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ORIGINAL ARTICLE

# Chronology of DIC technique based on the fundamental mathematical modeling and dehydration impact

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**Abstract** A chronology of mathematical models for heat and mass transfer equation is proposed for the prediction of moisture and temperature behavior during drying using DIC (Détente Instantanée Contrôlée) or instant controlled pressure drop technique. DIC technique has the potential as most commonly used dehydration method for high impact food value including the nutrition maintenance and the best possible quality for food storage. The model is governed by the regression model, followed by 2D Fick's and Fourier's parabolic equation and 2D elliptic-parabolic equation in a rectangular slice. The models neglect the effect of shrinkage and radiation effects. The simulations of heat and mass transfer equations with parabolic and elliptic-parabolic types through some numerical methods based on finite difference method (FDM) have been illustrated. Intel®Core™2Duo processors with Linux operating system and C programming language have been considered as a computational platform for the simulation. Qualitative and quantitative differences between DIC technique and the conventional drying methods have been shown as a comparative.

**Keywords** Mathematical modeling · DIC technique · Parabolic equation · Elliptic-Parabolic equation · Numerical methods · Finite difference method

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**Abbreviations**

| | |
|---|---|
| $A$ | Surface area (m$^2$) |
| $C_p$ | Specific heat (J/kgK) |
| $D_o$ | Pre- exponential factor Arrhenius equation (m$^2$/s) |
| $D_{eff}$ | Effective diffusivity coefficient (m$^2$/s) |
| $E_a$ | Activation energy (kJ/mol) |
| $H$ | Slice thickness (m) |
| $h_{fg}$ | Enthalpy of evaporation (J/kg) |
| $h_\infty$ | Convective heat transfer coefficient (m/s) |
| $h_m$ | Convective mass transfer coefficient (W/m$^2$ K) |
| $k$ | Thermal conductivity (W/m K) |
| $L$ | Slice length (m) |
| $M$ | Moisture content of drying specimen (g water/g dry matter) |
| $M_o$ | Initial moisture content of drying specimen (g/g dry) |
| $M_\infty$ | Air drying moisture content (g/g dry) |
| $P$ | Steam pressure (Pa) |
| $R$ | Gas constant, 8.3 J/mol K |
| $t$ | Processing time (sec) |
| $T$ | Temperature (°C) |
| $T_o$ | Initial temperature (°C) |
| $T_\infty$ | Air drying temperature (°C) |
| $t$ | Time (s) |
| $W$ | Moisture content (g/100g dry matter) |
| $x$ | Spatial coordinate in $x$ direction (m) |
| $y$ | Spatial coordinate in $y$ direction (m) |
| $z$ | Phytate content |

**Greek letter**

| | |
|---|---|
| $\rho$ | Density of drying |

## Introduction

Food dehydration is one of the most ancient and efficient preservation methods. In order to prevent the growth of bacteria, yeast, molds and bacterial action, the technique of removal moisture content of food to a certain level is necessary. Dehydration is a process where water vapor is removed from material surface into the surrounding space, resulting in a relatively dried form of the material (Chen and Mujumdar 2008). Recent years, the advances in dehydration techniques as well as the development of novel mathematical model to predict the drying methods have enabled the preparation of a wide range of dehydrated food to meet the quality, stability and functional requirements coupled with economy (Jayaraman and Das Gupta 1992).

The applicable methods available in food drying process industry are solar, spray, freeze, vacuum, osmotic, cabinet or tray, fluidized bed, spouted bed and microwave drying (George et al. 2004). DIC, which is another alternative of drying methods, has been used for swell-drying of fruits and vegetables drying (Louka and Allaf 2002; Al Haddad et al. 2008), texturing and drying various biological products by instant auto vaporization (Louka and Allaf 2004; Louka et al. 2004; Haddad and Allaf 2007; Nouviaire et al. 2008; Kristiawan et al. 2011), and microbiological decontamination (Setyopratomo et al. 2009), post harvesting and/or steaming paddy rice (Pilatowski et al. 2010) and essential oil extraction (Amor et al. 2008; Besombes et al. 2010).

DIC has been used for numerous new processes. The usage of DIC could greatly intensify the limiting transfer phenomenon, improve texture and usually maintain the product color, vitamins and flavor, reduce energy consumption and most importantly provide environmental friendly process (Al Haddad et al. 2008; Pilatowski et al. 2010). To ensure a high quality of food, the improved of both the kinetics and the capacity for dehydration and rehydration is emphasized. The abrupt pressure drop ($\Delta P/\Delta t > 0.5$ MPa/s) modifies the texture of the food material by increasing the material porosity and specific surface area, enhances the mass transfer and implies a rapid cooling of the product. Moreover, DIC process reduces diffusion resistance of moisture which reduce the energy usage and loss of food nutrition. This process also increases the water effective diffusivity of food material (Setyopratomo et al. 2009).

The motivation of this paper is to present the chronology of mathematical modeling using DIC technique quantitatively and qualitatively. Models are being established in order to predict some dependent and independent variables such as water losses, weight reduction, dehydration rates, temperature behavior, drying kinetics and others. The two important aspects simultaneously under consideration are heat and mass transfer. The four prevailing transport types of heat and mass transfer intervene during drying process are as following (Setyopratomo et al. 2009):

(i) Internal heat transfer transmits the energy by heat conduction condition,
(ii) External heat transfer carries out the energy based on contact, convection or radiation process,
(iii) Internal moisture mass transfer transports the water content in liquid and vapor phase,
(iv) External mass transfer process.

