COMPUTATIONAL MODELLING AND SIMULATION OF BIOMAGNETIC FLUID FLOW IN A STENOTIC AND ANEURYSMAL ARTERY

NURSALASAWATI BINTI RUSLI

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

MARCH 2013

TO MY BELOVED HUSBAND, MOM, DAUGTHERS AND FAMILY

ACKNOWLEDGEMENTS

First of all, thanks to Almighty Allah S.W.T for graciously bestowing me the perseverance to undertake this research. I sincerely have to thank all those who have supported and helped me writing this report. I wish to express my deepest gratitude to my supervisors, Professor Dr. Norsarahaida Saidina Amin, Dr. Airil Yasreen Mohd Yassin and Dr. Ahmad Kueh Beng Hong whose valuable time, expertise and guidance have been critical to my success.

I also would like to extend my sincere appreciation to all my friends who kindly provided valuable and helpful comments in the preparation of the thesis, and to those who have involved directly or indirectly in the preparation of this thesis, whom I have not mentioned above. Special thanks to Erwan Hafizi Kasiman for his willingness to be my *sifu*.

I wish to express deep appreciation and gratitude to Assoc. Prof. Dr. Sasa Kenjeres from Delft University of Technology, The Netherlands for fruitful discussions. Of course, special appreciation goes to two important institutions which have supported me, Universiti Malaysia Perlis (UniMAP) and Kementerian Pengajian Tinggi (KPT). Special thanks to my Dean and ex-Dean, Assoc. Prof. Dr. Abdul Wahab Jusoh and Assoc. Prof. Abdull Halim Abdul for their support and understanding throughout this study. I also deeply thank my parents and my family for their encouragement and patient support given in the preparation of this work.

To my husband, Muhamad Rasyidi Husin, for providing the necessary atmosphere of understanding and support during untold amount of hours at home required for writing this thesis. To my children, Husna, Nabilah, Hana and Danial, for your understanding and taking too much time to complete this research. To all of you, I say a most heartfelt thank you.

ABSTRACT

This thesis is concerned with the development of mathematical models for the computation and simulation of biomagnetic fluid flow in either a stenosed or aneurysmal artery in the presence of non-uniform magnetic fields. The mathematical models take into account blood rheology where blood behaves as a magnetic fluid due to the complex interaction of intercellular protein, cell membrane and haemoglobin. The present study involves steady three-dimensional biomagnetic fluid flow and mass transfer through a horizontal cylinder. Numerical solutions obtained through a finite volume method shows that the magnetisation force has an effect on the fluid concentration and exerts a greater influence on the secondary flow motions compared to that caused by the Lorentz force. Earlier studies allow only Lorentz force due to uniform magnetic fields but neglect the effect of magnetisation force. Here, a modified finite difference method has also been designed specifically to study separately the effect of magnetisation force, Lorentz force or both, on a steady two-dimensional channel flow. It is observed that a distorted asymmetric flow profile is presented near the magnetic source and that the Lorentz force gives little influence to the flow behaviours, vindicating previous proposition that the Lorentz force need not be considered in the model of biomagnetic fluid flow.

ABSTRAK

Tesis ini berkaitan pembangunan model matematik untuk pengiraan dan simulasi bagi aliran bendalir biomagnet di dalam arteri berstenosis atau beraneurisma di bawah pengaruh medan magnet tak seragam. Model matematik ini mengambil kira reologi darah bahawa darah bertindak sebagai suatu bendalir magnet disebabkan oleh interaksi kompleks antara sel protein, sel membran dan hemoglobin. Kajian ini melibatkan aliran mantap tiga matra bagi bendalir biomagnet melalui silinder mendatar dengan pemindahan jisim. Penyelesaian berangka yang diperoleh menggunakan kaedah isipadu terhingga menunjukkan daya pemagnetan memberi kesan kepada kepekatan bendalir dan mempunyai lebih pengaruh terhadap aliran sekunder berbanding aliran yang teraruh oleh daya Lorentz. Kajian sebelum ini tidak mempertimbangkan kesan daya pemagnetan, sebaliknya hanya mempertimbangkan daya Lorentz yang terhasil daripada medan magnet seragam. Di sini, kaedah beza terhingga telah diubahsuai khususnya untuk mengkaji secara berasingan kesan daya pemagnetan, daya Lorentz atau kedua-duanya terhadap aliran mantap dalam saluran dua matra. Hasil kajian menunjukkan profil aliran tak simetri berubah berhampiran sumber magnet, dan daya Lorentz pula mempunyai pengaruh yang kecil kepada kelakuan aliran. Keputusan ini mengesahkan hujah sebelum ini bahawa daya Lorentz boleh diabaikan bagi model aliran bendalir biomagnet.

