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Heat Transfer on Peristaltically Induced Walter's B Fluid Flow

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This paper look at the effects of heat transfer on peristaltic flow of Walter's B fluid in an asymmetric channel. The regular perturbation method is used to solve the governing equations by taking the wave number as the small parameter. Expressions for stream function, temperature distribution, and heat transfer coefficient are presented in explicit form. Solutions are analyzed graphically for different values of arising parameters. It has been found that these parameters affect considerably the considered flow characteristics. Results show that with an increase in the Eckert and Prandtl numbers, the temperature and heat transfer coefficient increase. Further, the absolute value of the heat transfer coefficient increases with an increasing viscoelastic parameter. Comparison with published results for viscous fluid is also presented. Heat Trans Asian Res, 41(8): 690–699, 2012; Published online 1 November 2012 in Wiley Online Library (wileyonlinelibrary.com/journal/htj). DOI 10.1002/htj.21021

Key words: heat transfer, peristaltic flow, Walter's B fluid, heat transfer coefficient

1. Introduction

Peristalsis is an inherited mode of fluid transport in living tracts induced by the wavy motion of the boundaries. This wavy motion is sinusoidal in nature [1] and responsible for mixing and transportation of the contents within the tracts. Peristalsis is involved in the motion of food in the gastrointestinal tract, motion of secretions in glandular ducts, transport of urine in the cervical canal, and others. In industrial applications, artificial hearts and ortho-pumps, heart lung machines, transport of toxic material, and waste inside the sanitary ducts involve peristalsis. The above-mentioned situations usually involve a fluid having non-Newtonian behavior with some elastic properties because most of them are a suspension of particles in Newtonian fluid with fading memory. Walter's B fluid model with limiting viscosity at low shear rates and short memory coefficient [2] is the best model for the above mentioned situations [3–5].

Heat transfer analysis of the flow of physiological fluids induced by the peristaltic waves is imperative in many realistic situations like oxygenation, hemodialysis, heat conduction, metabolic processes, internal heat generation, and others. It is also useful in the treatment of diseased tissues in

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cancer. Recent investigations on the peristaltic flows with heat transfer include numerous studies [6–12]. Peristaltic flow of Walters' B fluid in an inclined tube is considered by Nadeem and Akbar [13]. Moreover, Nadeem et al. [14] studied the peristaltic flow of Walters' B fluid in an endoscope. Much attention has been paid to heat transfer analysis of viscous and non-Newtonian fluids. However, no one has investigated the heat transfer analysis on the peristaltic flow of viscoelastic fluid in an asymmetric channel, which is of great interest in many physiological flows. Motivated by these facts, in the present problem, the heat transfer analysis of a peristaltic flow of Walter's B fluid in an asymmetric channel is studied.

2. Formulation of the Problem

A two-dimensional flow of an incompressible viscoelastic Walter's B fluid in an asymmetric channel is considered. The fluid motion is governed by the propagation of sinusoidal waves with different amplitudes and phases moving along the walls of the channel. The upper and lower walls are at distance d_1 and d_2 from the centerline ($Y = 0$) of the channel and maintained at temperature T_0 and T_1, respectively.

The continuity, momentum, and energy equations are given by

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \tag{1}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{U} = -\frac{\partial \bar{P}(\bar{X},\bar{Y},\bar{t})}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{X}\bar{X}}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{X}\bar{Y}}}{\partial \bar{Y}} \tag{2}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)\bar{V} = -\frac{\partial \bar{P}(\bar{X},\bar{Y},\bar{t})}{\partial \bar{Y}} + \frac{\partial \bar{S}_{\bar{X}\bar{Y}}}{\partial \bar{X}} + \frac{\partial \bar{S}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}} \tag{3}$$

$$\rho\xi\left(\frac{\partial}{\partial \bar{t}} + \bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{Y}}\right)T = k\left(\frac{\partial^2 T}{\partial \bar{X}^2} + \frac{\partial^2 T}{\partial \bar{Y}^2}\right) + \bar{U}_{\bar{X}}\bar{S}_{\bar{X}\bar{X}} + \left(\bar{U}_{\bar{Y}} + \bar{V}_{\bar{X}}\right)\bar{S}_{\bar{X}\bar{Y}} + \bar{V}_{\bar{Y}}\bar{S}_{\bar{Y}\bar{Y}} \tag{4}$$

