THE MACHINE EFFICIENCY OF
(*M*/*M*/1) MACHINE INTERFERENCE PROBLEM

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For my beloved family and Richard

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ABSTRACT

In many industrial processes a set of machines is served by a number of operatives. From time to time a machine breaks down and the operative repairs and restarts it. If the number of machine which stop exceeds the number of operatives, then some of these machines are forced to wait for repair and machine interference occurs. During the fifties and sixties, a number of empirical and semi-empirical interference formulae for the evaluation of diminished productivity have been suggested. In recent years a number of papers have appeared, dealing with the problem not only in calculating effective production rate but also in analysing the machine interference model. Therefore, this study is significant to the field of machine interference problem because it would extend the applicability of the theory of machine interference. The main objectives of this research are to illustrate the use of a theoretical solution of the machine interference problem by way of an example. Further, this research derives the equations for relationships between several factors affecting operative utilization or machine efficiency in a black box model, ($M/M1$) model and a patrolling system. This study shows that the number of machines (N) becomes an important factor not only in a black box model and ($M/M/1$) model but also in a unidirectional patrolled system, since the machine efficiency (E) is inversely proportional to the number of machines (N) (i.e. $E \propto \frac{1}{N}$). In ($M/M/1$) model, the method of calculating the machine efficiency when N becomes large has been simplified. Nevertheless, a more significant method can be developed in future research, where the simpler method for the calculation of the machine efficiency (E) when the number of machines (N) is large can be established.

ABSTRAK

Kebanyakan proses dalam industri melibatkan sebilangan mesin yang dikendalikan oleh sebilangan pengendali. Mesin-mesin tersebut mungkin akan rosak dan berhenti pada bila-bila masa sahaja. Justeru itu, pengendali akan bertugas untuk memperbaiki mesin-mesin tersebut dan membolehkannya berfungsi semula. Jika bilangan mesin yang rosak melebihi bilangan pengendali yang bertugas, mesin-mesin ini terpaksa menunggu giliran untuk diperbaiki. Ekoran daripada ini, gangguan mesin akan berlaku. Pada tahun 50-60an, rumus-rumus gangguan mesin untuk penafsiran penyusutan produktiviti telah dicadangkan secara empirikal dan separuh empirikal. Dewasa ini, terdapat kertas kerja yang bukan sahaja membincangkan masalah dalam pengiraan keberkesanan produktiviti, tetapi juga menganalisa model-model gangguan mesin. Oleh yang demikian, penyelidikan ini adalah amat bermanfaat untuk bidang-bidang yang berkaitan dengan masalah gangguan mesin kerana ia merupakan lanjutan kepada aplikasi teori gangguan mesin dalam industri. Objektif utama penyelidikan ini adalah menyelami penggunaan teori-teori gangguan mesin bagi menyelesaikan masalah gangguan mesin dalam situasi sebenar. Seterusnya, persamaan-persamaan untuk hubungan antara faktor-faktor yang mempengaruhi bilangan pengendali atau keefektifan mesin telah diperolehi bagi model kotak hitam, model ($M/M1$) dan model sistem rondaan. Penyelidikan ini menunjukkan bahawa bilangan mesin (N) merupakan suatu faktor yang penting bukan sahaja dalam model kotak hitam dan model ($M/M/1$) bahkan dalam model sistem rondaan searah (*unidirectional*). Ini kerana keefektifan mesin (E) adalah berkadar secara songsang kepada bilangan mesin (N) (iaitu $E \propto \frac{1}{N}$). Dalam model ($M/M/1$), proses pengiraan keefektifan engin telah dipermudahkan apabila bilangan mesin (N) menjadi besar. Walau bagaimanapun, kaedah yang lebih berkesan mungkin boleh diperolehi melalui penyelidikan selanjutnya.