TABLE OF CONTENT

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOLS:

ρ - Density
$\bar{u}$ - Velocity vector
∇ - Gradient operator
$\frac{D}{Dt}$ - Material derivatives
T - Temperature
Sh - Sherwood number
Sc - Schmidt number
ϑ - Diffusion constant
ΔC - Reference concentration difference
c - Concentration
Φ - Dissipation function
σ - Stress
τ - Viscous stress tensor
P - Pressure
μ - Dynamics viscosity
ν - Kinematic viscosity
χ - Magnetic susceptibility
M - Magnetisation
$\mathcal{L}$ - Differential operator
Re - Reynolds number
$\bar{\beta}$ - Thermal expansion
$\bar{g}$ - Acceleration due to gravity
E_c - Eckert number
ε - Temperature number

Mn_F - Magnetic number due to FHD
Mn_{M} - Magnetic number due to MHD
$\mathbf{F}^L$ Lorentz force
$\mathbf{F}^M$ Magnetisation force
$\overline{\sigma}$ - Electrical conductivity
$\overline{c_p}$ - Specific heat
k - Thermal conductivity
$\overline{h}$ - Reference length
$\overline{u_r}$ - Reference velocity
$\overline{H_r}$ - Reference magnetic field intensity
$\overline{H}$ - Magnetic field gradient
ϕ - Electrical potential
C_f - Skin friction coefficient
N - Stuart number

ABBREVIATIONS :

CFD - Computational fluid dynamics
BFD - Biomagnetic fluid dynamics
FDM - Finite difference method
FVM - Finite volume method
FEM - Finite element method
MDT - Magnetic drug targeting
FHD - Ferrohydrodynamics
MHD - Magnetohydrodynamics
SIMPLE - Semi-Implicit Method for Pressure-Linked Equations
TDMA - Tri-Diagonal Matrix Algorithm
ECG - Electrocardiogram
LDL - Low density lipoprotein
WSS - Wall shear stress

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

Blood is essential in maintaining life. It transports oxygen and nutrients to all parts of the body, relays chemical signals and moves metabolic waste to the kidneys for elimination. Despite more than 150 years of closed study, a concise, predictive model of blood flow is still of far reaching. A quantitative model of blood flow is important not only as it relates to clinical diagnosis of disease, but as an integral component of models of more complex structures. In this introductory chapter, some physiological background is provided in order to pave the way for the mathematical modelling and numerical simulations to be developed in later chapters.

To date, numerous mathematical models have been developed to describe blood flow in the circulatory system (Mahapatra *et al.* (2002), Midya *et al*. (2003), Maikap *et al*. (2005), Layek and Midya (2007), Chakravarty *et al.* (2007)). Following such a tradition but looking at different aspect, this thesis adopts the biomagnetic fluid model in representing the blood flow. A biomagnetic fluid is defined as a fluid that exists in a living creature and its flow is influenced by the presence of a magnetic field. The main reason in choosing this model to describe blood flow comes from the fact that blood behaves as a magnetic fluid (Tzirtzilakis, 2005). Due to the complex interaction of the intercellular protein, cell membrane and the hemoglobin, a form of iron oxides presents at a uniquely high concentration in the mature red blood cells (erythrocytes). Its magnetic property is affected by factors such as the state of oxygenation (Higashi *et al.* 1993).

During the last decades, extensive research has been done on the fluid dynamics of biological fluids in the presence of magnetic field. Numerous useful applications have been proposed in bioengineering and medical sciences (see Ruuge and Rusetski (1993), Plavins and Lauva (1993), Haik *et al*. (1999)). Among them are the development of magnetic devices for cell separation, targeted transport of drugs i.e. using magnetic particles as drug carriers, magnetic wound treatment and cancer tumor treatment, reduction of bleeding during surgeries and provocation of occlusion of the feeding vessels of cancer tumors and development of magnetic tracers.