where

$$\bar{S}_{\bar{X}\bar{X}} = \eta_0(4\bar{U}_{\bar{X}}) - k_0(4\bar{U}_{\bar{X}\bar{t}} + 4\bar{U}\bar{U}_{\bar{X}\bar{X}} + 4\bar{V}\bar{U}_{\bar{X}\bar{Y}} - 8\bar{U}_{\bar{X}}^2 - 4\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}} - 4\bar{V}_{\bar{X}}^2) \tag{5}$$

$$\bar{S}_{\bar{X}\bar{Y}} = \eta_0(2\bar{U}_{\bar{Y}} + 2\bar{V}_{\bar{X}}) - k_0(2\bar{U}_{\bar{Y}\bar{t}} + 2\bar{V}_{\bar{X}\bar{t}} + 2\bar{U}\bar{U}_{\bar{X}\bar{Y}} + 2\bar{U}\bar{V}_{\bar{X}\bar{X}} + 2\bar{V}\bar{U}_{\bar{Y}\bar{Y}} + 2\bar{V}\bar{V}_{\bar{X}\bar{Y}} - 6\bar{U}_{\bar{X}}\bar{U}_{\bar{Y}}$$
$$-2\bar{U}_{\bar{Y}}\bar{V}_{\bar{Y}} - 2\bar{U}_{\bar{X}}\bar{V}_{\bar{X}} - 6\bar{V}_{\bar{X}}\bar{V}_{\bar{Y}}) \tag{6}$$

$$\bar{S}_{\bar{Y}\bar{Y}} = \eta_0(4\bar{V}_{\bar{Y}}) - k_0(4\bar{V}_{\bar{Y}\bar{t}} + 4\bar{U}\bar{V}_{\bar{X}\bar{Y}} + 4\bar{V}\bar{V}_{\bar{Y}\bar{Y}} - 4\bar{U}_{\bar{Y}}^2 - 4\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}} - 8\bar{V}_{\bar{Y}}^2) \tag{7}$$

with boundary conditions, and the geometries of the walls $\bar{H}_1$ and $\bar{H}_2$ are defined by

$$\bar{U} = 0,\ T = T_0,\ \text{ at }\ \bar{Y} = \bar{H}_1 \tag{8}$$

$$\bar{U} = 0,\ T = T_1,\ \text{ at } \bar{Y} = \bar{H}_2 \tag{9}$$

$$\bar{H}_1(\bar{X},\bar{t}) = d_1 + a_1 \sin\left\{\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right\} \tag{10}$$

$$\bar{H}_2(\bar{X},\bar{t}) = -d_2 - a_2 \sin\left\{\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \phi\right\} \tag{11}$$

In Eqs. (1) to (11), ρ is the fluid density, $\bar{P}$ the pressure, T the fluid temperature, k the thermal conductivity, U the longitudinal velocity component, V the transverse velocity component, a_i (i = 1,

2) the wave amplitudes at upper and lower walls, respectively, λ the wavelength, c the wave speed, $\bar{t}$ the time, and ϕ is the phase difference varying in the range $0 \le \phi \le \pi$ ($\phi = 0$ leads us to a symmetric channel with waves out of phase and when $\phi = \pi$, the waves are in phase). Moreover, a_i, d_i ($i = 1, 2$) and ϕ satisfy the condition $a_1^2 + a_2^2 + 2a_1a_2\cos\phi \le (d_1 + d_2)^2$.

The extra stress tensor $\bar{\mathbf{S}}$ for Walter's B fluid (see Ref. 2) is given by

$$\bar{\mathbf{S}} = 2\eta_0\mathbf{e} - 2k_0\frac{\delta\mathbf{e}}{\delta\bar{t}} \tag{12}$$

$$\frac{\delta\mathbf{e}}{\delta\bar{t}} = \frac{\partial\mathbf{e}}{\partial\bar{t}} + \mathbf{V}.\nabla\mathbf{e} - \mathbf{e}\nabla\mathbf{V} - \nabla\mathbf{V}^T\mathbf{e} \tag{13}$$

where $\mathbf{e}$ is the rate of strain tensor, $\partial\mathbf{e}/\delta\bar{t}$ the convected differentiation of the rate of strain tensor in relation to the material in motion, η_0 the limiting viscosity at small shear rates, and k_0 is the short memory coefficient.

