

3D COORDINATE TRANSFORMATION USING MOLODENSKY BADEKAS TRANSFORMATION MODEL: MBT07

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ABSTRACT

Transformations between coordinate systems are routinely carried out in surveying. As satellite positioning techniques are more frequently used, transformations of three-dimensional (3D) coordinates will become a common practice. The main objective of this research is to develop a computer program (using Visual Basic 6.0) to determine the 7 parameters used in 3D coordinate transformations. The derived parameters are then back substitute into the transformation equation to perform coordinate transformation. The developed program is called Molodensky Badekas Transformation 2007 (MBT07). MBT07 was developed based on one of the most common transformation model for 3D coordinates which is Molodensky Badekas transformation model. Another short program was developed with MATLAB, to confirm the algorithms involved in Molodensky Badekas transformation before creating MBT07. The third software that is involved in this research is Geodetic Datum Transformation Suite (GDTS). This software is used to validate the computed results from MBT07 and MATLAB. Two sets of data were used for testing GDTS, MBT07 and MATLAB. The datasets contained coordinates of common points in two coordinate systems. The parameters derived from MBT07 and MATLAB are compared with the results obtained from GDTS. Even though the analysis shows a small difference for the parameters, it is still within the permitted standard deviation. Besides calculating parameters, MBT07 also contains three other modules related to transformation.

1.0 Introduction

Coordinate transformation is a mathematical operation that transforms the coordinates of a point from one system to another system. Hence, a mathematical method to establish a geometrical relationship between coordinates of a point in different reference frame is produced. Various transformation algorithms can be used according to the quality of the network, the accuracy required and other characteristics of the transformation models (Wu and Hill, 2005).

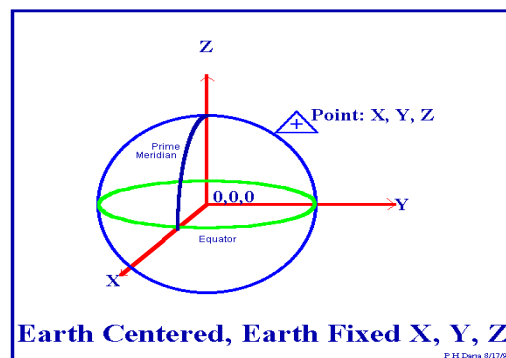


Figure 1: Earth Centered Earth Fixed X, Y, Z

A simple three dimensional (3D) parameter conversion between geodetic coordinates in different datum can be achieved by conversion through Earth Centered Earth Fixed (ECEF) XYZ Cartesian

coordinates (Figure 1) in one reference datum and three origin offsets. The offsets are for rotation, translation and scale. Therefore, the complete coordinate transformation is mostly based on seven parameters (DSMM, 2003). The seven parameters are three translation parameters (Figure 2), three rotation parameters (Figure 3) and a scale factor. Experts and researchers have developed the transformation model of 3D coordinate system for transforming from one datum to another.