The simple mathematical models to govern the DIC processes are using regression formula and first order PDE. Amor and Allaf (2009), Setyopratomo et al. (2009) and Pilatowski et al. (2010) adopted Crank's solution according to the geometry of the solid matrix (Crank 1975). This model focused on the basic dependent parameters. Then, Zarguili et al. (2009) solved the first order PDE of mass transfer equation by using integration method. SIGMAPLOT software was used to simulate the numerical method. Al Haddad et al. (2008) used Fick's type law to visualize the diffusion process by controlling the mass transfer of water within the material. The chronology of DIC technique is based on PDE and the enhanced mathematical modeling is from Haddad et al. (2007).

## DIC technique

The DIC technology was initially developed by Allaf et al. since 1988 in the University of La Rochelle, France. It was categorized as a high temperature high pressure-short time process (Haddad et al. 2007). The machine consists of three main parts. The diagrammatic layout of the DIC is shown in Fig. 1 (Haddad and Allaf 2007). The first part is the processing chamber where the moistened product is placed and exposed to steam pressure (up to $8 \times 10^5$ Pa) at high temperature (up to 170 °C) over a relatively short time (Fig. 1b). While the second part is a vacuum reservoir which is 50 times greater than the volume of the processing chamber (Fig. 1d). The third part is a valve with a large diameter (more than 200 mm) that can be opened rapidly in less than a second (Fig. 1c).

There are few stages involved in DIC drying which is shown in Fig. 2 (Pilatowski et al. 2010). At the first stage, the vacuum condition is created (Fig. 2a) followed by the next stage where the steam is injected to the material for several second (Fig. 2b and c). Then, the sudden pressure drop causes quick cooling of the treated material and massive evaporation of the water content (Fig. 2d). At the final stage, the pressure is released towards the atmospheric pressure (Fig. 2e and f).

The dehydration of DIC technique occurs by the instantaneous time for higher mechanical pressure and temperature which is related to PDE modeling with parabolic and elliptic combination. Hence, the rate of water removal from drying material is higher than the conventional drying and the abrupt pressure drop ($\Delta P/\Delta t > 0.5$ MPa/s) has been hypothesized to evaporate the water from inside to the outside of the material as liquid (Arenander and Wahren 1983). Thus, the benefit of this process is low energy usage

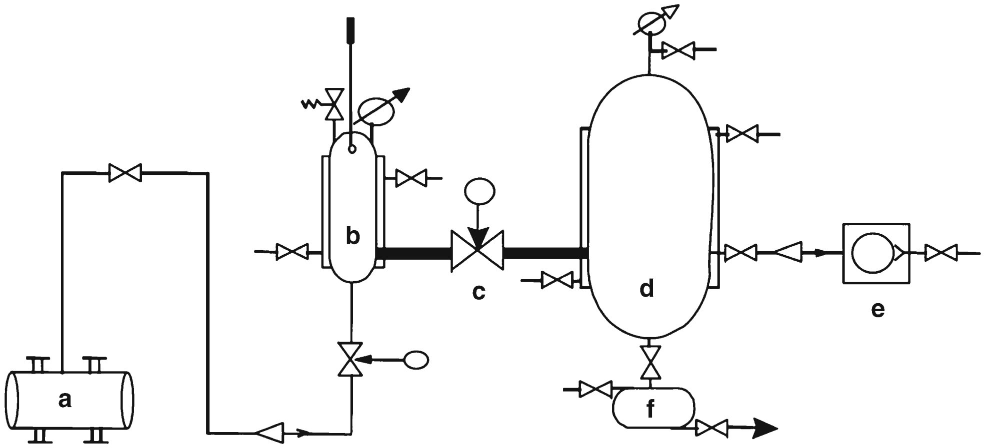


**Fig. 1** Schematic diagram of the DIC reactor which consists of (**a**) boiler, (**b**) DIC reactor, (**c**) decompression valve, (**d**) vacuum tank, (**e**) vacuum pump, and (**f**) condensation tank

and high commercial value of drying products in term of nutritional value, texture and product shape.

## Chronologies of the mathematical modeling and simulation

The mathematical modeling based on PDE with parabolic and elliptic types for multi-disciplinary applications were investigated by (Alias et al. 2009a, b, 2011a, b). The pioneer research on dehydration started in paper (Alias et al. 2009b). An experiment of the effect of DIC treatment on the phytate content equation of *Lupinus albus* was done by Haddad et al. (2007). The pilot result of dehydration process showed the phytate content decreased by 16 % after 1 min of DIC treatment as compared to steaming process where the phytate content decreased by 10 % after 30 min (De Boland et al. 1975). At the 5 % level of significance, α, the significant parameters for dehydration using DIC process are treatment pressure, processing time and initial water content. The phytate content equation of *Lupinus albus* is given by

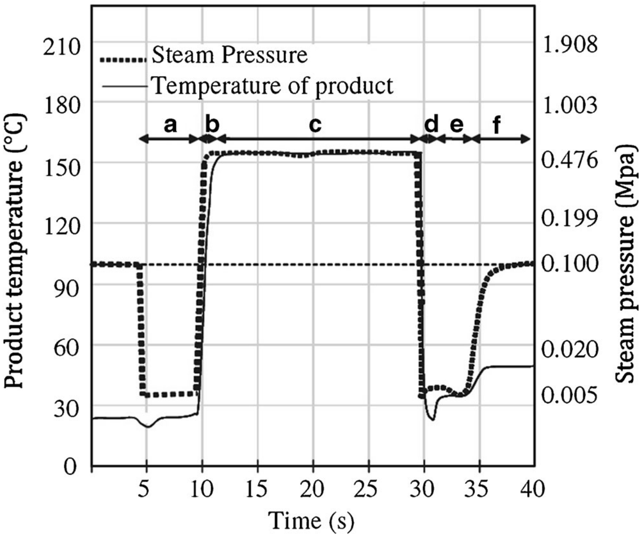


**Fig. 2** Temperature and pressure history of a cycle of the DIC at different stages: (**a**) initial vacuum, (**b**) saturated steam injection to reach the selected pressure, (**c**) constant temperature prior to saturated stem pressure, (**d**) abrupt pressure drop towards vacuum, (**e**) vacuum condition, and (**f**) return to atmospheric pressure

$$z(P, W, t) = a + bP + ct + dW + eP^2 + fPt + gPW + ht^2 + jtW + kW^2 \quad (1)$$

where constants $a$ to $k$ are shown in Table 1. The visualization of Eq. (1) is presented in Fig. 6. To increase the accuracy of the dehydration process prediction, Eq. (1) can be modified to 2D parabolic equation.