It is imperative to acknowledge that mathematical modelling and numerical simulations provide many important insights on the underlying interactions between blood flow and magnetic field, some of which are not directly assessible through experimental investigation. Computational simulations make possible the study of the feasibility of a medical technique before entering clinical trials, and simulations are useful for investigating the influence of various factors independently (Haverkort and Kenjeres (2008), Haverkort *et al.* (2009)). Since the human blood is slightly electrically conductive, it is important to formulate a mathematical model that mimics properly the effects of magnetisation and Lorentz force on the blood flow (Kenjeres and Opdam, 2009). The application of biomagnetic fluid will be considered in this research because the model is more realistic from the physiological point of view.

1.2 Problem Statement

Computational fluid dynamics (CFD) occupies one of the key physical disciplines that involve the description of fluid flow in terms of mathematical models which comprise convective and diffusive transports of matters. New techniques are needed when problems in real environments get more difficult and complex. Such complexities are encountered especially when the behavior of fluid in motion is no longer governed by the Navier-Stokes equations alone. In considering more realistic behaviour of the flow, such as biomagnetic fluid model in this study, other effects must be considered and this usually results in coupled partial differential equations.

One of the major difficulties in solving such a coupled problem is the handling of pressure terms in which singularity occurs. Having stated aforementioned matters, this study is therefore very general nature in regard to interactions between mathematical model, numerical simulation and blood flow behaviour in oversimplified geometries (artery is never a perfectly circular straight tube). Even so, we put concentration on the effects of two-dimensional and three-dimensional biomagnetic blood flow models when being imposed with non-uniform magnetic field. There are several issues pending to be addressed, all of which are stated in the followings. What are the effects on blood flow characteristics of anomalous geometry in stenosis and aneurysm compared to normal arteries in presence of magnetic field? What are the severities of the oxygen distribution along the artery for both geometries? What are the effects of aforementioned cases under the influence of non-uniform magnetic field? What are the effects on blood flow patterns when only magnetisation force is imposed? What happens to the flow when magnetisation and Lorentz forces are imposed? These research questions constitute the main concern of the present thesis.

1.3 Objectives of Research

The main concern of current study is to investigate the two-dimensional and three-dimensional blood flow through an artery by solving the following problems:

1. To investigate the blood flow patterns considering a three-dimensional channel flow in horizontal cylinder, cylindrical geometry with significant narrowing (mimicking stenosis) and cylindrical geometry with significant swelling (mimicking aneurysm) allowing magnetisation and Lorentz forces.
2. To determine the effect of mass and flow transport phenomena associated with disease on geometries stated in (1).
3. To derive and solve Navier Stokes differential equation allowing magnetisation force description using finite difference method considering a two-dimensional channel.

4. To derive and solve Navier Stokes differential equation allowing magnetisation and Lorentz forces description using finite difference method considering a two-dimensional channel.

1.4 Scope of Research

This study considers the nonlinear, steady, two-dimensional and three-dimensional blood flows through an artery in the presence of magnetic field. In addition, the symmetric and three-dimensional cylinder, stenotic and aneurysmal blood flows through an artery are investigated. It is imperative to emphasize also that this study only considers single horizontal flow without branching effect, treating all material properties as constant. In addition to the neutral conditions (0 Tesla), the investigated strengths of magnetic field include 5 Tesla and 10 Tesla. The finite volume method (FVM) will be used as the numerical technique for three-dimensional models. For two-dimensional modelling, a modified numerical algorithm based on the finite difference method (FDM) will be adopted.

1.5 Significance of Research

The significance of the study can be stated as follows:

1. To present more realistic mathematical models to interpret and enhance the understanding of blood flow characteristics under stenotic and aneurysmal conditions through mathematical behaviour.

2. The establishment of new numerical techniques will pave the way for the solution of more difficult bio-fluid and mathematical problems.

3. For diagnostic purposes, this study would help molding the fundamental understanding of bio-fluid behaviour and contribute to the advancement of medical science.

4. Development of simulation and visualization packages of real-time solution for application in medicine for physicists and biomedical engineers.

1.6 Outline of the Thesis

This thesis is divided into seven chapters including this introductory chapter. Chapter 1 presents some general introduction, research background, objectives, scope and significance of research. All the problems in this study are based on the Navier Stokes equations which is associated with varying magnetic field. Chapter 2 contains the general literature review on the significance of this study. Considered problems will be discussed in four chapters, namely Chapters 3 through 6. All of these problems are solved numerically using either the finite volume method or finite difference method.