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LIST OF SYMBOLS

M	-	Poisson arrival or departure distributions (or exponential inter arrival or service-time distribution)
m	-	Number of operatives
N	-	Number of machines
GI	-	General independent distribution of arrivals (or inter arrival times)
G	-	General distribution of departures (or service times)
P	-	Operative utilization
E	-	Machine efficiency
T	-	Time intervals (or shift time)
X	-	Random variable represents the time between successive arrivals to a given system
λ	-	Mean number of events occurring in a unit time interval (or mean number of stoppage occurred per unit running time per machine)
t	-	Time
μ	-	Expected service time
ρ	-	Servicing factor
T_o	-	Operative works on repair
T_r	-	Machines running
c	-	Mean repair time
n	-	Number of stoppage (or machines are stopped at time t)
k	-	Mean walking time
L	-	A round as starting at the point where the operative leaves a machine
R	-	Machines are stopped of the other $(N-1)$ machines

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CHAPTER I

INTRODUCTION

1.1 Introduction

In this world, everyone has the experience of queuing. The *Oxford Dictionary* defines that the word "queues" as a line of people, vehicles, etc., waiting for something or to do something (Christina Ruse, 1990). While this word has a somewhat negative connotation stemming from its association with waiting, it is a familiar experience of everyday life (Dshalalow, 1995). Drivers wait in line in their cars in traffic jams or at toll booths or at petrol stations; patients waiting for doctors at hospitals; customers wait in line at supermarkets and restaurants and so on. The time loss and the delays occur. This not only causes customers personal inconvenience, but also results in additional cost to the business. These are especially true in the case of machines in queue, waiting to be repaired.

1.2 Research Background

The term "machine interference" is generally used to describe a field of the application of the queuing theory. In many industrial processes a set of machines is served by a number of operatives. From time to time a machine breaks down and the operative repairs and restarts it. If the number of machines stop exceeds the number of operatives, then some of these machines are forced to wait for repair. Hence, in addition to the normal loss due to time spent in servicing machines, there is also the interference loss due to the time spent in waiting for the operative to reach the broken-down machine. The common objectives of the theory is to investigate the operative utilization and the machine efficiency for a given set of assumptions regarding the number of machines and the distributions of running repair times.

1.3 Problem Statement

In many industrial applications, the machine interference problem becomes important and a number of empirical and semi-empirical "interference formulae" for the evaluation of diminished productivity have been suggested. Palm (1947) gave a theoretical solution based on a very reasonable model. This solution is applicable to any number of machines and operatives. Feller (1950), following Palm's analysis, showed the machine interference problem to be a special case of the steady state "birth and death processes". Ashcroft (1950) presented the machine efficiency for variable repair times and extensive tables for constant repair times. Benson and Cox (1951) investigated the solution of the machine interference problem under more general assumptions. They have considered a practical situation, assumed exponentially distributed repair times and produced tables of machine efficiency and operative utilization in (*M*/*M*/1) and (*M*/*M*/*m*) machine interference model.

These papers assumed that the machines are repaired in the order in which they break down. Thus they have excluded any patrolling system, which might be convenient for the operative. Such patrolling problems, discussed qualitatively by Brunnschweiler (1954) were first solved quantitatively by Mack *et al*. (1957) who came out with a table of efficiency in the case of constant walking and repairing time in a unidirectional patrolling. Meanwhile Mack (1957) considered constant walking time but variable repair time.

However the solution of these papers are complicated when the problem involves a larger number of machines running. Therefore a more simple theoretical solution is suggested in the calculation of machine efficiency. There is no comparison between a patrolling system and non-patrolling system. In fact, this is necessary in order to determine the factors which affect these system. There are the factors which affecting both of these systems, but some only affecting one of these systems. Then a comparison is done between the values of machine efficiency in these systems. In the patrolling system, suggested theoretical solution is not applied to a real example. Thus several examples are created not only in a non-patrolling system but also a patrolling system to illustrate the use of the related theoretical solution.

1.4 Research Questions

The following questions arise.

i. How to estimate the loss of production when the number of machine is large (i.e. $N \to \infty$) in a simpler way?

ii. What are the factors affecting the machine efficiency?

iii. How to apply the theoretical solution in the real machine interference problems?

