# MODELING SEA SURFACE CURRENT CIRCULATION FROM SATELLITE ALTIMETRY DATA BY USING FIRST ORDER VOLTERRA MODEL

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## **ABSTRACT**

This paper introduces a new technique for acquiring accurate sea surface current from AVISO satellite altimetry data. The new technique was involved Volterra series expansion in order to transform the time series satellite altimetry data into a real ocean surface current. The basic equation of hydrodynamic has been solved by first order Volterra model. Then, the Volterra kernel inversion used to obtain the sea surface current velocity. The finite element model of Lax-Wendorff schemes was used to model the spatial current variations in the South China Sea during March 2003, April 2004 and March 2005. In situ sea surface current measurements were collected along the east coast of peninsular Malaysia by using Valeport electro-magnetic. The result shows that the integration between Volterra model and Lax-Wendroff scheme provides a means as a complementary tool to model sea surface current variations from AVISO satellite altimetry data with  $r^2$  value of 0.910 and root mean square error (RMSE) of  $\pm 0.4$  m/s.

KEY WORDS: Sea Surface Current, AVISO/Altimeter, Volterra, Lax-Wendrof Schemes

## 1.INTRODUCTION

Dynamics of ocean are playing tremendous role in designing coastal and offshore structures. The study of the dynamics of ocean surface is always a challenging and intriguing process. In fact, features of ocean surface are complex. It is not to easy to understand ocean surface dynamics because there are many factors controlling them (Maged 1994 and 2000). Most scientists identify the forces which induce sea surface dynamics to be the wind, tide, Corolois force and other factors such as density differences between water masses and usually cause vertical movements (Maged 1994). However, scientists cannot study these factors separately as this will give them imperfect scenario of coastal water hydrodynamics. Traditional methods which are involving in situ measurements by using sophisticated equipments are not able to study coastal water hydrodynamics within large scale size of the South China Sea for instance. Remote sensing technology which is based on altimeter sensors is able to provide excellent information of coastal hydrodynamics (Mohd et al. 2005). In addition, satellite altimeters provide long-term and continuous coverage of wave and wind fields of the world oceans. One of the first satellites that used altimetry technology in the measurement of sea levels was the Seasat 1978-64A. In the 1990s with more complex and advanced imaging and altimetry systems, radar remote sensing has become a more significant tool and has been utilized by the ERS-1. Geos-3 and TOPEX/Poseidon satellites. In 1992 TOPEX/Poseidon satellite altimetry mission was launched and its mission was ended in 2002. Since then it was replaced by Jason-1. Both satellite missions provide the most precise altimetry data when compared to others. Although

satellite altimetry records are still quite short compared to the tide gauge data sets, this technique appears quite promising for sea level change problem because it provides sea level measurement with large coverage. A precision of about 1 mm/year of measurement global change can be obtained. At present, few studies have been utilized different altimeter sensors for the sea surface investigations in the South China Sea (Mohd et al. 2005). Hu et al. (2001) investigated the sea surface height with a period of 3-6 months using six years TOPEX/POSEIDON altimeter data. They reported that the sea surface height variations are associated strongly with current and eddies features. Imawaki et al. (2003) introduces a new technique for acquiring highresolution mean surface velocity by combined use of TOPEX/POSEIDON and ERS-1/2 altimeter data and drifter data obtained from 1992 through 2001 to detect the undulation of mean-surface dynamic topography down to the oceanic current. In fact, the ocean current associated with geoid by the amount of sea surface dynamic topography is different compared to the mean -surface height acquired from satellite altimeter. In order to improve the hydrodynamic equation solutions of the mean sea-surface to invert accurate pattern of sea surface current we need to obtain altimeter data that is both more accurate and more widely distributed in time. Furthermore, we need to improve our knowledge of the long wavelength geoid so that its errors are substantially smaller than the corresponding values of the sea surface topography. The main contributions of this work is to design scheme to reduce the impact of Coriolis parameter in continuity equation. In fact low latitude zone such as Malaysia is dominated by weak Coriolis parameter. In this context, the geostrophic current might be produced must be weak. According to this prospective, this study attempt to implement Volterra model and Lax-Wendroff scheme to suppress the numerical solution of geostrophic continuity equation.

#### 2-DATA AND STUDY AREA

The study area is located the South China Sea and SCS is an equatorial, semi-enclosed sea with a complex topography that includes large shallow regions. The SCS is located between the Asian continent, Borneo, the Philippines, and Taiwan (Figure 1). The northeastern part adjoins a deep sea basin, while the southern part is a shelf sea with depths less than 200 m. The SCS has two features that have very important and interesting effects on the general circulation: (1) the Coriolis force decreases to zero at the equator where both nonlinear and frictional forces become very important; and (2) the monsoon regime exerts a strong effect on the SCS circulation. Neither of the above phenomena are studied in coastal regions of the world ocean at the present time (Wrytki 1962).