Chapter 3 discusses the blood flow patterns allowing magnetisation and Lorentz forces through a horizontal cylinder, cylindrical geometry with significant narrowing (mimicking stenosis) and cylindrical geometry with significant swelling (mimicking aneurysm). This chapter is divided into nine main sections, content of which includes the introduction, literature review, governing equations, numerical model description, numerical method, verification, steady blood flow through an artery subjected to the external magnetic field, flow distribution of velocity in two-dimensional view and conclusions. The discussion is focused on the effects of blood flow patterns when being imposed with different strengths of magnetic field which is 0 Tesla, 5 Tesla and 10 Tesla. The presentation of problem takes the form of a three-dimensional channel flow solved using a finite volume method.

In Chapter 4, problem related to the effect of mass and flow transport phenomena will be discussed. This problem is an extension from the first problem (Chapter 3) which also involves magnetisation and Lorentz forces. This chapter contains six sections. They are introduction, literature review, governing equations, numerical method, results and discussions and conclusions. In this chapter, the discussion concentrates on the effect of mass and flow transport phenomena associated with disease on geometries stated in Chapter 3 when exposed to various magnetic field values.

Chapter 5 explains a two dimensional channel flow allowing magnetisation force. This chapter is divided into seven sections: the introduction, literature review, governing equations, numerical method, verification, results and discussions and conclusions. A modified finite difference scheme based on staggered grid will be used to derive and solve Navier-Stokes differential equation. Three benchmark cases were considered for verification, i.e. lid driven cavity flow, Pouiselle flow and backward facing step flow. The discretization of the Navier Stokes equations allowing magnetisation force description in the numerical computation part is given afterward. This method has been found to be suitable and flexible to deal with varying magnetic field i.e. magnetisation force.

Finally, the last problem is discussed in Chapter 6 which concentrates on the effects of Lorentz force when associated with magnetisation force from Chapter 5. This chapter is divided into seven sections: the introduction, literature review, governing equations, numerical method, verification, results and discussions and conclusions. In this chapter, Navier-Stokes differential equation allowing magnetisation and Lorentz forces description will be derived and solved using the same method as described in Chapter 5. The model verification and validation are carried out in terms of results comparison with the analytical solution of 2D Hartman flow. The final and concluding chapter, Chapter 7, contains some concluding remarks, summary of research as well as some suggestions for future research.

REFERENCES

Alexiou, C., Arnold, W., Klein, R.J., Parak, F.G., Hulin, P., Bergemann, C., Erhardt, W., Wagenpfeil, S. and Lubbe, A.S. (2000). Locoregional Cancer Treatment with Magnetic Drug Targeting. *Cancer Research*. 60, 6641-6648.

Alexiou, C. Schmidt, A., Klein, R. Hulin, P., Bergemann, C. and Arnold, W. (2002). Magnetic Drug Targeting: Biodistribution and Dependency on Magnetic Field Strength. *Journal of Magnetism and Magnetic Material*. 252, 363-366.

Alexiou, C., Jurgons, R., Schmid, R., Hilpert, A., Bergemann, C., Parak, F., and Iro, H. (2005). In vitro and in vivo investigations of Targeted Chemotherapy with Magnetic Nanoparticles. *Journal of Magnetism and Magnetic Materials*. 293, 389-393.

Alexiou, C. Klein, R. J., Jurgons, R., Kremer, M., Wanner, G., Bergemann, C., Huenges, E., Nawroth, T., Arnold, W. and Parak, F.G. (2006). Targeting Cancer Cells: Magnetic Nanoparticles as Drug Carrier. *European Biophysics Journal*. 35, 446-450.

Alexander, D.L. (2011). Towards Numerical Modelling of Low-Density Lipoprotein Transport through the Multi-Layered Arterial Wall, Master's thesis, Delft University of Technology.

Anderson, J.D. (1995). *Computational Fluid Dynamics : The Basic with Applications*. McGraw-Hill, Inc.

Andersson, H.I., Halden, R. and Glomsaker, T. (2000). “Effects of surface irregularities on flow resistance in differently shaped arterial stenoses.” *Journal of Biomechanics*. 33, 1257-1262.

Armaly, B.F., Durst, F., Pereira, J.C.F., Schonung, B. (1983). Experimental and Theoretical Investigation of Backward-facing Step Flow. *Journal of Fluid Mechanics*, 127, 473-496.