Defining the following transformations

$$\bar{x} = \bar{X} - c\bar{t}, \quad \bar{y} = \bar{Y}, \quad \bar{u} = \bar{U} - c, \quad \bar{v} = \bar{V}, \ \bar{p}(\bar{x},\bar{y}) = \bar{P}(\bar{X},\bar{Y},\bar{t}) \tag{14}$$

and considering the non-dimensional relations

$$x = \frac{\bar{x}}{\lambda},\ y = \frac{\bar{y}}{d_1},\ u = \frac{\bar{u}}{c},\ v = \frac{\bar{v}}{c},\ p = \frac{d_1^2}{\eta_0\lambda c}\bar{p},\ h_1 = \frac{\bar{H}_1}{d_1}, h_2 = \frac{\bar{H}_2}{d_1}, \mathbf{S} = \frac{d_1}{\eta_0 c}\bar{\mathbf{S}},\ \delta = \frac{d_1}{\lambda}$$

$$\mathrm{Re} = \frac{\rho c d_1}{\eta_0},\ \kappa = \frac{k_0 c}{\eta_0 d_1},\ \eta = \frac{T - T_0}{T_1 - T_0},\ \mathrm{Er} = \frac{c^2}{\xi(T_1 - T_0)}, \mathrm{Pr} = \frac{\xi\eta_0}{k} \tag{15}$$

where ξ is the specific heat at constant volume, and then letting the stream function Ψ related to the velocity components u and v by the relations

$$u = \frac{\partial\Psi}{\partial y},\ v = -\delta\frac{\partial\Psi}{\partial x} \tag{16}$$

the continuity equation is satisfied and Eqs. (2)–(11) reduce to

$$\delta\,\mathrm{Re}\left\{\left(\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\Psi}{\partial y}\right\} = -\frac{\partial p}{\partial x} + \delta\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \tag{17}$$

$$-\delta^3\,\mathrm{Re}\left\{\left(\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\Psi}{\partial x}\right\} = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial S_{xy}}{\partial x} + \delta\frac{\partial S_{yy}}{\partial y} \tag{18}$$

$$\delta\,\mathrm{Re}\left\{\left(\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}\right)\nabla^2\Psi\right\} = \left(\frac{\partial^2}{\partial y^2} - \delta^2\frac{\partial^2}{\partial x^2}\right)S_{xy} + \delta\frac{\partial^2}{\partial x\partial y}(S_{xx} - S_{yy}) \tag{19}$$

$$\delta\,\mathrm{Re}\left(\frac{\partial\Psi}{\partial y}\frac{\partial\eta}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial\eta}{\partial y}\right) = \frac{1}{\mathrm{Pr}}\left(\delta^2\frac{\partial^2\eta}{\partial x^2} + \frac{\partial^2\eta}{\partial y^2}\right) + \mathrm{Er}\left\{\delta\Psi_{xy}S_{xx} + (\Psi_{yy} - \delta^2\Psi_{xx})S_{xy} - \delta\Psi_{xy}S_{yy}\right\} \tag{20}$$

where $\nabla^2 = \delta^2\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$.

$$S_{xx} = 4\delta\Psi_{xy} - \kappa(4\delta^2\Psi_y\Psi_{xxy} - 4\delta^2\Psi_x\Psi_{xyy} - 8\delta\Psi_{xy}^2 + 4\delta^2\Psi_{xx}\Psi_{yy} - 4\delta^4\Psi_{xx}^2) \tag{21}$$

$$S_{xy} = 2(\Psi_{yy} - \delta^2\Psi_{xx}) - \kappa(2\delta\Psi_y\Psi_{xyy} - 2\delta^3\Psi_y\Psi_{xxx} - 2\delta\Psi_x\Psi_{yyy} + 2\delta^3\Psi_x\Psi_{xxy} - 6\delta\Psi_{xy}\Psi_{yy}$$

$$+2\delta\Psi_{yy}\Psi_{xy}+2\delta^3\Psi_{xy}\Psi_{xx}-6\delta^3\Psi_{xx}\Psi_{xy}) \tag{22}$$