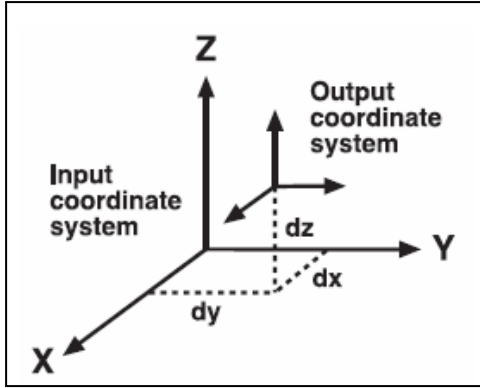


Figure 2: Three translation parameter

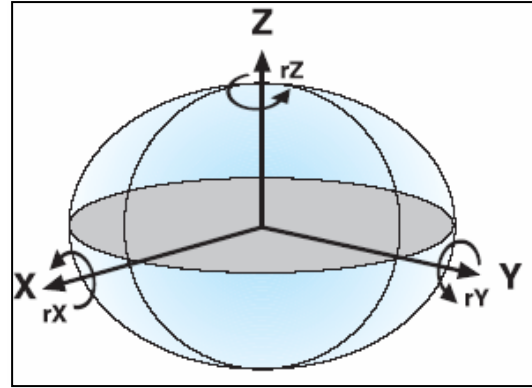


Figure 3: Three rotation parameter

The most widely used models for the 3D transformation are the Bursa Wolf (Equation 1) and Molodensky Badekas (2) models. In the Molodensky Badekas method, the transformation is referred to a local origin. This origin is obtained through the mean coordinate from all the common points in the network. Meanwhile, the Bursa Wolf method uses the reference system's origin (Ng, 1992). At a glance, both Equation (1) and Equation (2) look almost the same because both models share the same components. Both models contains the translation matrix ($\Delta X, \Delta Y, \Delta Z$ for Bursa Wolf model, and dX, dY , and dZ for Molodensky Badekas model), and a 3×3 rotation matrix (R_X, R_Y, R_Z). The additional component in Molodensky Badekas model (Equation 2) is the component for the origin of the network (X_m, Y_m, Z_m). During computation, the coordinates from the old system are first reduced according to this origin ($X_{old} - X_m$). By reducing the coordinates, the average shift of the origin and the effect of the change in the size and shape between the two reference systems are taken into consideration.

$$\begin{bmatrix} X_{New} \\ Y_{New} \\ Z_{New} \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} 1 + \Delta L & R_Z & -R_Y \\ -R_Z & 1 + \Delta L & R_X \\ R_Y & -R_X & 1 + \Delta L \end{bmatrix} \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} X_{New} \\ Y_{New} \\ Z_{New} \end{bmatrix} = \begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} + \begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} + \begin{bmatrix} 1 + \Delta L & R_Z & -R_Y \\ -R_Z & 1 + \Delta L & R_X \\ R_Y & -R_X & 1 + \Delta L \end{bmatrix} \begin{bmatrix} X_{old} - X_m \\ Y_{old} - Y_m \\ Z_{old} - Z_m \end{bmatrix} \quad (2)$$

The significance of coordinate transformation in Malaysia is becoming more obvious with the implementation of the new Geocentric Datum of Malaysia 2000 (GDM2000) in the year 2003. Transformation parameters are developed to relate this new datum to other existing datum such as the Malayan Revised Triangulation (MRT). Thus knowledge and insights about coordinate transformation is very important (Majid et al., 2003). This is because there will be more implementation of GDM2000 and GPS survey in the future.

2.0 Objective of Study

The main objective of this study is to create a computer program (called Molodensky Badekas Transformation 2007 (MBT07)) for 3D coordinate transformation based on the Molodensky Badekas model. Although Molodensky Badekas is one of the common transformation models, it is less appreciated here in Malaysia compared to Bursa Wolf. This is because the Department of Survey and Mapping Malaysia (DSMM) used the Bursa Wolf model to develop the transformation parameters for GDM2000 to other systems. The Bursa Wolf model sort of ‘overshadows’ the Molodensky Badekas model. The MBT07 can be used as a platform to introduce the Molodensky Badekas model as an alternative beside Bursa Wolf. MBT07 also gives users more flexibility in terms of functions and options. The program was created using Visual Basic 6.0 programming language. Visual Basic 6.0 was chosen because it is one of the fastest and easiest ways to create any Windows based program.

The second objective is to analyze and compare the results obtained from MBT07 with two transformation programs, i.e. Geodetic Datum Transformation Suite or GDTS (provided courtesy of Dr. Abd. Majid, 2007) and Datum Transformation (developed by Jan Hendrikse from ILWIS Development, Netherlands, and downloaded from the internet). Two sets of coordinates in two coordinate systems were used to do the analysis. The comparison was made for the computed parameters, the standard deviation and the standard error of unit weight. By doing the analysis and comparison, the reliability and efficiency of MBT07 is tested.

3.0 Literature Review

Coordinate system is the prime method for georeferencing that is used to locate the position of objects in 2D or 3D in correct relationship with respect to each other. Coordinate systems are also divided into local and global systems. Examples of local coordinate systems are Geocentric Datum Malaysia 2000 (GDM2000), Malayan Revised Triangulation (MRT) and Rectified Skew Orthomorphic (RSO). While World Geodetic System 1984 (WGS84) (shown in Figure 4) is an example of global coordinate system (NIMA, 1997). To suit the different application of these coordinate systems, algorithms are form to connect different coordinates systems. The coordinate systems are connected through a set of transformation parameters, and the relevant mathematical procedure is called coordinate transformation.