Aviles, M.O., Ebner, A.D., Chen, H., Rosengart, A.J., Kaminski, M.D., and Ritter, J.A. (2005). Theoretical Analysis of a Transdermal Ferromagnetic Implant for Retention of Magnetic Drug Carrier Particles. *Journal of Magnetism and Magnetic Materials*. 293(1), 605-615.

Baek, H., Jayaraman, M.V., Richardson, P.D., and Karniadakis, G.E. (2010). Flow Instability and Wall Shear Stress Variation in Intracranial Aneurysms. *Journal of The Royal Society Interface*. 7(47), 967-988.

Barber, R.W., Fonty, A. (2003). Comparison of Vortex-element and Finite-volume Simulations of Low Reynolds Number Flow over a Confined Backward Facing Step. Proceedings of 11th Annual Conference of the CFD Society of Canada, CFD2003, 2, 780-787.

Bartoszek, M. and Drzazga, Z. (1999). A Study of Magnetic Anisotropy of Blood Cells. *Journal of Magnetism and Magnetic Materials*. 196-197, 573-575.

Bashtovoy, V.G., Berkovsky, B.M. and Vislovich, A.N. (1988). *Introduction to Thermomechanics of Magnetic Fluids*. United Stated of America: Hemisphere Publishing Corporation.

Bozeman, J.D. (1973). Numerical Study of Viscous Flow in a Cavity. *Journal of Computational Physics*. 12, 348-363.

Bruneau, C.H. and Saad, M. (2006). The 2D lid-driven cavity problem revisited. *Computers & Fluids*, 35, 326-348.

Burggraf, O.R. (1966). Analytical and numerical studies of the structure of steady separated flows. Journal of Fluid Mechanics, 24, 113-151.

Chakravarty, S., Mandal, P.K. and Layek, G.C. (2007). Numerical Simulation if Unsteady Generalized Newtonian Blood Flow through Differently Shaped Distensible Arterial Stenoses. *Journal of Medical Engineering & Technology*. 40, 1268-1281.

Cheng, M. and Hung, K.C. (2006). Vortex structure of steady flow in a rectangular cavity. *Computers & Fluids*. 35, 1046-1062.

Cokelet, G.R. (1987). “The rheology and tube flow of blood.” *Handbook of Bioengineering, edited by R. Skalak and S. Chien (McGraw-Hill, NewYork),* 14.1–14.17.

Cruez, A.F. (2007, April 24). Nation’s No. 4 killer disease. *New Straits Times*. p.6.

Davies, P.F. (1995). “Flow-meditated Endothelial Mechanotransduction.” *Physiological Reviews*. 75. 519-560.

Dragos, L. (1975). *Magnetofluid Dynamics.* Abacus Press.

Driel, R.V. (2009). The Oxygen Dependency of Magnetic Susceptibility of Human Blood, Bachelor's thesis, Delft University of Technology.

Ferziger, J.H. and Peric, M. (2002). Computational Methods for Fluid Dynamics. 3rd Edition, Springer, USA.

Fung, Y.C. (1993). *Biomechanics: Mechanical Properties of Living Tissues*. Springer-Verlag, New York.

Ghia, U., Ghia, K.N. and Shin, C.T. (1982). High-Re Solutions for Incompressible Flow Using the Navier-Stokes Equations and a Multigrid Method. *Journal of Computational Physics*. 48, 387-411.

Gupta, M.M. and Kalita, J.C. (2005). A new paradigm for solving Navier-Stokes equations: Streamfunction-Velocity formulation. *Journal of Computational Physics*. 207, 52-68.

Haik, Y. (1997). *Development and Mathematical Modelling of Biomagnetic Fluid Dynamics*. Ph.D. Thesis. The Florida State University.

Haik, Y., Pai, V. and Chen, C.J. (1999). "Biomagnetic Fluid Dynamics." *Fluid Dynamics at Interfaces, edited by W. Shyy and R. Narayanan (Cambridge University Press, Cambridge)*, pp. 439-452.

Haik, Y., Pai, V. and Chen, C.J. (2001). Apparent Viscosity of Human Blood in a High Static Magnetic Field. *Journal of Magnetism and Magnetic Materials*. 225, 180-186.

Haik, Y., Chen, C.J. and Chatterjee, J. (2002). Numerical Simulation of Biomagnetic Fluid in a Channel with Thrombus. *Journal of Visualization*. 5, 187-195.

Haverkort, J.W. (2008). Analytical and Computational Analysis of Magnetic Drug Targeting in Simplified and Realistic Arterial Geometries. Master's thesis, Delft University of Technology, The Netherlands.