### Heat and mass transfer with 2D parabolic equation

The theoretical studies of heat and mass transfer with 2D parabolic equation are governed to describe the drying kinetics and heat conduction of food materials during the dehydration process using DIC technique. Many authors have studied the heat and mass transfer model to describe the conventional drying method (Wang and Brennan 1995; Ruiz-Lopez et al. 2004; Lagunez-Rivera et al. 2007; Ruiz-Lopez and Garcia-Alvarado 2007; Alias et al. 2009b; Villa-Corrales et al. 2010; Thuwapanichayanan et al. 2011; Ruiz-Lopez et al. 2012). The diffusion and capillary action of the mass transfer equation and heat transfer equations are referred to the Fick's second law and Fourier's law

**Table 1** The values of parameters for constants $a$ to $k$

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $a$ | 7.02534 | $f$ | −0.01375 |
| $b$ | 0.58773 | $g$ | 0.007080333 |
| $c$ | 0.132357 | $h$ | 0.00036531 |
| $d$ | 0.344761 | $j$ | −0.0028875 |
| $e$ | −0.00584031 | $k$ | −0.00272829 |

respectively. Four assumptions to solve the PDE with parabolic type are made based on Ramallo and Mascheroni (2011) and Villa-Corrales et al. (2010).

1. Solid temperature remained constant and equal to air temperature during drying process.
2. Uniform initial moisture and temperature distribution.
3. Fruits slice was considered as a thin slab of thickness $L_o=2l$. Both sides of the slice were exposed to uniform airflow at constant temperature.
4. External resistance to the mass transfer was negligible.
5. Moisture evaporation only for the upper surface.
6. Non-shrinkage and non-deformation of the slice.
7. Negligible radiation effects.
8. Moisture transfer inside the slice only by diffusion.

The 2D transient heat and moisture transfer equation which were governed from Villa-Corrales et al. (2010) are shown in Eqs. (2) and (3).

$$\frac{\partial M}{\partial t}=\frac{\partial}{\partial x}\left(D_{eff}\frac{\partial M}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_{eff}\frac{\partial M}{\partial y}\right) \tag{2}$$

$$\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\frac{k}{\rho C_p}\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k}{\rho C_p}\frac{\partial T}{\partial y}\right) \tag{3}$$

Given the initial and boundary conditions at $x=0$, $x=L$, $y=0$ and $y=H$ are as follows:
The initial conditions are:

$T(x,y,0)=T_o$ and $M(x,y,0)=M_o$.

The boundary conditions are:

$$\begin{aligned}&\frac{\partial T(0,y,t)}{\partial x}=\frac{\partial T(L,y,t)}{\partial x}=\frac{\partial T(x,0,t)}{\partial y}=0,\\&-h_{fg}\frac{\partial M(x,H,t)}{\partial t}=h_\infty A[T(x,H,t)-T_\infty]-kA\frac{\partial T(x,H,t)}{\partial y}\\&\frac{\partial M(0,y,t)}{\partial x}=\frac{\partial M(L,y,t)}{\partial x}=\frac{\partial M(x,0,t)}{\partial y}=0,\quad -D\frac{\partial M(x,H,t)}{\partial y}\\&\qquad=h_m[M(x,H,t)-M_\infty]\end{aligned} \tag{4}$$

The diffusion coefficient, $D_{eff}$ can be represented as

$$D_{eff}=D_o\exp\left(-\frac{E_a}{R(T+273)}\right). \tag{5}$$

## Heat and mass transfer with 2D elliptic-parabolic equation

There are numerous dependence and independence parameters involved in DIC process. Equation (2) and (3) is a coupled equation with different parameters. The changes parameters will observe the importance quality of the final product. The most influential parameters of the DIC technique are pressure, decompression time and the establishment of an initial vacuum (Louka and Allaf 2002, 2004). As presented in Heat and mass transfer with 2D parabolic equation section, the parabolic equations are used to generate the temperature and moisture behavior, and speed of air flow, without involving the main influential parameter which is pressure. This is the weaknesses of parabolic equations to represent the DIC process in correct manner.

To overcome this limitation, Alias et al. (2012) proposed an elliptic-parabolic equation where the model presented the dehydration process inside the food materials at high pressure. However, the parameters involved in the proposed model did not present the simultaneous heat and mass transfer movement. Therefore, we propose an elliptic-parabolic equation as Eqs. (6) and (7) where these equations represent the water removal and temperature behavior inside the food material during the dehydration at high pressure. Equations (6) and (7) involve pressure and weighted parameters to train the accurate prediction.

Elliptic-parabolic problems have been applied in many applications, for example, as a model of flow through porous media (Bear 1975; Diaz and de Thelin 1994); pressure equation in an injection molding process (Maitre 2002); and also in electromagnetic field theory (MacCamy and Suri 1987). The results from Eqs. (2) and (3) presented the differences with respect to time and space without pressure. Therefore, 2D heat and moisture transfer with elliptic-parabolic type are proposed which are presented in Eqs. (6) and (7).

$$\frac{\partial M}{\partial t}=D_{eff}\left(\frac{\partial M}{\partial x}+\frac{\partial M}{\partial y}\right)+\left(\frac{\partial P}{\partial x}+\frac{\partial P}{\partial y}\right) \tag{6}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\left(\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}\right)+\left(\frac{\partial P}{\partial x}+\frac{\partial P}{\partial y}\right) \tag{7}$$

The initial conditions are given as:

$$\begin{aligned}&M(x,y,0)=M_o, 0\le x\le 1, 0\le y\le 1,\\&T(x,y,0)=T_o, 0\le x\le 1, 0\le y\le 1.\end{aligned}$$

The boundary condition is given as:

$$P(x,y,t)=f(x,y,t), 0\le x\le 1, 0\le y\le 1, t\ge 0. \tag{8}$$

where $f$ is a known function. The heat and moisture transfer equations with parabolic and elliptic-parabolic types given by Eqs. (2), (3), (6) and (7) with related initial and boundary conditions are solved using FDM.

