

MODAL ANALYSIS OF CONCRETE BRIDGE DECKS SUBJECTED TO FREE
VIBRATION

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**SPECIALLY DEDICATED TO MY FATHER, MOTHER AND OTHER
FAMILY MEMBERS....**

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ABSTRACT

This paper reports a brief study on free vibration dynamic analysis for determining vibration parameters such as natural frequencies and mode shapes for two selected bridges in Johor. The vibration parameters for these bridges are studied using finite element analysis software package ANSYS 6.0. A three-dimensional finite element model is developed for the bridges based on design drawing provided by Public Works Department (PWD).

Separate element types with same nodal layer are used as a modeling technique in this finite element analysis. Mode shapes based on natural frequency analysis are obtained for the bridges from the models with and without diaphragms. It is found that a moderately coarse mesh density of 1m for every edge element length would be sufficient for a majority of practical structural applications.

ABSTRAK

Kajian ini menerangkan mengenai getaran bebas di dalam analisis dinamik bagi menentukan parameter getaran seperti frekuensi tabii dan bentuk ragam bagi dua buah jambatan di negeri Johor. Parameter getaran bagi jambatan-jambatan ini dikaji dengan menggunakan kaedah analisis unsur terhingga melalui penggunaan perisian program ANSYS 6.0. Model jambatan bagi analisis unsur terhingga ini dibina secara tiga dimensi berdasarkan plan lukisan yang diperolehi daripada Jabatan Kerja Raya (JKR) Johor.

Unsur elemen yang berbeza dengan satu lapisan nod yang sama telah dijadikan teknik pemodelan dalam analisis unsur terhingga ini. Bentuk ragam yang berdasarkan frekuensi tabii ini diperolehi melalui model dengan gegendang dan model tanpa gegendang. Selain daripada itu, kajian ini juga mendapati penggunaan jejaring kasar sebanyak 1 meter bagi setiap panjang elemen adalah mencukupi dan praktikal bagi menganalisis struktur.

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LIST OF NOTATIONS

- f - Driving force
- m - Mass system
- v - Velocity
- a - Acceleration
- $[M]$ - Structural mass matrix
- $\{\ddot{x}\}$ - The nodal acceleration matrix
- $[C]$ - Structural damping matrix
- $\{\dot{x}\}$ - The node velocity vector
- $[K]$ - The structural stiffness matrix
- $\{x\}$ - The node displacement vector
- $\{F\}$ - The applied time varying nodal load vector
- t - Time
- f_n - The frequency of vibration
- ω_n - Natural frequency
- A' - The amplitude of the vibration
- α - Phase angle
- λ - The eigenvalue
- θ - Angle of orientation

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CHAPTER I

INTRODUCTION

1.1 Background of the Study

Infrastructure ageing and deterioration have becoming a current issue in this modern world. The problem is severe in highway structures where increasing demands of heavier traffic loads that give the major effect on bridge deterioration. Faults in design or in construction stage also may cause damage in the bridge structures. At the same time, the external factors such as wind effect, seismic effect, flood, accident and others can contribute to the damage of these structures.

The defects developed in any one particular component in a bridge can extend and make weaken the bridge to resist the loads. The condition of bridge may become worse if the failure occurred in vital member and it will make the bridge structure totally collapse. Therefore, it is necessary that every parts of the bridge structure be kept under constant observation.

Bridge structures should regularly be inspected to ensure that they are in sound condition and fit for continued used. The inspection programs should consist

of both visual inspections and non-destructive testing methods where appropriate. In recent years, dynamic response monitoring has been effectively used to confirm overall structural integrity thus enabling visual inspections to be carried out less frequently (Creed, 1987).

The aim of dynamic analysis is to determine the predominant natural frequencies and mode shapes of a bridge. If a dynamic analysis is done in conjunction with the design of the bridge structure, the frequencies and mode shapes obtained through the dynamic test serve to verify the adequacy of the analytical model. However, in the majority of bridge design, a dynamic analysis is not required in the design process. For either situation, the dynamic test results contain useful information on the behaviour and condition of the overall structure. The dynamic test results do assess the current characteristics of the bridge and provide a benchmark for a comparison to subsequent evaluations of the modal parameters. There is a trend today towards dynamic testing of bridges as a means of determining structural deterioration (Morgan and Desterle, 1985)

According to Creed (1987) and Morgan and Desterle (1985), the natural frequencies are parameters which depend on the characteristics of the structure and therefore can be compared directly with those predicted at the design stage, to give an indication of how well the completed structure meets the design criteria.