$$S_{yy}=-4\delta\Psi_{xy}-\kappa(-4\delta^2\Psi_{y}\Psi_{xxy}+4\delta^2\Psi_{x}\Psi_{xyy}-4\Psi_{yy}^2+4\delta^2\Psi_{xx}\Psi_{yy}-8\delta^2\Psi_{xy}^2) \tag{23}$$

with boundary conditions

$$\Psi=\frac{F}{2},\quad \frac{\partial\Psi}{\partial y}=-1,\quad \eta=0,\qquad \text{at}\qquad y=h_1(x) \tag{24}$$

$$\Psi=-\frac{F}{2},\quad \frac{\partial\Psi}{\partial y}=-1,\quad \eta=1,\qquad \text{at}\qquad y=h_2(x) \tag{25}$$

$$h_1(x)=1+a\sin(2\pi x),\qquad \text{and}\qquad h_2(x)=-d-b\sin(2\pi x+\phi) \tag{26}$$

Here Re is the Reynolds number, κ the viscoelastic parameter, Er the Eckert number, Pr the Prandtl number, and $a=(a_1/d_1)$, $b=(a_2/d_1)$, $d=(d_2/d_1)$ satisfy the condition

$$a^2+b^2+2ab\cos\phi\le(1+d)^2 \tag{27}$$

and F is the dimensionless average flux in the wave frame defined by

$$F=\int_{h_2(x)}^{h_1(x)}\frac{\partial\Psi}{\partial y}dy=\Psi(h_1(x))-\Psi(h_2(x)) \tag{28}$$

which is related to θ the dimensionless average flux in the laboratory frame, by the relation

$$\theta=F+1+d \tag{29}$$

The dimensionless pressure rise per wavelength Δp_λ and heat transfer coefficient Z_{h_1} and Z_{h_2} at upper and lower walls are calculated by the relation

$$\Delta p_\lambda=\int_0^1\frac{dp}{dx}\Big|_{y=0}dx,\quad Z_{h_1}=\frac{\partial h_1}{\partial x}\frac{\partial\eta}{\partial y},\quad Z_{h_2}=\frac{\partial h_2}{\partial x}\frac{\partial\eta}{\partial y} \tag{30}$$

3. Solution of the Problem

Equations (17) to (23) with boundary conditions (24) and (25) are solved analytically using the regular perturbation method. For perturbation solutions we expand the flow quantities in terms of small wave number δ(δ << 1) as

$$\Psi=\sum_{i=0}^{\infty}\delta^i\Psi_i,\ \mathbf{S}=\sum_{i=0}^{\infty}\delta^i\mathbf{S}_i,\ \eta=\sum_{i=0}^{\infty}\delta^i\eta_i,\ Z=\sum_{i=0}^{\infty}\delta^i Z_i \tag{31}$$

After substituting Eq. (31) into Eqs. (17)–(23) with boundary conditions (24) and (25) and comparing the coefficients of powers of δ we obtain the system of equations as follows.

3.1 Zeroth-order system

The zeroth-order system is

$$\frac{\partial^2 S_{0xy}}{\partial y^2}=0 \tag{32}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2\eta_0}{\partial y^2}+2Er\Psi_{0yy}^2=0 \tag{33}$$

$$S_{0xx} = 0,\ S_{0xy} = 2\Psi_{0yy},\ S_{0yy} = 4\kappa\Psi_{0yy}^2 \tag{34}$$

where the boundary conditions are

$$\Psi_0 = \frac{F_0}{2}, \quad \frac{\partial \Psi_0}{\partial y} = -1, \quad \eta_0 = 0, \qquad \text{at} \qquad y = h_1(x) \tag{35}$$

$$\Psi_0 = -\frac{F_0}{2}, \quad \frac{\partial \Psi_0}{\partial y} = -1, \quad \eta_0 = 1, \qquad \text{at} \qquad y = h_2(x) \tag{36}$$

