

BEHAVIOUR OF REINFORCED CONCRETE FRAME AT
ULTIMATE LIMIT STATE

MOHD AZMIR ABU BAKAR

A technical project report submitted in partial fulfilment of the
requirements for the award of the degree of
Master of Engineering (Civil-Structure)

Faculty of Civil Engineering
Universiti Teknologi Malaysia

OCTOBER 2005

BEHAVIOUR OF REINFORCED CONCRETE FRAME AT ULTIMATE
LIMIT STATE

MOHD AZMIR ABU BAKAR

A project report submitted in partial fulfilment of the
requirements for the award of the degree of
Master of Engineering (Civil-Structure)

Faculty of Civil Engineering
Universiti Teknologi Malaysia

OCTOBER, 2005

To my dearly loved parents...

A caring mother

and a dedicated father...

ACKNOWLEDGEMENTS

In the name of Allah, Merciful and Sympathetic to his followers, without Him there will be never anything to be started with.

Given this scarce opportunity to recognise my gratitude to those people who are always there to support and encouraged me to completion of this project.

First and the most, my beloved parents who supported me as long as I can remember, I will never become what I am today without their caring and dedicated backing.

Secondly, to my darling wife, who always has to endure night sleep with light, manage my needs and time, may always be a sweetheart to me.

Thirdly to my committed supervisor, Assoc. Prof. Dr. Ahmad Baharuddin Abd Rahman, without his assistance and advice through out this project, I will never manage to finish it.

Fourthly, to my friend, Mohd Zaini Endut, who supported and willingly to lend hand as appropriate and fitted to completion of this project.

Last, but not the least, to those who may directly or not directly involved in completion of this project, my deepest thank to all and may Allah bless you all.

ABSTRACT

The objective of this study is to understand reinforced concrete beam-column connection behaviour toward ultimate limit state using ATENA 2D, a non linear finite element analysis. Based on previous studies, the behaviour of beam-column with semi rigid connection in steel structure has shown a very different behaviour during elastic range and beyond yield point, because of phenomenon called moment shedding. Using the same approach, analytical studies on the response of concrete framed structures at ultimate limit have been conducted. This study shows that when the reinforced concrete structures reach the ultimate limits state, the column will behave similar to an axial loaded column. This outcome is a result of redistribution of end moment near collapse condition, where moment has been transferred to nearby structure component, in this study to a beam component. With the phenomenon occurs near ultimate limit state, a simplified design approach can be formed for reinforced concrete designed.

ABSTRAK

Objektif penyelidikan ini dijalankan adalah untuk memahami sifat konkrit ketika menghampiri keadaan had muktamad menggunakan software ATENA 2D, sebuah perisian analisa elemen terhingga tidak linear. Berdasarkan penyelidikan terdahulu, sifat untuk rasuk-tiang dengan sambungan separuh tegar, menunjukkan sifat yang berbeza ketika had kenyal dan selepas melepasi had kritikal, disebabkan fenomena yang bernama momen berkurang (*moment shedding*). Penyelidikan ini menunjukkan apabila struktur konkrit bertetulang menghampiri keadaan had muktamad. Keputusan menunjukkan apabila konkrit bertetulang menghampiri keadaan had muktamad, tiang akan berubah seperti tiang yang hanya mengalami beban paksi. Ini terjadi akibat pembahagian moment hujung ketika keadaan mendekati tahap roboh, dimana momen akan dipindahkan ke komponen struktur berdekatan, dalam kes ini komponen rasuk. Dengan terjadinya fenomena ini ketika menghampiri keadaan had muktamad, satu kajian untuk kaedah mudah boleh dibentuk bagi rekabentuk konkrit bertetulang.

TABLE OF CONTENT

CHAPTER	TITLE	PAGE
	TITLE	i
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF TABLES	xi
	LIST OF FIGURES	xii
	LIST OF SYMBOLS	xv
	LIST OF APPENDICES	xvi
CHAPTER 1	INTRODUCTION	1
	1.1 General Introduction	1
	1.2 Statement of the Problem	1
	1.3 Objective and Aim of Present Study	2
	1.4 Scope of Study	3
	1.5 Significant of the Research	4
CHAPTER 2	LITERATURE REVIEW	5
	2.1 Reinforced Concrete	5
	2.1.1 Basis of design	5