## Finite difference method

FDM is the focus discretization used for mathematical model chronology proposed in Heat and mass transfer with 2D elliptic-parabolic equation section. FDM is restricted to handle rectangular shapes and simple alterations. It is an approximation to the PDE.

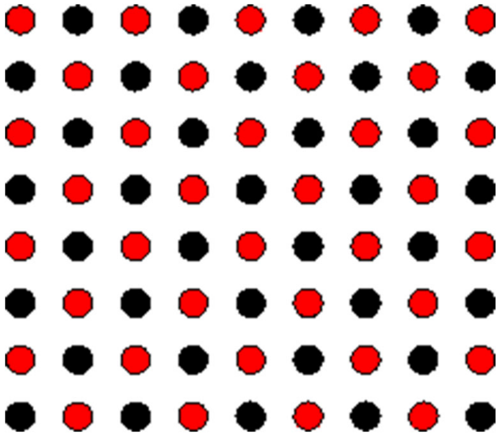


**Fig. 3** The grid for red and black points

### *Heat and mass transfer with 2D parabolic equation*

The discretization of Eqs. (2) and (3) are given by Eqs. (9) and (10) with $0 \le \theta \le \frac{1}{2}$ and $\frac{1}{2} \le \theta \le 1$,

$$\frac{M_{i,j}^{(k+1)} - M_{i,j}^{(k)}}{\Delta t} = \frac{D_{eff}}{(\Delta x)^2}\left[\theta\left(\delta_x^2 + \delta_y^2\right)M_{i,j}^{(k+1)} + (1-\theta)\left(\delta_x^2 + \delta_y^2\right)M_{i,j}^{(k)}\right] \tag{9}$$

and

$$\frac{T_{i,j}^{(k+1)} - T_{i,j}^{(k)}}{\Delta t} = \frac{k}{\rho C_p (\Delta x)^2}\left[\theta\left(\delta_x^2 + \delta_y^2\right)T_{i,j}^{(k+1)} + (1-\theta)\left(\delta_x^2 + \delta_y^2\right)T_{i,j}^{(k)}\right] \tag{10}$$

governing the 3 points formula;

$$\begin{aligned} &-\lambda\theta D_{eff}\left(M_{i+1,j}^{(k+1)} + M_{i-1,j}^{(k+1)} + M_{i,j+1}^{(k+1)} + M_{i,j-1}^{(k+1)}\right) \\ &\qquad + \left(1 + 4\lambda\theta D_{eff}\right)M_{i,j}^{(k+1)} \\ &= \lambda D_{eff}(1-\theta)\left(M_{i+1,j}^{(k)} + M_{i-1,j}^{(k)} + M_{i,j+1}^{(k)} + M_{i,j-1}^{(k)}\right) \\ &\qquad + \left[1 - 4\lambda D_{eff}(1-\theta)\right]M_{i,j}^{(k)} \end{aligned} \tag{11}$$

and

$$\begin{aligned} &\tfrac{-\lambda\theta k}{\rho C_p}\left(T_{i+1,j}^{(k+1)} + T_{i-1,j}^{(k+1)} + T_{i,j+1}^{(k+1)} + T_{i,j-1}^{(k+1)}\right) + \left(1 + \tfrac{4\lambda\theta k}{\rho C_p}\right)T_{i,j}^{(k+1)} \\ &= \tfrac{\lambda k}{\rho C_p}(1-\theta)\left(T_{i+1,j}^{(k)} + T_{i-1,j}^{(k)} + T_{i,j+1}^{(k)} + T_{i,j-1}^{(k)}\right) \\ &\qquad + \left[1 - \tfrac{4\lambda k}{\rho C_p}(1-\theta)\right]T_{i,j}^{(k)} \end{aligned} \tag{12}$$

where $i=1,2,\ldots,m$, $j=1,2,\ldots,m$ and $k=1,2,\ldots,T$ If $\Delta x=\Delta y$ then $\lambda = \frac{\Delta t}{(\Delta x)^2}$. Eqs. (11) and (12) can be considered for numerical simulation using three schemes. There are an explicit, Crank