Substituting Eq. (34) into Eqs. (32) and (33) and then solving the resulting system with boundary conditions (35) and (36), we obtain the solutions up to $O(S^0)$ as

$$S_{0xy} = 4A_3 + 12A_4 y \tag{37}$$

$$\Psi_0 = A_1 + A_2 y + A_3 y^2 + A_4 y^3 \tag{38}$$

$$\eta_0 = S_1 + S_2 y + S_3 y^2 + S_4 y^3 + S_5 y^4 \tag{39}$$

$$Z_{0h_1} = a\cos x(S_2 + 2S_3 y + 3S_4 y^2 + 4S_5 y^3) \tag{40}$$

$$Z_{0h_2} = -b\cos(x+\phi)(S_2 + 2S_3 y + 3S_4 y^2 + 4S_5 y^3) \tag{41}$$

3.2 First-order system

The first-order system is

$$\frac{\partial^2 S_{1xy}}{\partial y^2} + \frac{\partial^2 S_{0xx}}{\partial x \partial y} = \frac{\partial^2 S_{0yy}}{\partial x \partial y} + \mathrm{Re}\left(\Psi_{0y}\Psi_{0xyy} - \Psi_{0x}\Psi_{0yyy}\right) \tag{42}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 \eta_1}{\partial y^2} + Er\Phi_1 = \mathrm{Re}\left(\Psi_{0y}\eta_{0x} - \Psi_{0x}\eta_{0y}\right) \tag{43}$$

$$S_{1xx} = 4\Psi_{0xy} + 8\kappa\Psi_{0xy}^2 \tag{44}$$

$$S_{1xy} = 2\Psi_{1yy} - 2\kappa\Psi_{0y}\Psi_{0xyy} + 2\kappa\Psi_{0x}\Psi_{0yyy} + 4\kappa\Psi_{0xy}\Psi_{0yy} \tag{45}$$

$$S_{1yy} = 8\kappa\Psi_{0yy}\Psi_{1yy} - 4\Psi_{0xy} \tag{46}$$

$$\Phi_1 = 4\Psi_{0yy}\Psi_{1yy} - 2\kappa\Psi_{0y}\Psi_{0yy}\Psi_{0xyy} + 2\kappa\Psi_{0x}\Psi_{0yy}\Psi_{0yyy} + 6\kappa\Psi_{0xy}\Psi_{0yy}^2 - 6\kappa\Psi_{0yy}\Psi_{0xy} \tag{47}$$

where the boundary conditions are

$$\Psi_1 = \frac{F_1}{2}, \quad \frac{\partial \Psi_1}{\partial y} = 0, \quad \eta_1 = 0, \qquad \text{at} \qquad y = h_1(x) \tag{48}$$

$$\Psi_1 = -\frac{F_1}{2}, \quad \frac{\partial \Psi_1}{\partial y} = 0, \quad \eta_1 = 0, \qquad \text{at} \qquad y = h_2(x) \tag{49}$$

Using the zeroth-order solutions Eqs. (37)–(41) into first-order system Eqs. (42)–(47) and then solving the resulting system we obtain

$$S_{1xy} = L_{37} + L_{38} y + L_{39} y^2 + L_{40} y^3 + L_{41} y^4 + L_{42} y^5 \tag{50}$$

$$\Psi_1 = B_1 + B_2 y + B_3 y^2 + B_4 y^3 + B_5 y^4 + B_6 y^5 + B_7 y^6 + B_8 y^7 \tag{51}$$

$$\eta_1 = S_6 + S_7 y + S_8 y^2 + S_9 y^3 + S_{10} y^4 + S_{11} y^5 + S_{12} y^6 + S_{13} y^7 + S_{14} y^8 \tag{52}$$

$$Z_{1h_1} = a\cos x(S_7 + 2S_8 y + 3S_9 y^2 + 4S_{10} y^3 + 5S_{11} y^4 + 6S_{12} y^5 + 7S_{13} y^6 + 8S_{14} y^7) \tag{53}$$

$$Z_{1h_2} = -b\cos(x+\phi)(S_7 + 2S_8 y + 3S_9 y^2 + 4S_{10} y^3 + 5S_{11} y^4 + 6S_{12} y^5 + 7S_{13} y^6 + 8S_{14} y^7) \quad (54)$$

All the coefficients appearing in the above expressions of the solutions are calculated by the usual lengthy algebra that is involved in the regular perturbation method.

4. Discussion

The effects of viscoelastic parameter κ, Prandtl number Pr, Eckert number Er, and phase difference ϕ appearing in the solutions of stream function Ψ, temperature distribution η, and heat transfer coefficient Z are presented graphically in this section. The comparison of the present series solution for pressure rise per wave length Δp_λ for different flow rate θ is presented in Table 1. These results are found in good agreement with those reported by Mishra and Rao [15].