2.1.2	Material Properties	6
2.1.3	Design Strength	7
2.1.4	Loading	8
2.1.5	Characeristic Loading	8
2.1.6	Design Load	8
2.1.7	Stress-strain Curve	10
2.2	Beam Column Connection Behaviour	13
2.2.1	Beam Column Joint	14
2.2.2	Types of joints in frames	14
2.2.3	Forces acting on a beam column joint	15
2.2.4	Performance Criteria	17
2.2.5	Joint Mechanisms	17
2.2.6	Bond requirements	18
2.2.6.1	Interior Joint	18
2.2.6.2	Exterior Joint	19
2.2.6.3	Corner Joint	20
2.2.7	Factors affecting bond strength	21
2.3	Non Linear Analysis	22
2.3.1	Three type of Non Linearity	23
2.3.2	Non Linear Solution	24
2.3.3	Converge Criteria	27
2.5	Atena a tool for engineering analysis of fracture in concrete	28
2.5.1	Introduction	28
2.5.2	Material Models	29
2.5.2.1	Crack Band Model	29
2.5.2.2	Fracture-plastic model	32
2.6	Non Destructive Reliability Analysis of Concrete Structures Numerical concepts and material models for existing concrete structures	34
2.6.1	Nonlinear Simulation of Engineering Structures	34
2.6.1.1	Material models	35

2.6.1.2	Features and structure of ATENA software	38
CHAPTER 3 MODELLING OF 2D REINFORCED CONCRETE FRAME		40
3.1	Introduction	40
3.2	Software package ATENA	40
3.3	Atena 2D Software	43
3.4	Finite Element Modelling	43
3.4.1	Pre-Processing	46
3.4.2	Material Parameters	49
3.4.3	Geometrical Joint	55
3.4.4	Geometrical Lines	58
3.4.5	Geometrical Macro-Elements	60
3.4.6	Mesh Generation	63
3.4.7	Bar-reinforcement	64
3.4.8	Support and Action	68
3.4.9	Loading history and solution parameters	72
3.4.10	Monitoring points	76
3.4.11	FE non-linear analysis	78
3.4.11.1	Introduction	78
3.4.11.2	Starting analysis	79
3.4.11.3	Interactive window	80
3.4.11.4	Adding new load steps	82
CHAPTER 4 VERIFICATION OF FINITE ELEMENT MODEL		84
4.1	Introduction	84
4.2	Stress Block for Simply Supported Beam with Uniformed Distributed Load	85

4.3	Conclusion	86
CHAPTER 5	NON-LINEAR BEHAVIOUR OF REINFORCED CONCRETE FRAME	87
5.1	Non Linear Finite Element Analysis	87
5.1.1	Load – Deflection Characteristics of the Frame Structure	87
5.1.2	Load – Moment Characteristics of the Frame Structure	92
5.1.3	Moment – Rotation Characteristics of the Frame Structure	97
5.1.4	Behaviour of Stress Contour Areas of the Frame Structure	100
CHAPTER 6	CONCLUSION	116
6.1	Introduction	116
6.2	Conclusion	116
6.3	Recommendation for Future Research Work	117
	List of Reference	119

LIST OF TABLES

TABLE NO.	TITLE	PAGE
2.1	List of common Characteristic Strength of Concrete f_{cu}	6
2.2	Characteristic Strength of reinforcement f_y	7
2.3	Partial safety factors for Strength of Material	7
2.4	Load Combination	9
3.1	Coordinates of the geometrical joints reinforced Concrete Frame Type A	55
3.2	Coordinates of the geometrical joints reinforced Concrete Frame Type B.	56
3.3	Coordinates of the geometrical joints reinforced Concrete Frame Type C.	57

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
2.1	Examples of Ultimate Design Loadings	9
2.2	The distribution of stress in a uniform bar	10
2.3	Typical Stress-Strain Curve for Concrete	11
2.4	BS8110 modified stress-strain curve	11
2.5	Typical Stress-Strain Curve for Steel	12
2.6	BS8110 simplified stress-strain for steel	13
2.7	Simple Spring Problem	22
2.8	Non Linear Force-Displacement	24
2.9	Non Linear Stress-Strain	24
2.10	Pure Incremental Solution	25
2.11	Newton Raphson Solution	25
2.12	Load Steps, Substeps and Time Increments	26
2.13	Stress Crack Opening law according to Hordijk (1991)	31
2.14	Smearred crack model for tensile behavior of concrete.	35
2.15	Example - crack band in a shear wall	36
2.16	Concrete failure surface in 3D-stress state	37
3.1	Layered Structure of ATENA system	41
3.2	2D RC Frame (a) Type A, (b) Type B & (c) Type C	44
3.3	RC Frame Type A	45
3.4	RC Frame Type B	45
3.5	RC Frame Type C	46
3.6	Graphical user interface of ATENA 2D pre-processor	47
3.7	File Toolbar	47