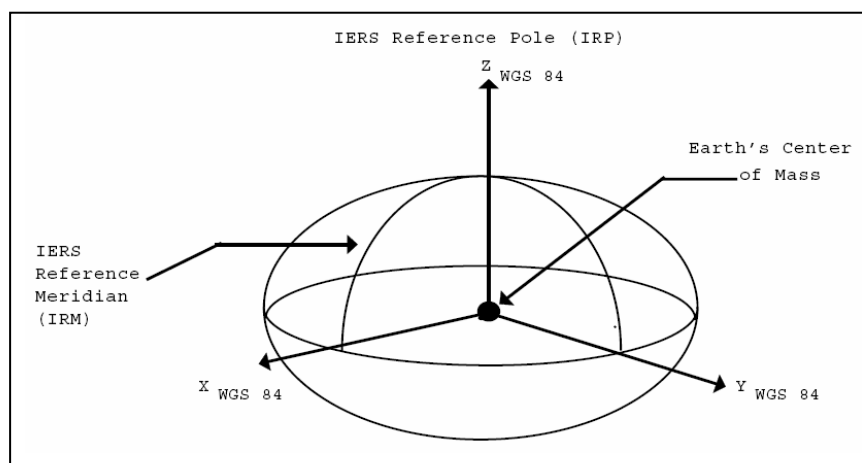


Figure 4: The WGS84 coordinate system definition.

There are a number of ways to mathematically transform coordinates from one system to another. Common points and the parameters of the reference ellipsoid are needed to perform this transformation.

Common points are surveyed points that have known positions in terms of both the local and global systems. Mujtaba (2004) also mentioned that most of the time when greater accuracy is required, more common points is needed. The common points should be a good sample of the true relationship between local and global datums.

Mitsakaki (2004) reveals that the choice of the appropriate coordinate transformation model depends on few factors. The first consideration is about the size of the areas of interest and the distortion of the local datum in the area. Then pay attention to the type of network (3D, 2D or even 1D) and the accuracy required.

According to the technical paper on GDM2000 written by Majid et. al. (2003), the Department of Survey and Mapping Malaysia (DSMM) uses Bursa Wolf model (Figure 5) to transform 3D Cartesian coordinates between two datums. This transformation model is more suitable for satellite datums on a global scale (DSMM, 2003). But Molodensky Badekas is also a 3D seven parameter transformation model. Krakiwsky and Thompson (1974) mentioned that both model are equivalent, whereby both models combine system and network rotations. However, Molodensky Badekas has a different design matrix from Bursa Wolf's.

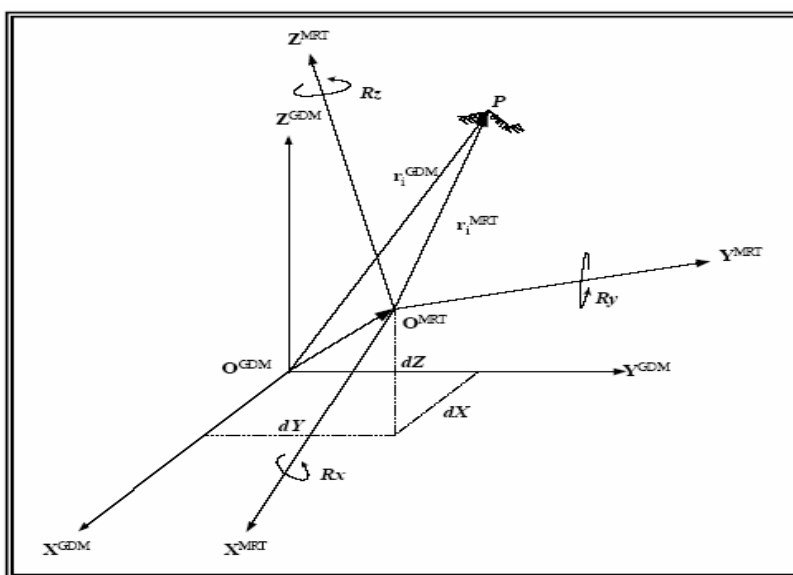


Figure 5: Bursa Wolf 3D transformation model

Many efforts were made to develop coordinate transformation programs to be used in Malaysia. Ng (1992) built a program using C programming language to determine the transformation parameters. While Lee (1999) created software package that included navigation and one way transformation from WGS84 to MRT, RSO and Cassini Soldner. He uses Visual Basic 5.0 to create the software mainly for its user friendly features. Shu (2004) also built a program called Coordinate Transformation Malaysia (CTM) using Visual Basic 6.0. The specialty of his program is that he covers all transformation for Peninsular Malaysia, Sabah and Sarawak.