Haverkort, J.W. and Kenjeres, S. (2008). Optimizing Drug Delivery using Non-Uniform Magnetic Fields: A Numerical Study. IFMBE Proceedings, 22, 2623-2627.

Haverkort, J.W., Kenjeres, S. and Kleijn, C.R. (2009). Computational Simulations of Magnetic Particle Capture in Arterial Flows. *Annals of Biomedical Engineering*. 37(12), 2436-2448.

Higashi, T., Yamagishi, A., Takeuchi, T., Kawaguchi, N., Sagawa, S., Onishi, S.and Date, M. (1993). Orientation of Erythrocytes in a Strong Static Magnetic Field. *Journal of Blood*. 82, 1328.

Higashi, T., Ashida, N. and Takeuchi, T. (1997). Orientation of Blood Cells in Static Magnetic Field. *Physica B*. 237-238. 616.

Hughes, F. and Young, F.J. (1966). *Electromagnetodynamics of Fluids*. John Wiley and Sons, Inc.

Ichioka, S., Minegishi, M., Iwasaka, M., Shibata, M., Nakatsuka, T., Harii, K., Kamiya, A., and Ueno, S. (2000). High-Intensity Static Magnetic Fields Modulate Skin Microcirculation and Temperature in Vivo. *Bioelectromagnetics*. 21, 183-188.

Jaspard, F. and Nadi, M. (2002). Dielectric Properties of Blood: An Investigation of Temperature Dependence. *Physiological Measurement*. 23, 547–554.

Jehenson, P., Duboc, D., Lavergne, T., Guize, L., Guerin, F., Degeorges, M. and Syrota, A. (1988). Change in Human Cardiac Rhythm Induced by a 2-T Static Magnetic Field. *Radiology,* 166, 227-230.

Johnston, H., Liu, J.G. (2002). Finite Difference Schemes for Incompressible Flow Based on Local Pressure Boundary Condition. *Journal of Computational Physics*. 180, 120-154.

Jou, L.D., Shaltoni, H.M., Morsi, H., and Mawad, M.E. (2010). Hemodynamic Relationship Between Intracranial Aneurysm and Carotid Stenosis: Review of Clinical Cases and Numerical Analyses. Neurological Research. 32(10), 1083-1089.

Kaazempur-Mofrad, M. R., Wada, S., Myers, J. G., and Ethier, C. R. (2005). Mass Transport and Fluid Flow in Stenotic Arteries: Axisymmetric and Asymmetric Models. *International Journal of Heat and Mass Transfer*, 48, 4510-4517.

Kasiman, E. H. (2012), Mixed Formulation for Navier Stokes Equations with Magnetic Effect in Rectangular Channel, Master’s thesis, Universiti Teknologi Malaysia.

Kenjeres, S. (2008). Numerical analysis of blood flow in realistic arteries subjected to strong non-uniform magnetic fields. *International Journal of Heat and Fluid Flow*, 29, 752-764.

Kenjeres, S. and Opdam, R. (2009). Computer Simulations of a Blood Flow Behaviour in Simplified Stenotic Artery subjected to Strong Non-Uniform Magnetic Fields, 4th European Conference of the International Federation for Medical and Biological Engineering, IFMBE Proceedings, 22(22), 2604-2608.

Khashan, S.A. and Haik, Y. (2006). Numerical simulation of biomagnetic fluid downstream an eccentric stenotic orifice. *Physics of Fluids*. 18, 113601.

Kolk, A.G., Hendrikse, J., Zwanenburg, J.J.M., Visser, F. And Luijten, P.R. (2011). Clinical Applications of 7 T MRI in the Brain. *European Journal of Radiology*. In Press, Corrected Proof.

Kyriacou, S.K. and Humphrey, J.D. (1996). Influence of Size, Shape and Properties on the Mechanics of Axisymmetric Saccular Aneurysms. *Journal of Biomechanics*. 29 (8), 1015-1022.

Layek, G.C. and Midya, C. (2007). Effect of constriction height on flow separation in a two-dimensional channel. *Communications in Nonlinear Science and Numerical Simulation*. 12, 745–759.

Lee, J.W.K. and Shannon, S.P. (2007). Tesla Magnetic Resonance Imaging (MRI) - Is it Ready for Prime Time Clinical Applications? *Canadian Journal of Medical Radiation Technology*. 38 (4), 37–50.