3.8	Solution Toolbar	47
3.9	Zoom and View Toolbar	48
3.10	Selection Toolbar	48
3.11	General data table shows general information about the structure	49
3.12	The editing dialog for general data appears after selecting the Edit button from the General data table.	49
3.13	The Materials table, from which new materials can be added or existing materials can be modified or removed.	50
3.14	Selection of plane stress elastic isotropic material for the steel plates.	50
3.15	The dialog for the definition of material properties for the steel plates.	50
3.16	Selection of material model for the bar reinforcement.	51
3.17	The dialog for the definition of reinforcement material parameters.	51
3.18	Selection of SBETA material model for the concrete beam. The SBETA model corresponds to the material formulation, which was implemented in the program SBETA. SBETA was a previous DOS version of ATENA.	52
3.19	Default values of material parameters are generated based on the cube strength of concrete.	52
3.20	The dialog window for the definition of basic properties for SBETA material. The parameters were generated based on the concrete cube strength.	52
3.21	The dialog window for the tensile properties for SBETA material.	53
3.22	The dialog window for the compressive properties of SBETA material.	53
3.23	The dialog window for the shear properties of SBETA material.	54
3.24	The dialog window for the miscellaneous properties of SBETA material.	54

3.25	The three materials, which were defined previously, can be viewed or modified from the Material table windows.	54
3.26	The dialog for specifying the coordinates and properties for the newly created joints.	55
3.27	The line prototype dialog box appears after clicking the button . In this dialog a mesh refinement method or line springs can be specified. All subsequently created lines will use this set of prototype properties.	59
3.28	In the graphical mode, a geometrical line is defined by first selecting a line beginning and a line end joint by mouse. The order of end points is not important in ATENA.	59
3.29	Program display after the definition of the first boundary line.	60
3.30	Program display at the beginning of the macro-element definition.	61
3.31	The dialog window, which appears after the selection of the button from the toolbar for graphical input and editing. This dialog is used for the definition of macro-element prototype, the properties of which will be used for the subsequently created macro-elements.	62
3.32	The dialog for changing the macro-element prototype properties for the beam region, where concrete material model should be used.	62
3.33	The program display after the definition of the macro-element with the concrete material.	63
3.34	Generated finite element mesh using the element size of 0.08 m	64
3.35	The program window at the beginning of the reinforcement bar definition.	65
3.36	The dialog for the definition of reinforcement bars contains two property sheets. The sheet Properties is used for the definition of material model and reinforcement cross-sectional area.	66
3.37	The sheet Topology is used for the definition of bar geometry. A reinforcement bar is composed of segments, and each segment can be a line, arc or a circle.	66

3.38	This figure shows the definition of the bar end point.	67
3.39	The program display after the definition of the reinforcement bar.	67
3.40	The load-case definition starts by highlighting the Load-cases item in the data access tree and clicking the Add button in the Load cases table.	69
3.41	The first load case will contain the horizontal and vertical supports.	69
3.42	The list of created load-cases in the Load cases table.	70
3.43	An appropriate active load case must be selected before the definition of supports. Supports should be in the load case 1.	70
3.44	The definition of the vertical support at selected joint.	71
3.45	The definition of horizontal support.	71
3.46	The definition of the prescribed displacement in load case 2.	72
3.47	The program display with the table of solution parameters. Standard solution parameters can be examined by clicking the button Show. New set of solution parameters can be created using the button Add.	73
3.48	The first property sheet for the new set of solution parameters.	73
3.49	The second property sheet for the new set of solution parameters.	74
3.50	The table with the newly created solution parameters.	74