4.0 Problem Statement

Transformations between coordinate systems are often carried out in GPS survey. Results of GPS observation are in terms of WGS84. But the specifications of the survey often wanted the positions to be supplied in terms of local datum. This will create the problem of transforming the positions from WGS84

to the local datum. Most part of the world has its own local geodetic systems. There will certainly be a requirement to continue to use these local systems for a long time into the future. Take MRT and RSO as example, which existed since 1940s and still being used until now. There are many systems for different purposes, for example Cassini Soldner system (used in cadastre survey) and RSO system (for mapping purposes). Therefore GPS results must be appropriately transformed into required local systems.

When GDM2000 was launched, DSMM employs Bursa Wolf transformation model for deriving transformation parameters for GDM2000 and WGS84. Resulting from this, most computer programs that are developed for coordinate transformation use Bursa Wolf model. But these programs only apply the parameters into the model to perform coordinate transformation. MBT07 is different from other transformation programs. MBT07 not only uses the alternative transformation model from Bursa Wolf which is the Molodensky Badekas model, but also uses Molodensky Badekas to derive the parameters and then use the parameters again to transform the coordinates.

Ng (1992) built a coordinate transformation program based on the Molodensky Badekas model. Ng's program was created in C programming language. He uses a partitioning method while developing his program. By using this method, it will partitioned the huge matrix A and B (refer to Appendix A) into separate parts. Each part was calculated separately and then combined to form the matrix again. It is practical because during that time, some software is not custom to do large scale computation. MBT07 is different from the program developed by Ng. MBT07 was written in Visual Basic 6.0 and it was develop without using partitioning method. There are improvements in MBT07 compared to Ng (1992). The execution of MBT07 is windows based, while Ng's program was in DOS environment. MBT07 also has interesting Graphic User Interface (GUI) and easy to understand command buttons that will be a convenience for users to operate the program.

5.0 Scope of Study

The Molodensky Badekas transformation model is only applicable for 3D Cartesian coordinate transformation. If the input is in geographical form, then it has to convert to Cartesian coordinates before going further into the computation. Therefore the first application in MBT07 is the ability to perform the conversion from geographical to Cartesian and vice versa.

$$X = (v + h)\cos\phi\cos\lambda \quad (3)$$

$$Y = (v + h)\cos\phi\sin\lambda \quad (4)$$

$$Z = (v(1 - e^2) + h)\sin\phi \quad (5)$$

$$\phi = \arctan\left[\frac{Z + e'^2 b \sin^3 \beta}{p - ae^2 \cos^3 \beta}\right] \quad (6)$$

$$\lambda = \arctan\left[\frac{Y}{X}\right] \quad (7)$$

$$h = p \cos\phi + Z \sin\phi - \frac{a^2}{v} \quad (8)$$

Equation (3), Equation (4) and Equation (5) are used to convert geographical coordinates to Cartesian coordinates. But different algorithm such as Equation (6), Equation (7) and Equation (8) are

used to convert Cartesian coordinates back to geographical. The coordinates can be transformed from one system to another system as long as both systems are compatible.

The scope of MBT07 also includes the calculation of the transformation parameters using Molodensky Badekas transformation model. MBT07 is able to calculate seven transformation parameters. Unfortunately MBT07 is unable to calculate or apply four or six transformation parameters. MBT07 also incorporates Least Square Adjustment technique for parameters estimation. The combined mathematical model was involved in the computation. By referring to Appendix A, it shows how Least Square Adjustment gets involved in the computation process. After obtaining the parameters, they are back substitute into the model to perform transformation for other coordinates that are not used as common points.

MBT07 was developed using Visual Basic 6.0. But prior to that, a short program was written in MATLAB environment to validate the algorithm and procedure. The MATLAB program is a very basic program to test out the calculation. When the calculations are confirmed, the MATLAB program was used to verify the results from MBT07. Therefore a program language (Visual Basic 6.0) and technical software (MATLAB) were used in completing this project.