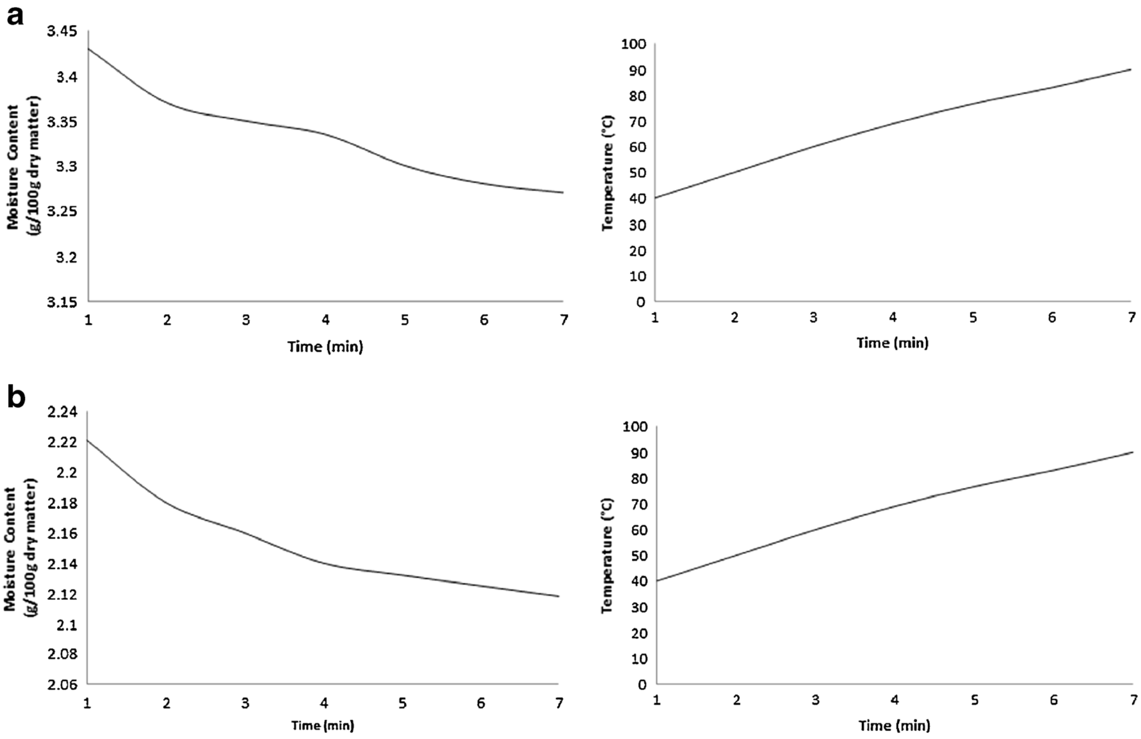


**Fig. 4** Experimental results for the moisture content and temperature behaviour of (**a**) *psidium guajava* and (**b**) *citrus suhueinsis*

Nicolson and implicit scheme. The equation depends on the value of variables, $\theta = 0, \frac{1}{2}$ or 1 respectively. Jacobi (JB), Gauss Seidel (GS), Red Black Gauss Seidel (RBGS) and Successive Over Relaxation (SOR) methods are the selected schemes for solving the discretization of Eqs. (11) and (12). The calculations of these methods are as follows:

(1) JB method is a simple and fundamental iterative method. Jacobi method computes the value of $M$ for each component respect to $x$ and $y$.

$$M_i^{(k+1)} = \left(b_i - \sum a_{ij} M_j^{(k)}\right) / a_{ii}, i = 1, 2, 3, \ldots, m.$$

(2) GS method is an enhanced version of the Jacobi method. The calculation of this method is as follows

$$M_i^{(k+1)} = \left(b_i - \sum_{j>i} a_{ij} M_j^{(k)} - \sum_{j<i} a_{ij} M_j^{(k+1)}\right) / a_{ii}, i = 1, 2, 3, \ldots, m.$$