- 3.51 Load steps are specified using the button Add from the table of Analysis steps. This table appears in the table window after highlighting the Analysis steps item in the data access tree. 75
- 3.52 Each step will be composed of load cases 1 and 2. The multiplier 3 will be used to multiply the applied actions and the newly created solution parameters will be used during the load steps. 75
- 3.53 The Analysis steps table after the definition of twenty load steps with the above parameters. It is possible to add more load steps later during the analysis. 75
- 3.54 The dialog window for the definition of a monitoring point. 76
- 3.55 The program display after the definition of monitoring points. 77
- 3.56 The way, in which the program selects the closest node for monitoring becomes more apparent after zooming at the middle section of the beam. The button returns the view to the state when the whole structure is displayed. 77
- 3.57 The dialog box for activating the display of finite element node and element numbers. 78
- 3.58 The finite element mesh along with node and element numbers. The size of characters can be modified from the menu item Options | Settings. 79
- 3.59 The dialog window before the finite element analysis. 80

3.60	The interactive window for monitoring the progress of non-linear analysis.	81
3.61	The interactive window after selecting a different format of the L-D diagram. The L-D diagram in this figure shows the iterative changes of the monitored quantities.	81
3.62	The dialog box for the new analysis steps. Same properties are used as in Section 3.4.9	82
3.63	The table with the analysis steps after the definition of additional 20 steps.	82
4.1	Simply Supported Beam	84
4.2	The result show that the ATENA 2D finite element model is in good aggrement with the theoretical value	85
5.1a	Load-Deflection curve for RC Frame Type 1A	88
5.1b	Load-Deflection curve for RC Frame Type 1B	89
5.1c	Load-Deflection curve for RC Frame Type 1C	90
5.2a	Load-Deflection curve for RC Frame Type 2A	91
5.2b	Load-Deflection curve for RC Frame Type 2B	91
5.2c	Load-Deflection curve for RC Frame Type 2C	92
5.3a	Load-Moment characteristic for RC Frame Type 1A	93
5.3b	Load-Moment characteristic for RC Frame Type 1B	93
5.3c	Load-Moment characteristic for RC Frame Type 1C	94
5.4a	Load-Moment characteristic for RC Frame Type 2A	95
5.4b	Load-Moment characteristic for RC Frame Type 2B	95
5.4c	Load-Moment characteristic for RC Frame Type 2C	96

5.5a	Moment-rotation characteristic for RC Frame Type 1A	97
5.5b	Moment-rotation characteristic for RC Frame Type 1B	98
5.5c	Moment-rotation characteristic for RC Frame Type 1C	98
5.6a	Moment-rotation characteristic for RC Frame Type 2A	99
5.6b	Moment-rotation characteristic for RC Frame Type 2B	99
5.6c	Moment-rotation characteristic for RC Frame Type 2C	100
5.7a	RC Frame Type 1A Stress Contour Areas Step 1	101
5.7b	RC Frame Type 1A Stress Contour Areas Step 3	101
5.7c	RC Frame Type 1A Stress Contour Areas Step 6	102
5.7d	RC Frame Type 1A Stress Contour Areas Step 9	102
5.7e	RC Frame Type 1A Stress Contour Areas Step 12	103
5.8a	RC Frame Type 1B Stress Contour Areas Step 1	103
5.8b	RC Frame Type 1B Stress Contour Areas Step 6	104
5.8c	RC Frame Type 1B Stress Contour Areas Step 12	104
5.8d	RC Frame Type 1B Stress Contour Areas Step 18	105
5.8e	RC Frame Type 1B Stress Contour Areas Step 20	105
5.9a	RC Frame Type 1C Stress Contour Areas Step 1	106
5.9b	RC Frame Type 1C Stress Contour Areas Step 6	106
5.9c	RC Frame Type 1C Stress Contour Areas Step 12	107
5.9d	RC Frame Type 1C Stress Contour Areas Step 18	107
5.9e	RC Frame Type 1C Stress Contour Areas Step 21	108

5.10a	RC Frame Type 2A Stress Contour Areas Step 1	108
5.10b	RC Frame Type 2A Stress Contour Areas Step 5	109
5.10c	RC Frame Type 2A Stress Contour Areas Step 9	109
5.10d	RC Frame Type 2A Stress Contour Areas Step 13	110
5.10e	RC Frame Type 2A Stress Contour Areas Step 14	110
5.11a	RC Frame Type 2B Stress Contour Areas Step 1	111
5.11b	RC Frame Type 2B Stress Contour Areas Step 5	111
5.11c	RC Frame Type 2B Stress Contour Areas Step 9	112
5.11d	RC Frame Type 2B Stress Contour Areas Step	112
5.11e	RC Frame Type 2B Stress Contour Areas Step 14	113
5.12a	RC Frame Type 2C Stress Contour Areas Step 1	113
5.12b	RC Frame Type 2C Stress Contour Areas Step 5	114
5.12c	RC Frame Type 2C Stress Contour Areas Step 9	114
5.12d	RC Frame Type 2C Stress Contour Areas Step 11	115
5.12e	RC Frame Type 2C Stress Contour Areas Step 15	115