Loukopoulos, V. and Tzirtzilakis, E.E. (2004). Biomagnetic channel flow in spatially varying magnetic field. *International Journal of Engineering Science.* 42, 571–590.

Mahapatra, T.R., Layek, G.C., and Maiti, M.K. (2002). Unsteady laminar separated flow through constricted channel. *International Journal of Non-Linear Mechanics*. 37, 171-186.

Maikap, T.K., Mahapatra, T.R., Niyogi, P. and Ghosh, A.K. (2005). Numerical Investigation of Laminar Separated Flow through a Channel with Symmetric Double Expansion. *Acta Mechanica*. 179, 197-210.

McGill, H.C., McMahan, C.A., Zieske, A.W. (2000). Associations of coronary heart disease risk factors with the intermediate lesion of atherosclerosis in youth. *Arteriosclerosis, Thrombosis, and Vascular Biology*. 20(8), 1998–2004.

Merrill, A.M., Margetts, W.G., Cokelet, G.R., and Gilliland, E.W. (1965). The Casson Equation and Rheology of Blood Near Zero Shear. In Copler, A., editor, Symposium on Biorheology, p. 135-143. Interscience Publishers.

Meyers, P.M., Schumacher, H.C., Higashida, R.T., Barnwell, S.L., Creager, M.A., Gupta, R,, McDougall, C.G., Pandey, D.K., Sacks, D., and Wechsler, J.R. (2009). American Heart Association: Indications for the performance of intracranial endovascular neurointerventional procedures: a scientific statement from the American Heart Associatino Council on Cardiovascular Radiology and Intervention. Stroke Council, council on Cardiovascular Surgery and Anesthesia, Interdisciplinary Council on Peripheral Vascular Disease, and Interdisciplinary Council on Quality of Care and Outcomes Research. *Circulation.* 119, 2235-2249.

Midya, C., Layek, G.C., Gupta, A.S., and Mahapatra, T.R. (2003). Magnetohydrodynamic Viscous Flow Separated in a Channel With Constrictions. *Journal of Fluid Engineering*. 125, 952-962.

Neofytou, P. and Tsangaris, S. (2006). Flow Effects of Blood Constitutive Equations in 3D Models of Vascular Anomalies. *International Journal for Numerical Methods in Fluids*. 51, 489-510.

Opdam, R.C.G. (2008). Numerical 3D Simulation of Blood Flow in a Symmetrical Stenotic Artery influenced by a Strong Non-uniform Magnetic Field. Bachelor’s thesis, Delft University of Technology, The Netherlands.

Pai, V.M., Chen, C.J., and Haik, Y. (1999). Microscopic Flow Visualization System for Fluids in Magnetic Field. *Journal of Magnetism and Magnetic Materials*. 194, 262–266.

Pan, F. and Acrivos, A. (1967). Steady flows in rectangular cavities. *Journal of Fluid Mechanics*. 28, 643-655.

Pauling, L. and Coryell, C.D. (1936). The Magnetic Properties and Structure of Hemoglobin, Oxyhemoglobin and Carbonmonoxy Hemoglobin. *Proceedings of the National Academy of Sciences*. 22, 210.

Plavins, J. and Lauva, M. (1993). Study of colloidal magnetite binding erythrocytes: Prospects for cell separation. *Journal of Magnetism and Magnetic Materials*. 122, 349–353.

Pedley, T.J. (1980). *The Fluid Mechanics of Large Blood Vessels.* Cambridge University Press, Cambridge.

Ferziger, J.H. and Peric, M. (2002). *Computational Methods for Fluid Dynamics.* Springer-Verlag, Berlin.

Perktold, K., Peter, R. and Resch, M. (1989). Pulsatile Non-Newtonian Blood Flow Simulation Through a Bifurcation with an Aneurism. *Biorheology*. 26, 1011-1030.

Petersson, N.A. (2001). Stability of Pressure Boundary Conditions for Stokes and Navier-Stokes Equations. *Journal of Computational Physics*. 172, 40-70

Reggio, M., Camarero, R. (1986). Numerical Solution Procedure for Viscous Incompressible Flows. *Numerical Heat Transfer*. 10, 131-146.

Ritter, J.A., Ebner, A.D., Daniel, K.D. and Stewart, K.L. (2004). Application of High Gradient Magnetic Separation Principles to Magnetic Drug Targeting. *Journal of Magnetism and Magnetic Materials*. 280, 184-201.