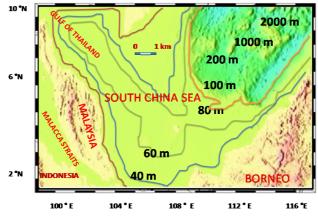


Figure 1. Location of the South China Sea.

The AVISO/Altimeter data of Merged Sea Level Anomaly (MSLA) were acquired on October 2003, April 2003 and March 2005, respectively. The study area was between 102° E to 114° E and 1° N to 10° N. In fact the in situ sea surface current measurements are collected between 102° 5'E to 105° 10'E and 2° 5'N to 6° 10'N (Figure 2). The in situ current measurements were collected by using Valeport electro-magnetic current meter which was lowered down from the sea surface to water depth of 50 m. The in situ sea surface current measurement were conducted by the research vessel UNIPERTAMA VII. These in situ current measurements were acquired on October 6 – 12 2003, April 6-12 2004, and March 1-8 2005.

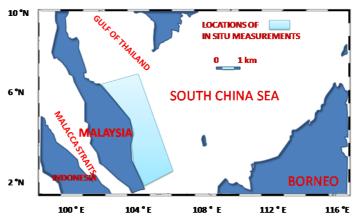


Figure 2. Location of In Situ Measurements along east coast of Malaysia.

## 3- MODEL DESCRIBITION

#### 3.1 Volterra Model

The Volterra model can be used to express the geostrophic current velocity obtained from AVISO altimeter data as a series of nonlinear filters on the ocean surface current. This means that the Volterra model can be used to study the geostrophic current energy variation as a function of parameters such as the current direction, or the current waveform. A generalized, nonparametric framework to describe the input—output x and y geostrophic current components relation of a time-invariant nonlinear system. In discrete form, the Volterra series for input, X(n), and output, Y(n) as given by Inglada and Garello (1999) can be expressed as:

$$Y(n) = h_0 + \sum_{i_1=1}^{\infty} h_1(i_1)X(n-i_1) + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} h_2(i_1,i_2)X(n-i_1)X(n-i_2) +$$

$$\sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} \sum_{i_3=1}^{\infty} h_3(i_1,i_2,i_3)X(n-i_1)X(n-i_2)X(n-i_3) + \dots +$$

$$\sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} \dots \sum_{i_3=1}^{\infty} h_k(i_1,i_2,\dots,i_k)X(n-i_1)X(n-i_2)\dots X(n-i_k)$$

$$(1)$$

where, n,  $i_1$ ,  $i_2$ ,..., $i_k$ , are discrete time lags. The function  $h_k(i_1,i_2,...,i_k)$  is the kth-order Volterra kernel characterizing the system. The  $h_1$  is the kernel of the first order Volterra functional, which performs a linear operation on the input and  $h_2$ ,  $h_3$ ,..., $h_k$  capture the nonlinear interactions between input and output AVISO sea level variation. The order of the non-linearity is the highest effective order of the multiple summations in the functional series.

According to Imawaki et al. (2003) the instantaneous surface geostrophic velocity  $U_g$  can be obtained from temporal mean velocity  $\bar{U}(x,t)$  and anomaly of sea-surface geostrophic velocity  $\bar{U}(x,t) = (u',v')$  by following formula:

$$U_{\sigma}(x,t) = \bar{U}(x) + \bar{U}(u',v')$$
 (2)

where (u',v') are the sea surface geostrophic components in x and y directions which are estimated from the anomaly  $\xi$  of sea surface dynamic topography which practically equivalent to the sea-surface height anomaly  $\zeta_s$ . It might be the following Volterra model can be used to express the instantaneous surface geostrophic velocity  $U_{\sigma}$  as follows;

$$U_{g}(x,t) = N(\xi,\zeta_{s}) + \sum_{i=1}^{\infty} \int_{p} h_{i}(\bar{\tau},\xi) \prod_{i=1}^{i} \bar{U}_{a}(u'-\tau_{j})(v'-\tau_{i})d\tau$$
(3)

where  $N(\xi, \zeta_s)$  is the geoid height which is estimated from sea surface dynamic topography  $\xi$  and the sea-surface height anomaly  $\zeta_s$  as described by Imawaki et al (2003). Following, Inglada and Garello (1999), the mathematical expressions for first –order Volterra kernels  $(D_{1x} and D_{1y})$  of instantaneous surface geostrophic velocity  $U_g$  are as follows;