LIST OF SYMBOLS

f_{cu}	-	Characteristic Strength of Concrete
f_y	-	Characteristic Strength of Steel
γ_m	-	Partial safety factors for Strength of Material
F_k	-	Characteristic Loads
γ_f	-	Partial safety factor for Loads
G_k	-	Dead load
Q_k	-	Imposed Load
W_k	-	Wind Load
σ	-	Stress
F, P	-	Force
A	-	Area
u	-	Deflection
f_y	-	Steel Tensile Strength
f_t	-	Concrete Tensile Strength
w	-	Width of Stress Crack
w_{i0}	-	Initial Width of Stress Crack
G_f	-	Fracture Energy
ε	-	Strain
P	-	Force
P_u	-	Ultimate Force
σ_1	-	Stress at x-axis
σ_2	-	Stress at y-axis
σ_3	-	Stress at z-axis
σ_o	-	Total Acting Stress
τ_o	-	Total Acting Torque

E	-	Modulus of Elasticity
σ_y	-	Stress at y-axis
R_{cu}	-	Concrete Compression Strength
MU	-	Poisson's Ratio
f_t	-	Concrete Tensile Strength
f_{cu}	-	Concrete Compressive Strength
w_d	-	Critical Compressive Displacement
Rho	-	Specific Material Weight
α	-	Coefficient of Thermal Expansion

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A	Deflection Report for RC Frame 1A	
B	Moment Lines Report for RC Frame 1A	
C	Deflection Report for RC Frame 1B	
D	Moment Lines Report for RC Frame 1B	
E	Deflection Report for RC Frame 1C	
F	Moment Lines Report for RC Frame 1C	
G	Deflection Report for RC Frame 2A	
H	Moment Lines Report for RC Frame 2A	
I	Deflection Report for RC Frame 2B	
J	Moment Lines Report for RC Frame 2B	
K	Deflection Report for RC Frame 2C	
L	Moment Lines Report for RC Frame 2C	

CHAPTER 1

INTRODUCTION

1.1 General Introduction

Concrete has been a major construction material for centuries. Flexibility, economical and easy to find source material is an attraction for engineer which ever ready to ease their mind. Although commonly used in construction section, behaviour of concrete near ultimate limit state is known very little, but still interest many researchers to fully understand it.

1.2 Statement of the Problem

Based on previous parametric studies, the behaviour of beam-column with semi rigid connections has shown that a very different behaviour during elastic range and beyond yield point. This phenomenon called moment shedding is the reason behind the unique behaviour. Previous study showed that moment shedding not only happen after yielding point, but also during elastic period. It may not significantly influence during elastic state, but after yielding point, moment

shedding impact is significant, where the moments are relaxed to near zero moment.

By this phenomenon finding, researcher has found a simplified design method, but in condition that, when the structure in collapse load condition, it still in equilibrium condition. This study is to try to find out if this phenomenon is also happen in reinforced concrete frame. By validating this, simplified design method can be implementing into reinforced concrete design, thus producing more efficient and practical approach to design problem.

1.3 Objective and Aim of Present Study

The primary aim of the thesis is to investigate the non linear aspect of reinforced concrete frame structures.

- (i) To investigate the behaviour of reinforced concrete frame in non linear state in term of load-deflection, load-moment and moment-rotation.
- (ii) To study the respond of moment of reinforced concrete column near Ultimate Limit State.

1.4 Scope of study

The study will be concentrating on non linear finite element analysis of reinforced concrete frame to obtain the collapse state, which the ultimate load of the frame.

The study is limited to the following scopes.

- (i) Only reinforced concrete framed is considered.
- (ii) The frame is considered as 3 type
 - Type A – Single bay frame
 - Type B – Double bay frame
 - Type C – Double storey frame
- (iii) The frame is subjected to 2 types of loading.
 - Type 1 - Incremental load at all column head with static uniformly distributed load (UDL)
 - Type 2 - Incremental load at all middle span of beam

1.5 Significant of Research

- (a) To understand behaviour of reinforced concrete near Ultimate Limit State.
- (b) To validate previous study on moment shedding too happen in reinforced concrete frame.
- (c) To promote more research on the subject, and finally to encourage research on simplified design method for reinforced concrete design.