6.0 Method and implementation

The method comprises of several important stages and tasks, as shown in Figure 6. The main outcome from literature review is the complete algorithm for coordinate transformation computation (as shown in Appendix A).

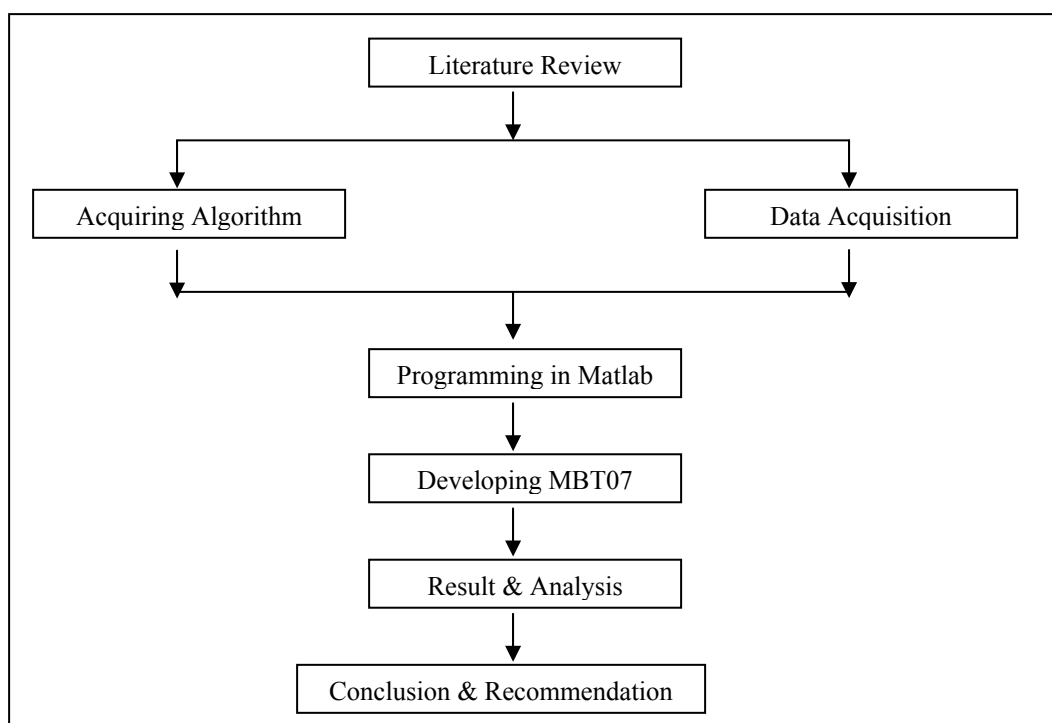


Figure 6: Method

To test the algorithms, a simple program was first written using MATLAB. After confirming the algorithm with MATLAB, the same procedures are repeated in Visual Basic, for the development of MBT07. Basically, different levels of computation and transformation were separated into modules and the modules were combined to form the main program. Microsoft Visual Basic 6.0 proves to be the best choice of programming language for this project because it is able to create 'simple to use' interfaces and perform computations as well.

Two datasets were used in this project. Each dataset contains coordinates in different systems for 10 and 11 common points respectively. For verification purpose, the results from MBT07 are compared with other transformation programs such as Geodetic Datum Transformation Suite (GDTS) and Datum Transformation.

The main outcome from this project is a computer program called Molodensky Badekas Transformation 2007 (MBT07) (Figure 7). There are four main modules in MBT07: (i) Coordinate Conversion – From Geographical to Cartesian Coordinates, (ii) Coordinate Conversion – From Cartesian to Geographical Coordinates, (iii) Coordinate Transformation – User Define Parameters, (iv) Coordinate Transformation – Calculate Parameters (Figure 8).