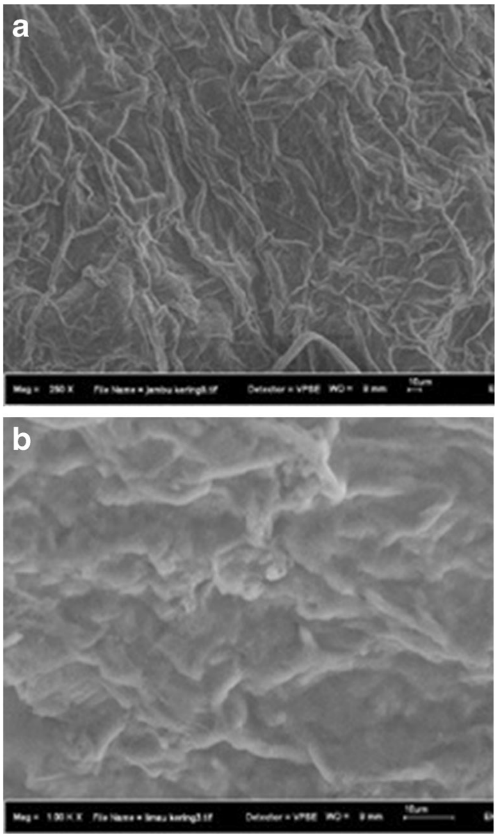


**Fig. 5** FESEM image for dried (**a**) *psidium guajava* and (**b**) *citrus suhueinsis*

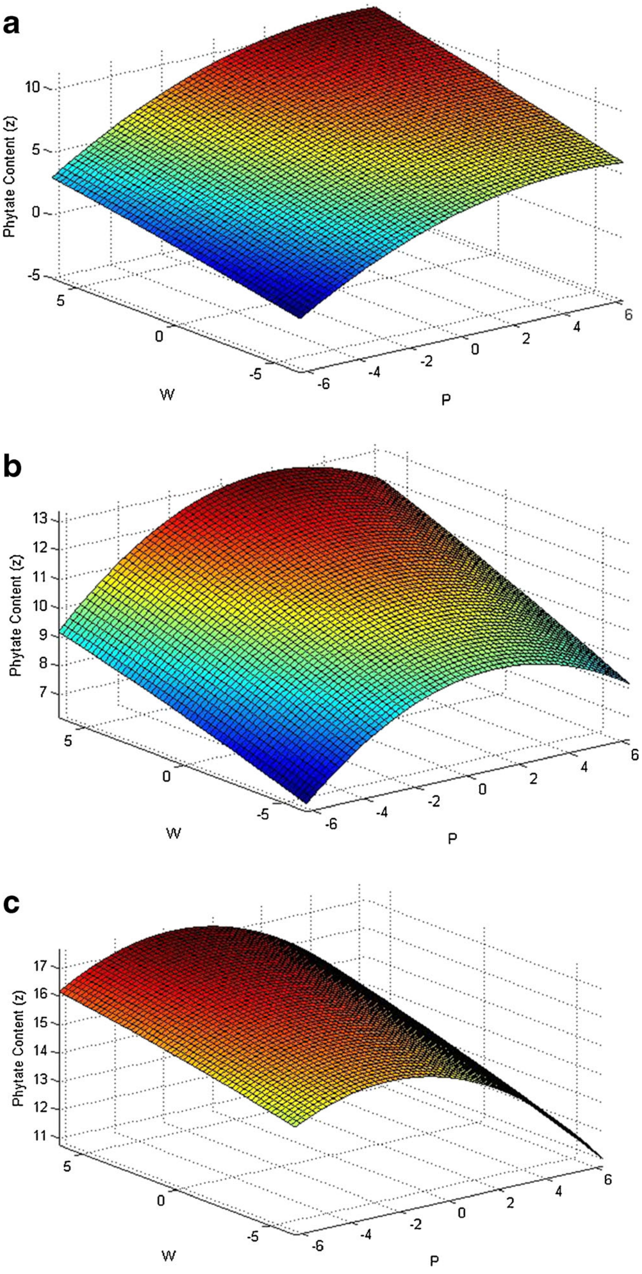


**Fig. 6** The visualization of the exact solution, Eq. (1) when (**a**) $t=1s$, (**b**) $t=30s$ and (**c**) $t=60s$

(3) RBGS method contains two subdomain, $\Omega^R$ and $\Omega^M$. Red point depends on the black point, and vice-versa. The loop starts by computing the odd points, from the bottom left and then going up to the next row and so on. When the all odd points are finished, do the black ones. The red black grid is illustrated in Fig. 3.

a. The grid calculation at $\Omega^R$

$$M_i^{(k+1)} = \left(b_i - \sum_{j>i} a_{ij} M_j^{(k)} - \sum_{j<i} a_{ij} M_j^{(k+1)}\right) / a_{ii}, i = 1, 3, 5, \ldots, m-1.$$

b. The grid calculation at $\Omega^M$

$$M_i^{(k+1)} = \left( b_i - \sum_{j>i} a_{ij} M_j^{(k)} - \sum_{j<i} a_{ij} M_j^{(k+1)} \right) / a_{ii}, i = 2, 4, 6, \ldots, m.$$

(4) SOR method is a variant of the GS method resulting in faster convergence. However, for $\omega$=1 the SOR method reduces to GS method. The formula is given by

$$M_i^{(k+1)} = (1-\omega) M_j^{(k)} + \omega \left( b_i - \sum_{j>i} a_{ij} M_j^{(k)} - \sum_{j<i} a_{ij} M_j^{(k+1)} \right) / a_{ii}, 0 < \omega < 2, i = `1, 2, 3, \ldots, m.$$

These methods are repeated until it reaches the stopping criterion such that $\left| M_i^{(k+1)} - M_i^{(k)} \right| \leq \varepsilon$ where *$\varepsilon$ is the convergence criterion.*

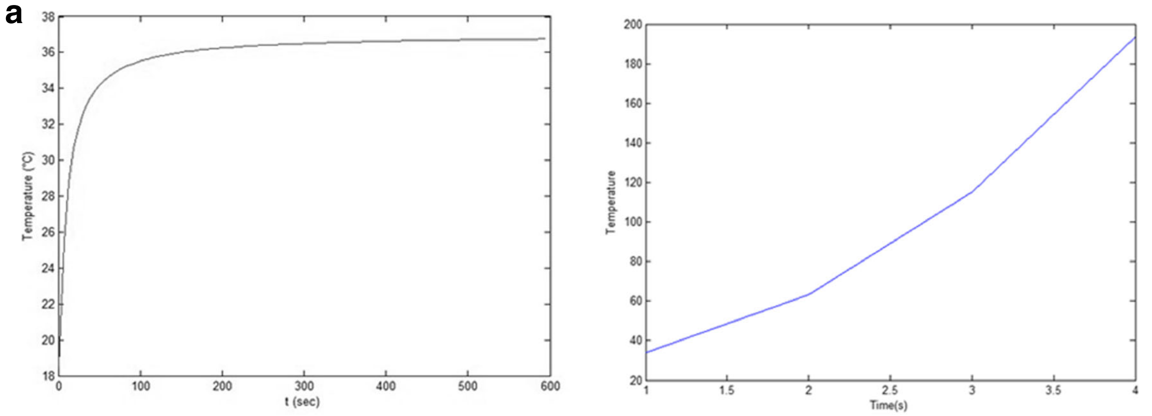
### *Elliptic-parabolic equation*

The discretization of Eqs. (6) and (7) which are using two points forward differences scheme becomes

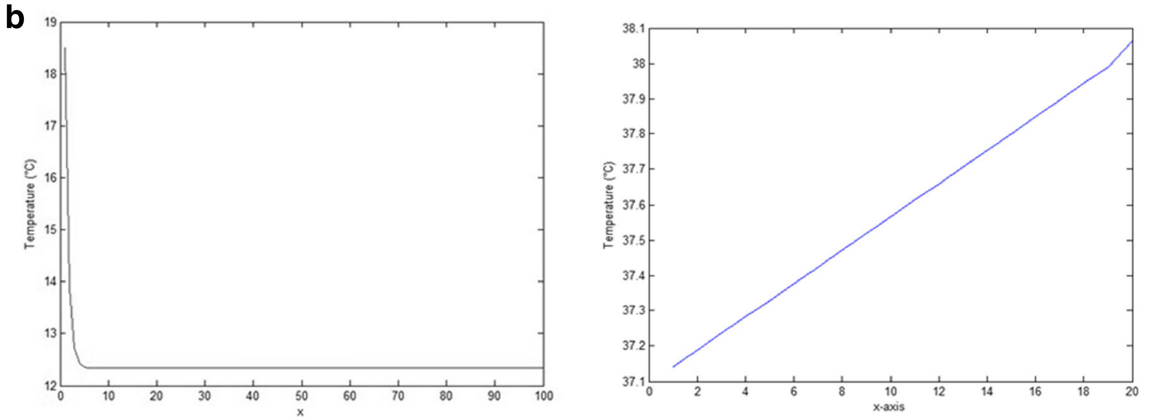
$$M_{i,j}^{(k+1)} = \lambda D_{eff} \left( M_{i+1,j}^{(k)} + M_{i,j+1}^{(k)} + P_{i+1,j}^{(k)} - 2P_{i,j}^{(k)} + P_{i,j+1}^{(k)} \right) + \left(1 - 2\lambda D_{eff}\right) M_{i,j}^{(k)} \tag{13}$$