- 4) Re-analyse the frame structure using different lengths of column, to monitor the relationship between length and initial moment shedding occurrence.

REFERENCES

- 1) Edward G. Nawy, Alan Moreton, Bryant Mather, Mohan Malhotra, Michael Sprinkel, *Concrete Properties*, A2E03: Committee on Properties of Concrete
- 2) Nawy, E. G. *Fundamentals of High-Performance Concrete*, 1st ed., Ch. 12. Longman, United Kingdom, 1996.
- 3) Vladimir Cervenka, Jan Cervenka and Radomir Pukl, *ATENA a tool for engineering analysis of fracture in concrete*, *Sādhanā* Vol. 27, Part 4, August 2002, pp. 485–492.
- 4) Alfred Strauss & Konrad Bergmeister, Radomir Pukl & Vladimir, *Non Destructive Reliability Analysis of Concrete Structures Numerical concepts and material models for existing concrete structures*, International Symposium (NDT-CE 2003), Non-Destructive Testing in Civil Engineering 2003
- 5) Pukl R., Novak D., Vorechovský M. and Cervenka V. *Statistical nonlinear analysis size effect of concrete beams* in proceedings: FraMCoS – Fracture Mechanics of Concrete and Concrete Structures, Pages 823–830, 2001
- 6) Vladimír Červenka, Libor Jendele and Jan Červenka *ATENA Program Documentation Part 1 Theory* Prague, Oct. 4, 2000
- 7) Bažant and B.-H. Oh, *Crack band theory for fracture of concrete*, *Materials and Structures* 16(93), 155-177, 1983.

- 8) Hillerborg A, Modeer M, Petterson PE *Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements*. Cem Concr Res 6:773–782, 1976
- 9) Hordijk, *Local approach to fatigue of concrete*, Ph. D Thesis, Delft University of Technology, pp.1-207, 1991
- 10) Rots J, *Computational modeling of concrete fracture*, Ph. D Thesis, Delft University of Technology, Delft, The Netherlands, 1988
- 11) Vecchio, F.J. and Collins, M.P., *The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear*, ACI Journal, *Proceedings V. 83* No. 2, March-April 1986, pp. 219-231.
- 12) Menétrey, Ph. and Willam, K., *A Triaxial Failure Criterion for Concrete and its Generalization*, ACI Structures Journal, Vol. 92, pp. 311-318, 1995
- 13) Bažant, Z.P., Caner, F.C., Carol, I., Adley, M.D. and Akers, S.A. *Microplane model M4 for concrete (Part I)*, Journal of Engineering Mechanics, ASCE 126(9), 944–952, 2000
- 14) Borst, R. , *Computational modelling of concrete structures*, Euro-C Int. Conf., Pineridge press, Swansea, U.K, 1986.
- 15) A.B. Abd-Rahman, P.A. Kirby & J.B. Davison, *Simplified Semi-rigid Design of Beam-Column in Non-sway Steel Frames*, First International Conference on steel & Composite Structure, Pusan, Korea, June 14-16, 2001.
- 16) A.B. Abd Rahman, *Worked Examples on Simplified Design of Semi-rigid Non-sway Frames*, Department of Civil and Structural Engineering, University of Sheffield, 1999.

- 17) Weaver W Jr, Johnson PR, (1987) *Structural Dyanamics by Finite Elements*, Prentice Hall, 1987
- 18) Kotsovos,M.D. and Pavlovic, M.N. (1995) *Structural Concrete : Finite Element Analysis for Limit State Design*, London, Thomas Telford
- 19) Yam, Lloyd, C.P. (1981) *Design of Composite Steel-Concrete Structure*, London Surrey University Press
- 20) Bathe, K.J. (1982) *Finite Element Procedures in Engineering Analysis*, Englewood Cliffs, New Jersey.
- 21) Bedard, C. and Kotsovos, M.D. (1985) *Application on NLFEA to concrete structure*, Journal of Structural Engineering. 111:2691-2707
- 22) Carlton, D (1993) *Application of the finite element Method to Structural Engineering Problem*, The Structural Engineering. 71:55-59
- 23) Weaver W.J., Johnson PR, (1984) *Finite Element for Structural Analysis*, Englewood Cliffs, New Jersey. Prentice-Hall, Inc.