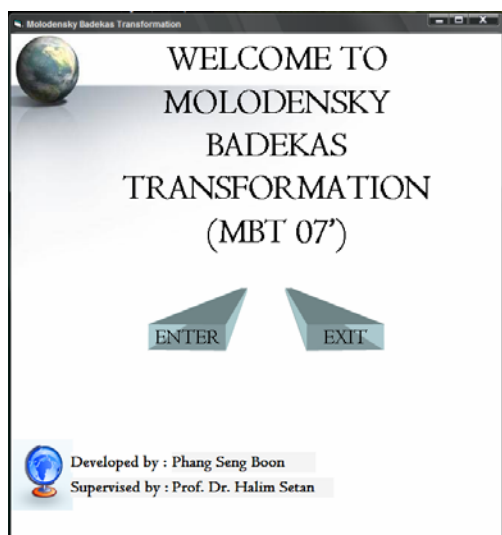


Figure 7: Front page of MBT07

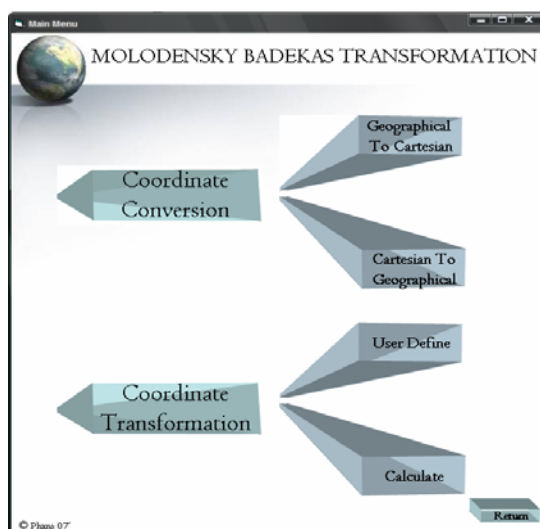


Figure 8: The main Menu containing four modules

In each of the module, the user is given the flexibility to choose the input method either via keyboard entry or from file (Figure 9). If the input is from a file, a short briefing about the format of the input (Figure 10) is available to the user. This will help avoid input errors to occur while using the program. The computed parameters are shown on the screen (Figure 11) but at the same time, a default output file called Parameters Out will be created.

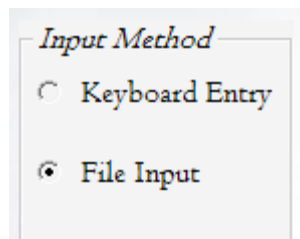


Figure 9: Option for input method

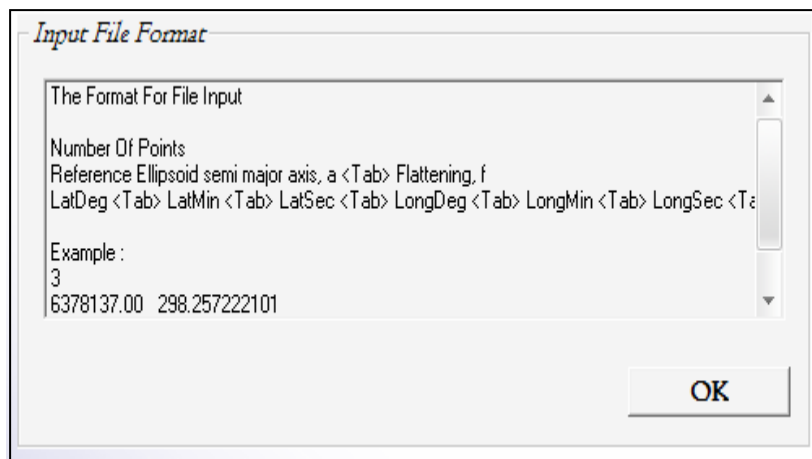


Figure 10: Short briefing about the input file format

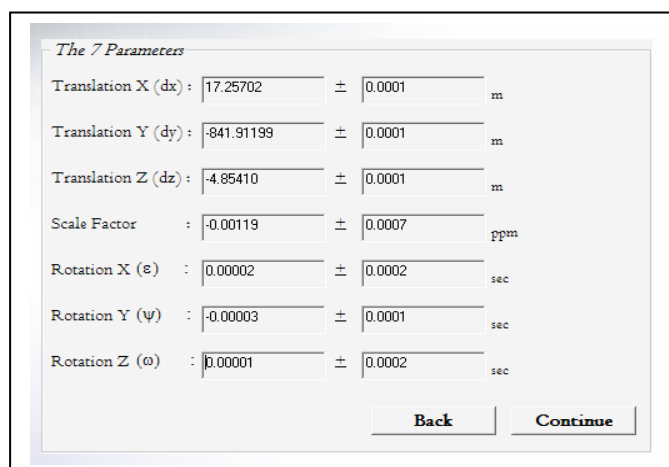


Figure 11: Parameters display

7.0 Analysis

The results from MBT07 were analyzed and compared with the result from the MATLAB program and Geodetic Datum Transformation Suite (GDTS).