Furthermore, Creed (1987) stated that the overall integrity of a structure can be monitored by regular measurement of the natural frequencies. If damage occurs which is sufficient to cause a change in overall stiffness in the structure, the natural frequencies and mode shapes will change. Thus serious damage to a structure can be detected and in some structures, approximately located.

Recently, several incidents involving vibration-induced damage in structures occurred in Malaysia as a consequence of the Sumatra earthquakes. Although the

damage reported is not significant, the potential danger caused by the tremor should be a concern for the owners and authorities of landmark structures including bridges and buildings. The determination of baseline dynamic measurement or vibration signature of our structures should be made for their health monitoring. The after effects of loadings and earthquake forces can be detected on the structures using dynamic analysis and more research is required to study the application of vibration techniques for structural health monitoring.

1.2 Problem Statement

Previously, there have been some bridges failures through excessive vibration induced either by wind loading or by traffic (TRRL, 1990). The bridge failures have motivated researchers to investigate the dynamic behavior of bridges. Throughout the years, the Transport and Road Research Laboratory (1990) has carried out research in the measurement of vibration on existing bridges, comparison with predicted performance based on improved analytical methods and studies of human reaction to structural vibration. From these researches, improvements in design and construction can be done.

Many old bridges are still in use today but obviously designed for live loads quite different from the vehicular traffic they are subjected to today. To continue using these bridges, it is necessary to evaluate their load-bearing capacity so that traffic loads are managed to ensure their continued safe operation (Spyrakos et al., 1999).

Moses, Lebet and Bez (1994) also stated that the problem in evaluating older bridge is the difficulty in identifying the existing properties as well as making judgement of adequate safety, especially when considering heavier loads due to new

trucks weight regulation. One approach in safety verification is to use inexpensive bridge tests. From the test, typical dynamic characteristics and damages of the tested bridges can be defined.

Moreover, it has been common practice that to test the new bridges before putting them into service and after several years on service for future checking. Thus, the equipment and field experience as well as test interpretation need to be widely developed. Besides, bridge tests are also being conducted to investigate any correlation that may exist between the repair works and changes in dynamic characteristics of the bridges. This is important for development in the field of bridge maintenance and repairs.

In general, structural damage of an element is reflected in the reduction of its stiffness. In a blind experiment, damage is randomly introduced in the bridge and time histories of the dynamic response due to an impulse excitation are generated from the governing differential equations of the structures. Given the time histories and estimates of the structural masses, the method yields information on the stiffness distribution and thus on the integrity of the investigated structure (Topole, 1994).

The inability to detect critical local damage in some bridges as well as large errors in estimating the available capacities of other parameters indicate a lack of reliability in our conceptual understanding of bridge behavior. Therefore, a systematic study of typical bridges is needed (Raghavendrachar and Aktan, 1992). This study must focus on identifying realistic analytical model that can predict the actual behavior of the bridge structures as well as diagnostic technique that will be permit identifying critical local damage. Besides providing more efficient and economical design, together with the trend in aesthetic taste favoring slender lines, bridge testing also can be a tool in investigating and minimizing the factors which affect vibration in highway bridges (Ling, 2002).

Ambient vibration tests are interesting alternative successfully applied to a variety of civil engineering structures. This method only requires the measurement of the structural response under ambient excitation, usually due to wind or traffic. It can lead to accurate estimates of the dynamic parameters quickly and inexpensively.

Though damping factors can be also identified using ambient vibrations test, the corresponding estimates are often not so accurate and this may be a major point of concern in some applications especially in large cable-stayed or suspension bridges, in which the structural damping plays a crucial role (Jones et al., 1998). Therefore, it is particularly appropriate in such cases to perform a free vibration test, introducing an initial perturbation that can induce a free vibration response significantly higher than the ambient response (Ventura et al., 1996).

In these works, the ambient structural response due to wind or traffic loads has been proven to be useful for determining the dynamic characteristics of bridges. Since the finite element model of a bridge is usually constructed from the highly idealized engineering drawings, quite often, significant differences were found between the prediction and the measured dynamic data. Under this situation, the finite element model would need to be calibrated in order to eliminate the differences as much as possible. This process is customarily termed “modal updating” (Chang, 2001).

Generally, the finite element analysis gives a more detailed but not necessarily accurate description of the dynamic characteristics of the bridge, since the modal is constructed based on the highly idealized design drawings. The field vibration measurement, on the other hand, can serve as a valuable source of information for validating the assumption and the accuracy of the finite element model. As careful as one can be, discrepancies between these two approaches seem unavoidable. These discrepancies could come from the finite element modeling errors, the ambient vibration measurement and post processing errors, or both (Chang, 2001). In this study, an attempt was made to minimize these frequency

differences by updating the finite element model tools based on the measured results.