Rosensweig, R.E. (1985). *Ferrohydrodynamics*. Cambridge University Press, Cambridge.

Ross, R. (1999). Atherosclerosis - An Inflammatory Disease. *The New England Journal of Medicine*, 340(2), 115-126.

Ruuge, E.K. and Rusetski, A.N. (1993). Magnetic Fluid as Drug Carriers: Targeted Transport of Drugs by a Magnetic Field. *Journal of Magnetism and Magnetic Materials*. 122, 335–339.

Rusli, N., Ahmad Kueh Beng Hong, Erwan Hafizi Kasiman, Airil Yasreen Mohd Yassin, and Norsarahaida Amin (2012). “Numerical Computation of a Two-Dimensional Biomagnetic Channel Flow”, International Journal of Modern Physics: Conference Series, Vol 9, 178-192.

Sicilian, J.M. and Hirt, C.W. (1984). An Efficient Computation Scheme for Tracking Contaminant Concentrations in Fluid Flows. *Journal of Computational Physics*. 56, 428–447.

Strikwerda, J.C., (1997). High Order-accurate Schemes for Incompressible Viscous Flow. *International Journal for Numerical Methods in Fluids*. 24, 715-734.

Strong, J. P., Malcom, G. T., McMahan, C. A., Tracy, R. E., Newman, W. P., Herderick, E. E., and Cornhill. J. F. (1999). Prevalence and Extent of Atherosclerosis in Adolescents and Young Adults. *The Journal of the American Medical Association*. 281(8), 727-735.

Sutton, G.W. and Sherman, A. (1965). *Engineering Magnetohydrodynamics*. McGraw-Hill, Inc., New York.

Takeuchi, T., Mizuno, T., Higashi, T., Yamagishi, A., and Date, M. (1995). Orientation of Red Blood Cells in High Magnetic Field. *Journal of Magnetism and Magnetic Materials*. 140-144, 1462–1463.

Tenforde, T.S. (1992). Interaction Mechanisms and Biological Effects of Static Magnetic Felds. Automedica, 14, 271–293.

Tzirtzilakis, E.E. (2005). “Mathematical Model for Blood Flow in Magnetic Field. *Physics of Fluids*. 17, 077103.

Tzirtzilakis, E.E. and Loukopoulos, V.C. (2005). Biofluid Flow in a Channel under the Action of a Uniform Localized Magnetic Field. *Computational Mechanics.* 36, 360-374.

Tzirtzilakis, E.E. (2007). A Simple Numerical Methodology for BFD Problems using Stream Function Vorticity Formulation. *Communications in Numerical Methods in Engineering.* 24(8), 683-700.

Tzirtzilakis, E.E. (2008). Biomagnetic Fluid in a Channel. *Pysica D*. 237, 66-81.

Versteeg, H.K. and Malalasekera, W. (1995). An Introduction to Computational Fluid Dynamics: The Finite Volume Approach. Pearson, UK.

Warmoeskerken, M. M. C. G. and Janssen, L. P. B. M. (2006). Transport Phenomena Data Companion, VSSD, Delft, The Netherlands.

Wiebers, D.O. (1998). Unruptured Intracranial Aneurysms – Risk of Rupture and Risks of Surgical Intervention. *The New England Journal of Medicine*, 339, 1725-1733.

Xu, C., Schuster, E., Vazquez, R., and Krstic, M. (2008). Stabilization of Linearized 2D Magnetohydrodynamic Channel Flow by Backstepping Boundary Control. *Systems & Control Letters*. 57, 805-812.

Yamamoto, T., Nagayama, Y., and Tamura, M. (2004). A Blood-Oxygenation-Dependent Increase in Blood Viscosity due to a Static Magnetic Field. *Physics in Medicine and Biology*. 49, 3267-3277.

Zendehbudi, G.R. and Moayeri, M.S. (1999). Comparison of physiological and simple pulsatile flows through stenosed arteries. *Journal of Biomechanics*. 32, 959-965.

Zhang, K.K.Q., Shotorban, B., Minkowycz, W.J., and Mashayek, F.A. (2006). Compact Finite Difference Method on Staggered Grid for Navier–Stokes Flows. *International Journal for Numerical Methods in Fluids*, 52, 867-881.

Zogheib, B. (2006). Velocity-Pressure Coupling in Finite Difference Formulations for the Navier-Stokes Equations. (Windsor, Ontario, Canada, 2006).