Two datasets were used to carry out the analysis. The first dataset contains 11 common points with coordinates in WGS84 and MRT. The transformation is from WGS84 to MRT. Table 1 shows the comparison of parameters obtained from the MATLAB program, MBT07 and GDTS. The second dataset contains 10 common points. The coordinates are in WGS84 and GDM2000. Transformation is from WGS84 to GDM2000. Table 2 shows the comparison of parameters obtained from the MATLAB program, MBT07 and GDTS.

Parameters	MATLAB	MBT07	GDTs
Translation X (dx), m	15.25702	17.25702	17.25702
Translation Y (dy), m	-841.91199	-841.91199	-841.91199
Translation Z (dz), m	-4.85410	-4.85410	-4.85410
Scale Factor, ppm	-0.00119	-0.00119	-0.00119
Rotation X, sec	0.00002	0.00002	0.00002
Rotation Y, sec	-0.00003	-0.00003	-0.00003
Rotation Z, sec	0.00001	0.00001	0.00001

Table 1: Analysis for transformation parameters from WGS84 to MRT

Parameters	MATLAB	MBT07	GDTs
Translation X (dx), m	-1.15564	-1.15564	-1.15705
Translation Y (dy), m	0.39672	0.39672	0.40286
Translation Z (dz), m	-1.25417	-1.2418	-1.25397
Scale Factor, ppm	-3.71305	-0.37131	-0.37057
Rotation X, sec	-0.01486	-0.01486	-0.03250
Rotation Y, sec	0.03433	0.03433	0.03113
Rotation Z, sec	0.10340	0.10342	0.07415

Table 2: Analysis for transformation parameters from WGS84 to GDM2000

Stn	$\Delta \phi$ (sec)	$\Delta \lambda$ (sec)	Δh (sec)
1	0.00312	0.08672	0.00009
2	0.00298	0.07594	0.00025
3	0.00172	0.07696	0.00031
4	0.00092	0.05833	0.00047
5	0.00604	0.00603	0.00049
6	0.00328	0.01933	0.00045
7	0.00147	0.0255	0.00027
8	0.00087	0.00198	0.00022
9	0.00243	0.00404	0.00046
10	0.00184	0.00209	0.00044
11	0.00566	0.02315	0.00003

Table 3: The difference of coordinates obtained from Datum Transformation with known coordinates in dataset 1 using parameters derived from MBT07.

The estimated parameters were used in Datum Transformation to check the parameters. The parameters were applied to the coordinates of the first system in both datasets. The objective here is to get back the same coordinates as system 2 in both datasets. Table 3 show the difference of latitude, longitude

and height from known coordinates in dataset 1 and Datum Transformation using parameters derived from MBT07. The same analysis is carried out on dataset 2 and it is shown in Table 4.

Stn	$\Delta \phi$ (sec)	$\Delta \lambda$ (sec)	Δh (sec)
1	0.00261	0.02526	0
2	0.00143	0.00276	0.013
3	0.00365	0.05616	0.009
4	0.0029	0.02713	0.013
5	0.00114	0.01083	0.009
6	0.00256	0.00208	0.085
7	0.00221	0.08682	0.012
8	0.00058	0.06805	0.023
9	0.004	0.00408	0.047
10	0.00355	0.03554	0.007

Table 4: The difference of coordinates obtained from Datum Transformation with known coordinates in dataset 2 using parameters derived from MBT07.

8.0 Conclusion and Recommendation

The developed MBT07 in this project can be used to determine seven transformation parameters between any two types of coordinate systems, i.e. transformation between local geodetic system and global reference system.

There are many ways to improve MBT07; one of them is to add graphical representation of the transformation. Meaning users are able to view the way that the transformation parameters are applied. It is also recommended to add more functions such as Bursa-Wolf transformation model to MBT07 to make it a complete transformation program.