1.5 Research Objectives

The main objectives of this research are as follows.

i. To simplify or modify the method of calculating the machine efficiency in $(M/M/1)$ model when the number of machine is large (i.e. $N \to \infty$).

ii. To derive equations of relationships between several factors and operative utilization or machine efficiency in a black box model, $(M/M/1)$ model and a patrolling system.

iii. To illustrate the use of theoretical solution of machine interference problem by a real example in:

(a) a black box model,

(b) ($M/M/1$) model, where machines running times and repair times are of negative exponential distribution, and

(c) ($M/M/1$) model where the machine unidirectional patrolled by one operative when walking time and repair time are constants.

1.6 Research Scope

The study mainly focuses on the machine efficiency in a black box model and ($M/M/1$) machine interference model. *T*-shift, operative repair time, total machines running time and number of machines running are considered in a black box model. Even though the number of machines running and servicing factor play a major role in ($M/M/1$) model, these factors are discussed only in a certain range. Meanwhile a regular patrolling system is observed where the constant walking time and repair time of an operative, machine running time and number of machines are being considered.

1.7 Research Significance

In recent years a number of papers have appeared dealing with the problem not only in calculating effective production rate but also in analysing the machine interference model in a more complicated way. For example, Hsieh Yi-Chih (1996) investigated multiple machine repair problems with one repairman. Then, he described a procedure to determine the optimal number of machines, which should be assigned to a repairman. Meanwhile Gupta (1996) discussed the machine interference problem with warm spares, server vacations and exhaustive service. Wai Ki Ching (1998) considered repairing models (more than one phase) for production systems. Jaejin Jang *et al.* (1999) presented a new procedure to estimate waiting time in ($GI/G/2$) system.

The above examples have revealed clearly that research on machine interference problem conducted in the fifties and sixties are valuable for further expansion on the theoretical and applicability aspects. Therefore, this study is significant to the field of machine interference problem because it extends the applicability of the theory of machine interference.

1.8 Organization of the Dissertation

In Chapter II, the queuing theory is introduced generally. First, the history of queuing theory is presented. The presentation covers the application of queuing theory in the machine interference problem. Secondly, the applications of queuing theory are discussed. Finally, the components of queuing system are explained systematically.

Poisson process is considered since the machine interference problem discussed in this thesis assumes that the arrivals follow a Poisson distribution and the service time is exponentially distributed. This is essentially White *et al.* (1975) s' work, but Bunday (1986, 1996) showed the arrival pattern and the service mechanism in a detail, which is important in a machine interference problem. Subsequently the theoretical development of steady-state is discussed because the solution of queuing problems is considered in the steady-state results.

A comparison of the basic assumptions on the machine interference problem used by various authors is shown. These assumptions are quite similar to the assumption used by Bunday (1986, 1996).

In Chapter III, theoretical development of the machine interference problem is given for the following parts:

(a) a black box model,

(b) ($M/M/1$) model, where machine running time and repair time follows the negative exponential distribution, and

(c) ($M/M/1$) model where the machine unidirectional patrolled by one operative when walking time and repair time are constants.

There are two parts in Chapter IV. In section 4.1, several real examples are suggested and solved by utilizing the theoretical solutions in different cases. Factors which affect the operative utilization (P) and machine efficiency (E) are observed. Then *Microsoft Excel* and *Matlab* programmes are used to simplify the calculation to obtain related tables and graphs. In section 4.2, the results obtained from experiments presented in section 4.1 are discussed. These results are used to obtain the relationship between the machine efficiency and the related factors. Then, numerical results of machine efficiency obtained from different method when the number of machines approaching infinitive are compared. Furthermore, the numerical results of machine efficiency in a patrolling system and non-patrolling system are compared.

Finally, in Chapter V conclusions and recommendation is established base on the results and analyses given in previous chapters.

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