1.3 Aim

Generally, this study is carried out to understand thoroughly the application of dynamic analysis in evaluating the deterioration of the bridge structures and it can be achieved through experimental and numerical work. In this study, the aim is focused on numerical work using available finite element software.

For further understanding of dynamic analysis of bridges, two vibration parameters of dynamic response has been studied namely natural frequencies and mode shapes. The ANSYS 6.0 package program is performed to model the actual case study of bridges. The results obtained from this modeling can be used as a guideline for further research in bridge health monitoring.

1.4 Objectives

The main objectives for the study of modal analysis of concrete bridge decks subjected to free vibration are stated as follows:

- a. To obtain suitable model for modal analysis of two selected bridges.
- b. To determine natural frequencies and mode shapes of the bridge model.
- c. To study the effect of cross bracing in the bridge models.

1.5 The Scope of the Study

The scope of this study is limited to the determination of the structural dynamic properties such as natural frequencies and mode shapes. Two reinforced concrete bridges in Johor are selected and the dynamic analysis is performed based on free vibration circumstances.

The ANSYS 6.0 package program is used as a tool of this finite element analysis. The bridge is modeled as simply supported bridge and it consists of deck slab and beams system. Isotropic materials are used over the bridge structures. It comprised of a cast in-situ and pre-cast prestress concrete within linear elastic analysis. All data specifications and detail drawings are based on the document provided by Public Works Department (PWD) of Johor.

1.6 Description of the Bridges

Two reinforced concrete bridges are selected in Johor. There are located at main road with heavy traffic load. The performance of these bridges should regularly be inspected to ensure the safety from any deterioration during its service life.

1.6.1 Description of Bridge 1

Bridge 1 is the multi-level junction bridge at junction Jalan Persekutuan1-Jalan Tampoi-Jalan Pengkalan Rinting, Johor. It was constructed in 1995 and opened to traffic in 1998. It is an eight-span simply supported bridge and has three-lanes with double carriageway which crosses the main road Kuala Lumpur-Johor Bahru. The bridge has 23 pre-cast I Beam spaced at 1.43m c/c, tied by 4 pre-cast concrete diaphragms 152mm thick, spaced at 5m c/c and two end diaphragms 305mm thick for each span. The bridge supports a 200mm thick cast in-situ concrete slab with 50mm thick asphaltic concrete wearing course. Each span is 25m and 35.75m wide. There are concrete crash barriers on either side of the concrete slab. The pre-cast concrete beams are supported by elastomeric bearing pads. These beams are fixed on one end and free at the other end. The bridge is supported on concrete abutments at the end supports and on concrete piers at the inner supports as shown in Figure 1.1.



(a) Longitudinal View of the Bridge



(b) General View of the Bridge

Figure 1.1: Multi-level Junction Bridge at Junction Jalan Persekutuan 1-Jalan Tampoi-Jalan Pengkalan Rinting, Johor

1.6.2 Description of Bridge 2

Bridge 2 is the Sungai Skudai Bridge at Route F0094 of Jalan Kulai-Kota Tinggi, Johor. It was constructed in 1998 and opened to traffic in 2000. It is a three-span simply supported bridge and has two-lanes with double carriageway which crosses the Sungai Skudai. The bridge has 8 pre-cast I Beam spaced at 1.50m c/c, tied by 3 pre-cast concrete diaphragms 152mm thick, spaced at 4.36m c/c and two end diaphragms 381mm thick for each span. The bridge supports a 200mm thick cast in-situ concrete slab with 50mm thick asphaltic concrete wearing course. Each span is 18.3m and 11.45m wide. There are concrete crash barriers on either side of the concrete slab. The pre-cast concrete beams are supported by elastomeric bearing pads. These beams are fixed on one end and free at the other end. The bridge is

supported on concrete abutments at the end supports and on concrete piers at the inner supports as shown in Figure 1.2.



(a) Longitudinal View of the Bridge



(b) General View of the Bridge

Figure 1.2: Sungai Skudai Bridge at Route F0094 of Jalan Kulai-Kota Tinggi, Johor

instrumentation would involve the use of shaker or hammer for excitation and set of accelerometers to measure the frequencies.

The results obtained from the experimental and numerical approach can be used as a basis for verifying and updating numerical models of the real life structures. The modal updating from the experiment and numerical approach can be used as a baseline vibration signature for the studied bridges. The baseline for the bridge will play an important role in the structural health monitoring. Any changes in the bridge behaviour due to traffic and environment loading will detect by monitoring the difference in frequency of baseline.

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