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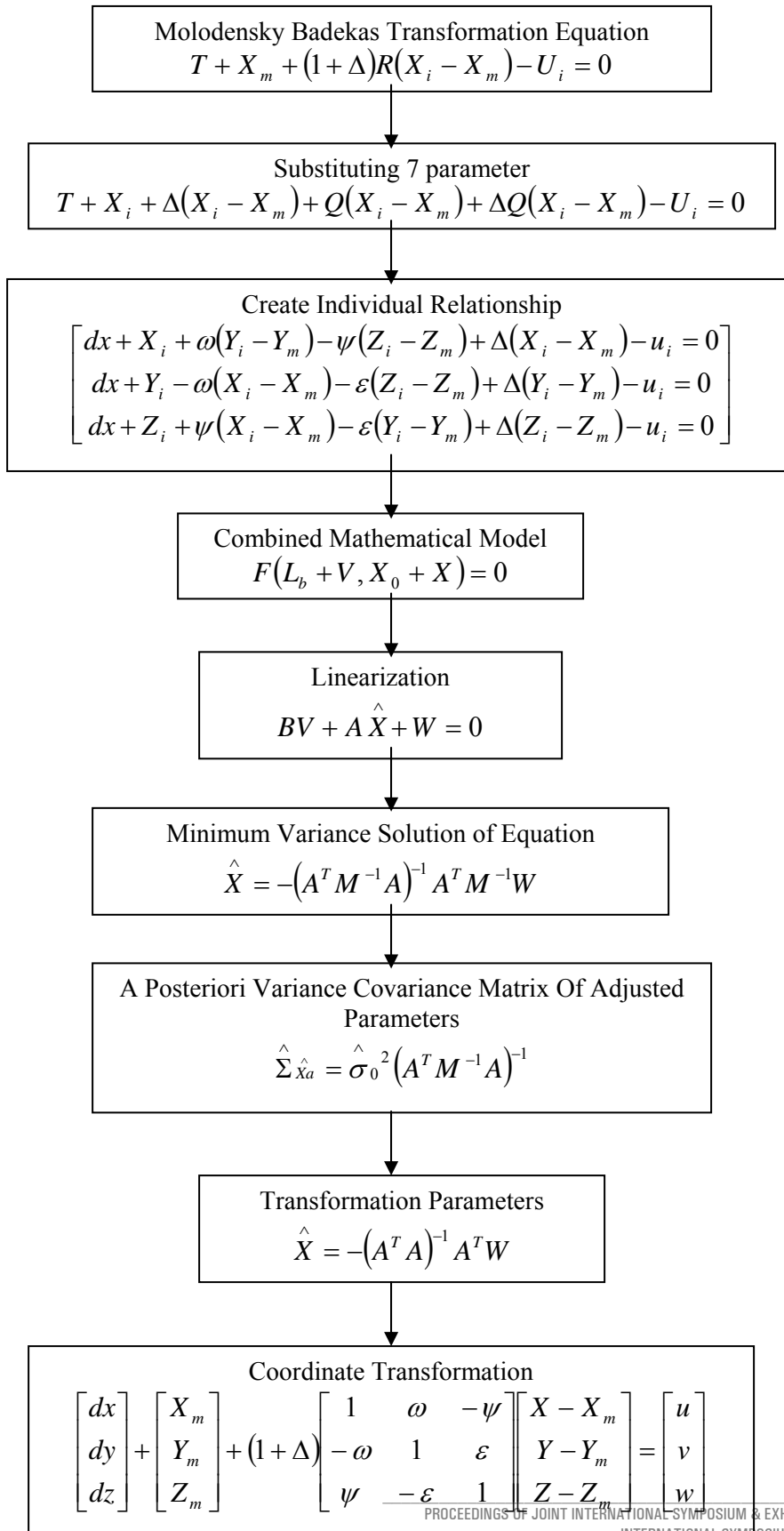
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Appendix A. Flow Chart of Coordinate Transformation Computation



Notation:

$(u, v, w), (X, Y, Z)$	- Cartesian Coordinates
(dx, dy, dz)	- Translation
$(1 + \Delta)$	- Scale factor
$(\varepsilon, \psi, \omega)$	- Rotation angle
$\hat{\sigma}_0^2$	- A posteriori variance of unit weight