Figures 1 and 2 display the changes in temperature distribution η against y due to variation of parameters Er, Pr, and θ. As expected, the temperature profiles are almost parabolic concaved downward. These temperature profiles are greater near the upper wall which parabolically decreases moving toward the lower wall. It is worth mentioning that in Fig. 1(i) with the absence of viscous dissipation (Er = 0) the temperature profile is linear. Further, it is noticed in this figure and Fig. 1(ii) that the temperature profiles increase with an increasing Pr and Er. Figure 2(i) shows that temperature profiles decrease with an increasing θ(< 1), whereas in Fig. 2(ii) the temperature profiles increase with an increasing θ(> 1).

Finally, Figs. 3 to 6 present the coefficients of heat transfer Z_{h_1} and Z_{h_2} for different values of Er, Pr, κ, and ϕ at the upper and lower walls, respectively. An oscillatory behavior is observed in all figures which is due to the propagation of peristaltic waves along the walls of the channel. Further, it is noticed that the absolute value of the heat transfer coefficient increases with an increasing Er, Pr, and κ. It is also noted in Fig. 6 that the absolute value of the heat transfer coefficient increases at the upper wall but decreases at the lower wall of the channel with an increasing ϕ.

Table 1. Comparison of Pressure Rise Per Wavelength Δp_λ for Different Values of Flow Rate θ When $a = 0.7$, $b = 1.2$, $d = 2$, κ = 0, δ = 0, Re = 0

	$\Delta p_\lambda(\phi = 0)$		$\Delta p_\lambda(\phi = \pi)$	
θ	Mishra and Rao [15]	Present results	Mishra and Rao [15]	Present results
-1.0	*4.8126*	*4.812560*	*0.5431*	*0.543106*
-0.5	*3.8514*	*3.851390*	*0.3014*	*0.301358*
0.0	*2.8902*	*2.890210*	*0.0596*	*0.059609*
0.5	*1.9290*	*1.929030*	*-0.1821*	*-0.182139*
1.0	*0.9679*	*0.967850*	*-0.4239*	*-0.423888*
1.5	*0.0067*	*0.006672*	*-0.6656*	*-0.665637*
2.0	*-0.9545*	*-0.954507*	*-0.9074*	*-0.907385*
2.5	*-1.9157*	*-1.915690*	*-1.1491*	*-1.149130*
3.0	*-2.8769*	*-2.876860*	*-1.3909*	*-1.390880*

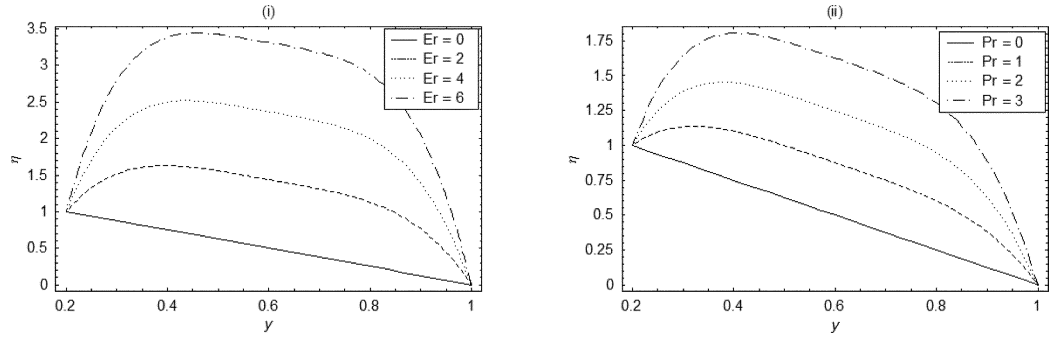

Fig. 1. Temperature profile η for fixed $a = 0.5$, $b = 1.2$, $d = 1$, $\phi = \pi/2$, $\theta = 1$, $\kappa = 0.1$, $\delta = 0.01$, Re = 5, $x = 0.5$ (i) Pr = 5 (ii) Er = 4.