and

$$T_{i,j}^{(k+1)} = \frac{\lambda k}{\rho C_p} \left( T_{i+1,j}^{(k)} + T_{i,j+1}^{(k)} + P_{i+1,j}^{(k)} - 2P_{i,j}^{(k)} + P_{i,j+1}^{(k)} \right) + \left( 1 - \frac{2\lambda k}{\rho C_p} \right) T_{i,j}^{(k)}. \tag{14}$$

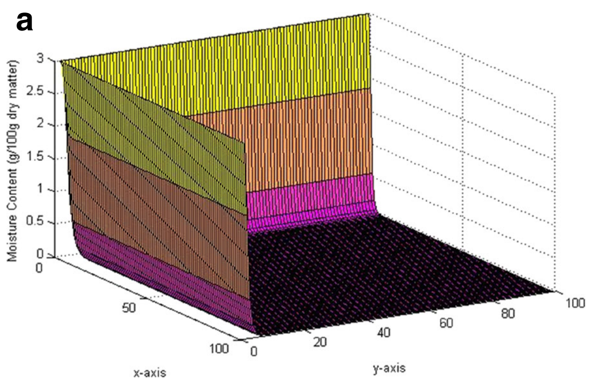


Equation 3 Equation 7

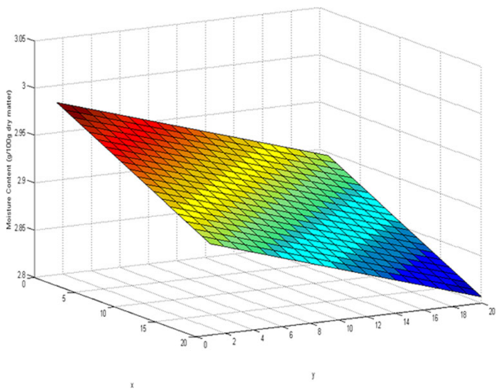


Equation 3 Equation 7

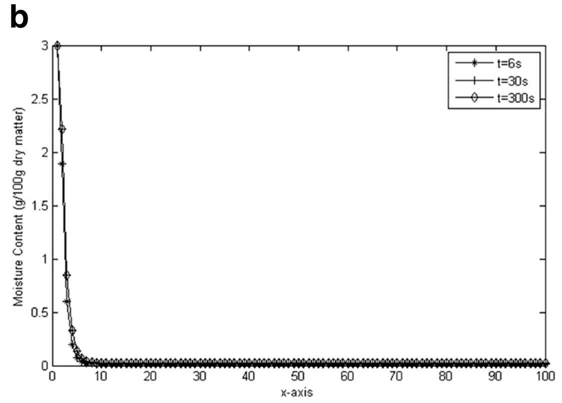
**Fig. 7** Temperature behavior inside the food material during drying for parabolic equation (Eq. 3) and elliptic-parabolic equation (Eq. 7) with respect to (**a**) time, and (**b**) x-axis



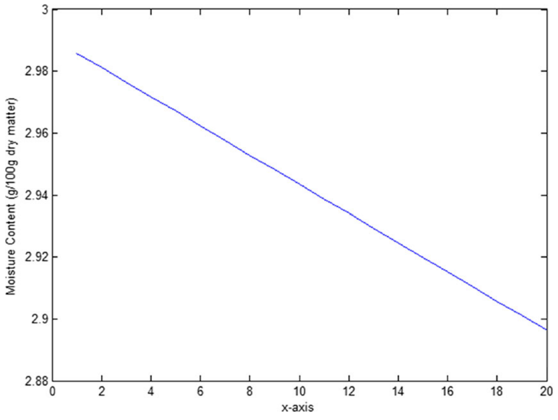
Equation 2



Equation 6



Equation 2



Equation 6

◀ **Fig. 8** Drying curves of moisture transfer inside the food material during drying for parabolic equation (Eq. 2) and elliptic-parabolic equation (Eq. 6) (**a**) in two dimensional and (**b**) for variation with drying time (seconds)

Equations (13) and (14) are solved by using JB and GS methods as in Heat and mass transfer with 2D parabolic equation section since both are first order equations.

## Conventional drying process

The chronology of DIC technique using mathematical modeling is compared from the laboratory experiments. The dehydration of some tropical fruits sample; the *psidium guajava* which is one of the guava fruit species and *citrus suhueinsis* as one of the sweet orange species can be referred to Fig. 4 for convective drying. Results are supported by Field Emission Scanning Electron Microscope (FESEM) image of two types of tropical fruits (Fig. 5).

## Results and discussion

The dehydration process of the exact solution (Eq. 1) is shown in Fig. 6. The figures show that the pressure and the water content decreased in respect of the increasing time. The model provides a good representation of the experimental measurements ($R^2$=81.23 %).

Estimated temperature behavior and water diffusivities for the parabolic Eqs. (2) and (3) and elliptic-parabolic Eqs. (6) and (7) are presented in Figs. 7 and 8. The higher the pressure inserted into the chamber, the higher the temperature, which promotes higher water mobility in food system. The platform for this computation is based on Intel®Core™2Duo processors with Linux operating system and C programming language. Table 2 shows the data configuration for the simulation of heat and mass transfer equation.