$$D_{1x}(f_x, f_y) = \left[ \xi_x \, \bar{f} + \frac{\partial N(\vec{\xi})}{\partial \vec{\xi}} . (\bar{U}_{t_0} - \beta(\frac{\partial \zeta_s}{\partial y} - \frac{\partial \xi}{\partial y})) \right] \tag{4}$$

$$D_{1y}(f_x, f_y) = \begin{bmatrix} \bar{\xi}_y & \bar{f} + \frac{\partial N(\vec{\xi})}{\partial \vec{\xi}} \cdot (\bar{U}_{t_0} - \beta(\frac{\partial \zeta_s}{\partial x} - \frac{\partial \xi}{\partial x}) \end{bmatrix}$$
 (5)

where  $f_x, f_y, f$  are the frequency domain in x and y dimensions while f is average frequency domain. Furthermore,  $\beta$  is the ratio of gravity acceleration to the Coriolis parameter expressing the effect rotation of the earth. In order to estimate the sea surface current from the altimeter data, we assume that there is a linear relationship between 2-D Fourier transform of mean surface slope variations obtained from AVISO data  $(F(f_x, f_y))$  and the first –order Volterra kernels  $D_{1x}$  and  $D_{1y}$  as follows:

$$F(f_x, f_y) = U_x(f_x, f_y) \cdot D_{1x}(f_x, f_y)$$
(6)

$$F(f_x, f_y) = U_y(f_x, f_y) \cdot D_{1y}(f_x, f_y)$$
 (7)

Equations 6 and 5 can be used to estimate the current at x and y directions where the inverse Volterra model is used to estimate the current flow as described by Inglada and Garello (1999). The current direction can be obtained by using the traditional tan formula as follows:

$$\theta = \tan^{-1}(\frac{U_{y}}{U_{x}}) \tag{8}$$

# 3.2 Lax-Wendrof Scheme

The second-order accurate dispersive Lax-Wendrof scheme is used for sea surface current flow in AVISO/ altimeter data which can be written in predictor–corrector form in a staggered grid. The staggered grid consists of a primary grid where points are labeled with (i, j, k) and a dual grid where points are labeled with (i + 12, j + 12, k + 12). In 2D, the centers of the cell edges in the primary grid are given by (i + 12, j), etc. Following Liska and Wendroff (1998), 2D, numerical flow for the two-step forms of Lax-Wendrof scheme is

$$U_{i,j}^{n+1} = U_{i,j}^{n} + \frac{\Delta t}{2\Delta x} \left( v(U^{n+1/2}_{i+1/2,j+1/2}) + v(U^{n+1/2}_{i+1/2,j-1/2}) - v(U^{n+1/2}_{i-1/2,j+1/2}) \right) - v(U^{n+1/2}_{i-1/2,j-1/2}) + \frac{\Delta t}{\Delta y} \left( G(U^{n+1/2}_{i+1/2,j+1/2}) + G(U^{n+1/2}_{i-1/2,j+1/2}) - G(U^{n+1/2}_{i+1/2,j-1/2}) - G(U^{n+1/2}_{i+1/2,j-1/2}) \right)$$

$$G(U^{n+1/2}_{i-1/2,i-1/2})$$

where v and G are smooth functions as described by Liska and Wendroff (1998). The values at the center of all faces of the primary cell on time level n + 1/4 are computed using the analog of 2D predictor by

$$U_{i,j+1/2,k+1/2}^{n+1/4} = \frac{1}{4} (U_{i,j,k}^{n} + U_{i,j+1,k}^{n} + U_{i,j,k+1}^{n} + U_{i,j+1,k+1}^{n}) + \frac{\Delta t}{4\Delta y} (G(U_{i,j,k+1/2}^{n+1/6}))$$
(10)

The finite difference scheme model was applied with traditionally rectangular grids due to most their simplicity. Model numerical solutions was enhanced by using the curvilinear nearly orthogonal, coastline-following grids. This technique was implemented to overcome the problems raised due to complicated SCS topography, and boundary conditions.

## 4-RESULTS AND DISCUSSION

The geostrophic current patterns have been modeled by Volterra- Lax-Wendrof Scheme are shown in Figures 3, 4 and 5. Figure 3 shows the typical water circulation during the intermonsoon period of October 2003 of the South China Sea. During October 2003, non fixed current patterns basically dominate the northern and center part of the South China Sea. The maximum current magnitude was 1.0 m/s was found in the northern and eastern coast of Malaysia. The minim current velocity was 0.4 m/s and observed in the Gulf of Thailand. It is interested to find that the stream current pattern was a dominant feature in Malacca Straits with current magnitude of 0.8 m/s. The east coast of Malaysia was dominated by southwards current flow which moving parallel to coastline. Part of these currents was turned south-east and moving parallel to Borneo coastline with current velocity of 1.0 m/s.