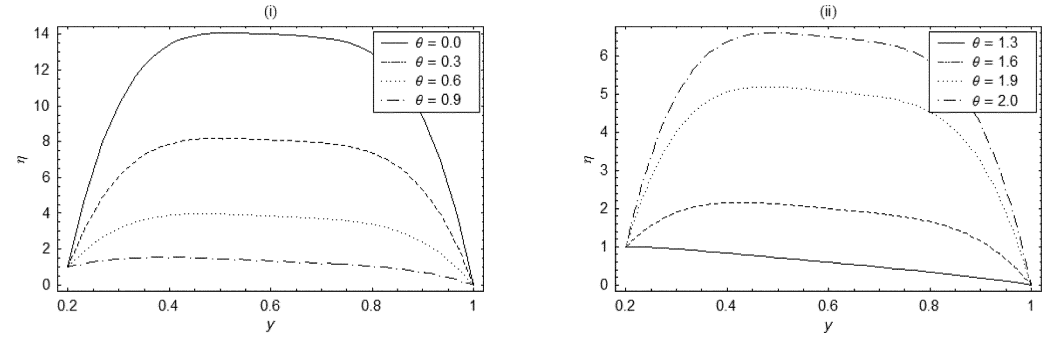

Fig. 2. Temperature profile η for fixed $a = 0.5$, $b = 1.2$, $d = 1$, $\phi = \pi/2$, $\kappa = 0.1$, $\delta = 0.01$, Re = 5, $x = 0.5$, Pr = 1, Er = 4 (i) $\theta < 1$ (ii) $\theta > 1$.

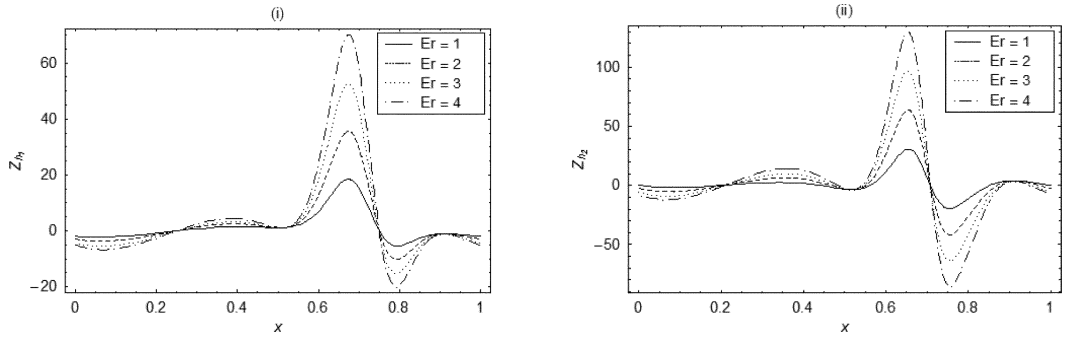

Fig. 3. Heat transfer coefficient (i) Z_{h_1} at upper wall (ii) Z_{h_2} at lower wall for fixed $a = 0.4$, $b = 1.2$, $d = 1.5$, $\phi = \pi/12$, $\theta = 1$, $\kappa = 0.1$, $\delta = 0.01$, Re = 1, Pr = 1.

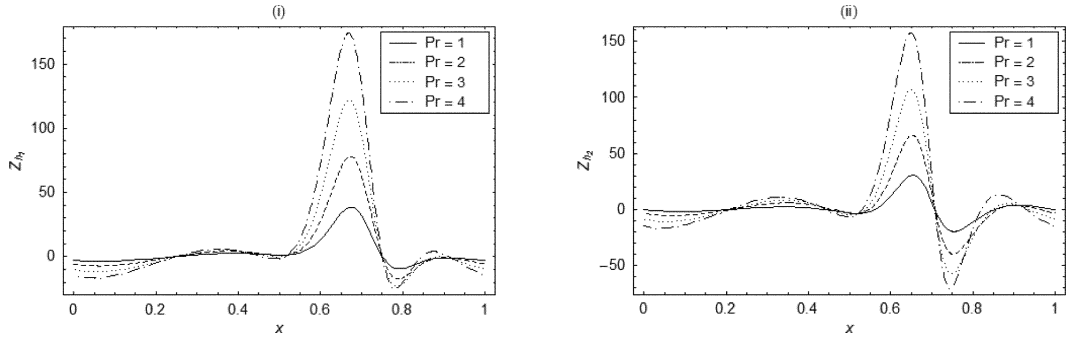

Fig. 4. Heat transfer coefficient (i) Z_{h_1} at upper wall (ii) Z_{h_2} at lower wall for fixed $a = 0.4$, $b = 1.2$, $d = 1.5$, $\phi = \pi/12$, $\theta = 0.5$, $\kappa = 1$, $\delta = 0.01$, Re = 1, Er = 2.