By comparing Figs. 6 and 8(a) for Eq. (6), the graphs show similarity in the form of the curve for moisture content. The moisture content during drying decreased linearly with respect to time. According to Chronologies of the mathematical modeling and simulation section, the saturated steam pressure is injected to reach the selected pressure. The increasing pressure causes the increase in temperature in a short time (Fig. 7). By comparing the graphs in Fig. 7, the elliptic-parabolic equation took less seconds to reach the high temperature. Thus, the elliptic-parabolic equation can be used as an alternative model to represent the dehydration process using DIC technique. The pressure can be presented by the combination of elliptic and parabolic equation where the elliptic approach highlights the fall of moisture content

**Table 2** Data configuration for simulation of heat and mass transfer equation

| | |
|---|---|
| $C_p$ | 1005.04 J/kgK |
| $D_{eff}$ | 1.06e-9 |
| $k$ | 0.576 W/mK |
| $L$ | 5 mm |
| $M_o$ | 3.0 kg/kg |
| $\rho$ | 1.141 kg/m$^3$ |
| $t$ | 600 s |

due to the high pressure. The elliptic equation will generate the solid, bounded and sharp curve while the parabolic equation control the time dependency and multidimensional of region. Comparing the graphs in Fig. 8, we can see that the graph become more precise in predicting the dehydration process by changing parameters.

### Numerical analysis

Based on the weighted finite difference approximation, a computer program was developed and implemented in C programming language on Linux based to simulate the food drying process using some numerical methods which are Jacobi, Gauss Seidel, Red Black Gauss Seidel and SOR methods. The CPU is supported by Intel®Core™ based on Dualcore processors. The initial and boundary conditions are stated in Chronologies of the mathematical modeling and simulation section while the properties of fruits and physical conditions are illustrated in Table 2. The numerical analysis of the weighted finite difference schemes for each method is illustrated in Table 3. The analysis of the iterative methods is based on the time execution, number of iteration, computational cost, maximum error and root mean square error (RMSE). The RMSE is a measure of the differences between the new values at the new time step and the old values at the previous time step. The formula for RMSE is given by

$$RMSE = \sqrt{\sum_{i}^{N} \left( M_i^{(k+1)} - M_i^{(k)} \right)^2 / N}.$$

From Table 3, SOR method shows the best performance which is more accurate and performs faster than RBGS, GS and Jacobi. SOR provides the lowest number of iteration that is 68 iterations and the shortest time of execution to converge which is 0.495658 s for mass simulation analysis. The accuracy method is determined by computing RMSE. The lowest value of RMSE represents the most accurate method. Absolute errors can also describe the accuracy of each method. It is shown that SOR is the most accurate method due to the smallest value of RMSE. Therefore, from the obtained results, SOR is the alternative method with the lowest computational complexity, time of execution and number of iterations.

## Conclusion

The chronology of mathematical model in DIC was started using regression model. The basic heat and mass transfer using 2D Fick's and Fourier's parabolic equation were governed to simulate the drying process inside the rectangular slices that only affected by temperature, humidity, speed of air flow and solid physical form. However, the parabolic

**Table 3** Numerical analysis of heat and mass transfer with parabolic equation for different iterativel methods based on the time execution, number of iteration, computational complexity (Comp. Comp.), maximum error and root mean square error (RMSE)

| Numerical analysis | | Mass simulation | | | |
|---|---|---|---|---|---|
| $N$=100, $\theta$=0.5 Tolerance=1.0e-5 | | JB | GS | RBGS | SOR |
| Time execution (s) | | 0.661715 | 0.575745 | 0.579582 | 0.495658 |
| No. of iteration | | 100 | 81 | 83 | 68 |
| Comp. | Mult. | 1800 | 1458 | 2988 | 1360 |
| Comp. | Add. | 1100 | 891 | 1826 | 884 |
| Maximum error | | 1.74628754e-4 | 9.87947188e-6 | 9.57743152e-6 | 8.83848514e-6 |
| RMSE | | 4.63781e-5 | 2.73411e-6 | 2.81913e-6 | 2.40718e-6 |
| Numerical analysis | | Heat simulation | | | |
| $N$=100 $\theta$=0.5 Tolerance=1.0e-5 | | JB | GS | RBGS | SOR |
| Time execution (s) | | 0.701816 | 0.691348 | 0.698586 | 0.697315 |
| Iteration | | 100 | 100 | 100 | 100 |
| Comp. | Mult. | 2600 | 2600 | 4200 | 2800 |
| Comp. | Add. | 1200 | 1200 | 2400 | 1400 |
| Maximum error | | 6.7355941e-3 | 1.25554675e-1 | 6.78360179e-2 | 1.26322808e-1 |
| RMSE | | 3.61165e-5 | 0.0406959 | 0.0210217 | 0.0204961 |

equations were not adequate for describing the moisture movement process and the temperature behavior during drying because the most crucial parameter of DIC was pressure. Therefore, the contribution of this paper was to improve the mathematical model from parabolic to elliptic-parabolic equation. By referring to the obtained results, the elliptic-parabolic equation was good to represent the DIC process involving pressure, water content, time dependency, dimension of region and temperature behavior parameters. The programming supported involving changes of parameters. The pressure can be presented by the combination of elliptic and parabolic equation where the elliptic approach highlighted the fall of moisture content due to the high pressure and the parabolic equation controlled the time dependency and multidimensional of region. The numerical results obtained have proved that the numerical methods were capable to visualize the heat and mass transfer during dehydration process. The future research for this study is to implement the parallel algorithm to simulate the mathematical model of food dehydration for large sparse problems. Parallel computing is an execution of the same task on multiple processors in order to obtain high speed and accurate predictions.

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