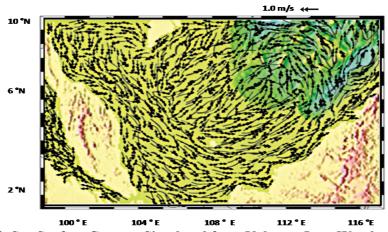


Figure 3. Sea Surface Current Simulated from Volterra-Lax- Wendrof Scheme During October 2003.

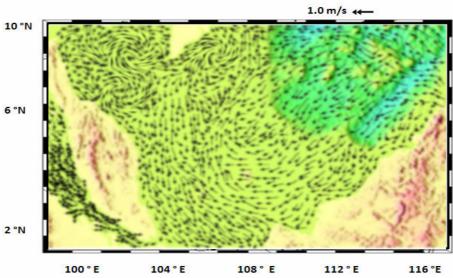


Figure 4. Sea Surface Current Simulated from Volterra-Lax- Wendrof Scheme During April 2004.

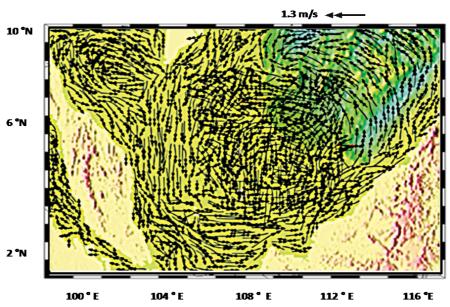


Figure 5. Sea Surface Current Simulated from Volterra-Lax- Wendrof Scheme During March 2005.

In April 2004, there was weak current flow along the east coast of Malaysia with magnitude 0.3 m/s. This current was moved towards southward direction. In Gulf of Thailand there were two eddies were moved opposite to each other with maximum velocity of 0.3 m/s. The southern eddy was moved anticlockwise while the north eddy was moved clockwise. It is obvious that there was southward current flow along the Borneo coastline with current velocity of 0.5 m/s. This current was turned to north and formed small scale eddy in the southern part of Vietnam coastline. The results confirm the study of Wryki (1962), Maged (1995) and Alejandro (1996). In March 2005, the current magnitude is increased and generate the eddy in the center of the South China sea. The maximum current magnitude was 1.2 m/s. the dominate current direction was northward. The Gulf of Thailand dominated by anticlockwise eddy with maximum velocity of 1.0 m/s. Figure 5 shows the regression relationship between average in situ current measurements and the simulated current flow from Volterra- Lax-Wendrof Scheme. The

scatter points are more closed to the regression line with  $\rm r^2$  value of 0.91 and RMSE of  $\pm 0.4$  m/s which is indicated high correlation between in situ measured current and simulated one from Volterra-Lax-Wendrof Scheme. The Volterra algorithm was shown an overcome of the Coriolis force problem. It does mean that the Volterra algorithm has provided solution for low geostrophic current velocity always appears along Malaysian coastal water due to weakness of Coriolis parameter involves in continuity equation. In addition, Lax-Wendrof scheme can handle mixed sub and super-critical current flows directly, with no regard for directional nature of the computation (left to right or vice versa) (Liska and Wendrof, 1998).

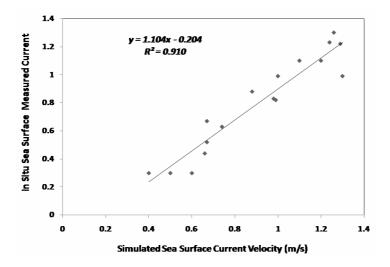


Figure 5. Regression of Simulated Averaged Current and Measured In situ Current.

# **5-CONCLOUSIONS**

This work has demonstrated the two-dimensional sea surface current flow modelling from AVISO satellite altimetry data in the South China. The new technique was used to reduce the impact of Coriolis parameter in the continuity equation of geostrophic current. Two procedures were involved: first order Volterra Kernel and Lax-Wendrof Scheme. It is interested to name this algorithm as Volterra-Lax-Wendrof scheme. The results showed good correlation between in situ current measured and Volterra-Lax-Wendrof scheme with high r<sup>2</sup> of 0.91. It can be concluded that Volterra-Lax-Wendrof scheme could be used as good model to acquire fine resolution of sea surface current in such sea basin of the South China Sea.

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