines.

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