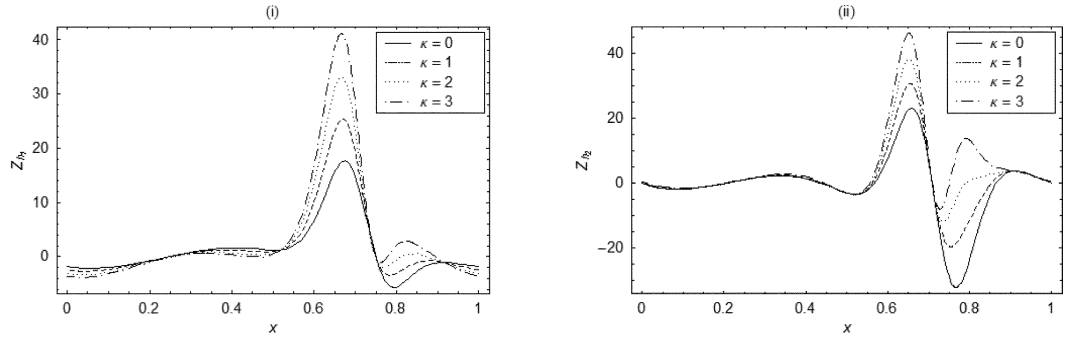

Fig. 5. Heat transfer coefficient (i) Z_{h_1} at upper wall (ii) Z_{h_2} at lower wall for fixed $a = 0.4$, $b = 1.2$, $d = 1.5$, $\phi = \pi/12$, $\theta = 0.5$, $\delta = 0.01$, Re = 1, Er = 2, Pr = 1.

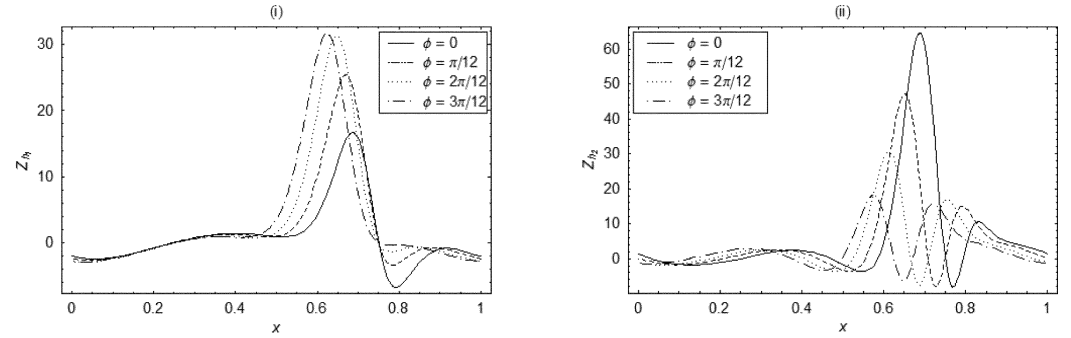

Fig. 6. Heat transfer coefficient (i) Z_{h_1} at upper wall (ii) Z_{h_2} at lower wall for fixed $a = 0.4$, $b = 1.2$, $d = 1.5$, $\theta = 0.5$, $\kappa = 1$, $\delta = 0.03$, Re = 1, Er = 1, Pr = 1.

5. Conclusion

In this study the problem of heat transfer on the peristaltic flow of a Walter's B fluid in a two-dimensional asymmetric channel is considered. The resulting equations containing viscoelastic parameter κ, Eckert number Er, Prandtl number Pr, and phase difference ϕ are solved analytically by taking the wave number as the small parameter. It is concluded that the temperature profiles are parabolic and significant variations lie in the center of the channel due to the viscous dissipation. In the absence of viscous dissipation the temperature profile becomes linear. Temperature of the fluid increases with an increasing Eckert and Prandtl number. The behavior of the heat transfer coefficient is oscillatory due to the propagation of peristaltic waves. Further the absolute value of the heat transfer coefficient increases with an increasing viscoelastic parameter.

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