MATHEMATICAL MODELING OF GOUNDWATER FLOW WITH THE EFFECTS OF ADVECTION AND DISPERSION IN A POROUS MEDIUM

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To my beloved Mak and Abah, for your love and support

To my family members, for your consideration and care

To my friends, for your concern, intelligence and guidance in life

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ABSTRACT

Groundwater is one of the most important fresh water resources in our earth .This dissertation presents the mathematical model of groundwater flow with the effect of advection and dispersion in a porous medium. The purpose of this dissertation is to review in detail the one-dimensional advection-dispersion equation (ADE) for two different groundwater problems that is continuous injection and instantaneous injection. In this study, we are formulating a one dimensional mathematical model of a groundwater flow with only the effect of advection and dispersion. We are using Laplace transform to solve the advection dispersion equation for both cases but in a different way. For continuous injection we introduce dependent variable, $C(x,t) = r(x,t)\exp\left(\frac{ux}{2D} - \frac{u^2 t}{4D}\right)$ and apply the Duhamel's theorem. While for instantaneous injection, after transform the general equation into Laplace form we change it into characteristic form and solve it using by applied the boundary condition. Lastly, we solved the continuous injection problem used numerical method via Mathematica programming. Results of the advection dispersion model of this problem are presented by the graphical figures.

ABSTRAK

Air bawah tanah merupakan salah satu sumber air tawar terpenting di bumi kita. Disertasi ini membentangkan model matematik pengaliran air bawah tanah dengan kesan olahan dan penyebaran dalam medium berliang. Tujuan disertasi ini adalah untuk mengkaji secara terperinci persamaan satu dimensi olahan-penyebaran atau singkatannya ADE untuk dua masalah air bawah tanah yang berbeza iaitu suntikan berterusan dan suntikan serta-merta. Dalam kajian ini, kami hanya fokus kepada model matematik satu dimensi aliran air bawah tanah dengan hanya kesan olahan dan penyebaran. Penyelesaian tepat diperolehi dengan menggunakan jelmaan Laplace untuk menyelesaikan persamaan penyebaran olahan bagi kedua-dua masalah tetapi dengan cara yang berbeza. Un tuk suntikan berterusan, kami memperkenalkan pemboleh ubah, $C(x,t) = r(x,t)\exp\left(\frac{ux}{2D} - \frac{u^2 t}{4D}\right)$ dan menggunakan teorem Duhamel's bagi menyelesaikan masalah ini. Sementara, untuk suntikan serta merta selepas menukarkan persamaan umum kepada Laplace dan persamaan karakteristik, seterusnya selesaikan dengan menggunakan keadaan sempadan yang telah juga di ubah kepada Laplace. Akhir sekali, kami menggunakan kaedah berangka melalui pengaturcaraan Mathematica bagi menyelesaikan masalah suntikan berterusan. Keputusan model penyebaran olahan masalah ini dibentangkan secara graf.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

A pollutant or leachate may enter the groundwater zone directly to a landfill site from an industrial site such as nuclear power plants, chemical industries, construction industries etc by dumping of toxic waste, agriculture and waste disposal. Radioactive waste is disposed under the ground and gets released into groundwater because of advection, dispersion and diffusion.

In recent year's groundwater flow and contaminant transport model play an important role in many projects dealing with groundwater exploitation, protection and remediation. A groundwater flow model is essential for the development of a contaminant transport model. Groundwater flow models are used to calculate the rate and the direction of the movement of groundwater through aquifers and confining units in the subsurface. Scientists facing lot of problem in assessing extend of groundwater pollution and effectiveness remedial action.

In developing a groundwater flow or solute transport model the analyst begins by preparing a conceptual model consisting of a list of physical and chemical processes of the system under study (solute transport by advection, dispersion and diffusion). The next step is to translate the conceptual model into mathematical model consisting of one or one partial difference equations and set of initial and boundary conditions. Advection-dispersion equation (ADE) describes the solute transport due to combined effect of dispersion and convection in a medium. Environmentalists, hydrologists, civil engineers and mathematical modelers concentrate more on the advection-dispersion equations to ensure the safe hydro-environment. The analytical and numerical solutions of the advection-dispersion equations are useful to assess the time and position at which the concentration level of the pollutants will start affecting the health of the habitats in the polluted water eco-system. Also such solutions help estimate and examine the rehabilitation process and management of a polluted water body after elimination of the pollution. The analytical solution of dispersion problems in ideal conditions is obtained by reducing the advection-dispersion equation into a dispersion equation by eliminating the convective term.

The analytical solutions of advection-dispersion equation have been discussed by many researchers like Bastian and Lapidus (1956), Ogata and Banks (1961), Banks and Ali (1964), Lai and Jurinak (1971), Marino (1974), Al-Niami and Rushton (1977), Atul Kumar et al. (2009), Dilip Kumar et al (2011) etc.

For finding the analytical or numerical solutions of dispersions problems, the decision parameters of the model must have crisp value. But in groundwater hydrology, uncertainties arise due to insufficient data or imprecise information in the identification and classification of aquifers properties. Also, in real-world applications certainty, reliability and precision of data are often not possible and it involves high information costs. These uncertainties are due to insufficient data or imprecise information. In this project, we model the uncertain parameters as interval numbers.

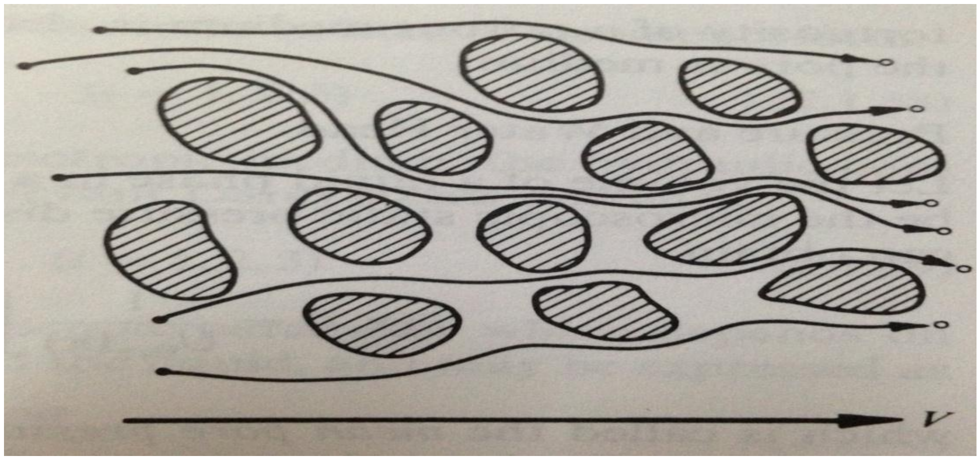

Figure 1: Microscopic movement of fluid particles in pores.

1.2 Background of the Problem

Groundwater is one of the most prominent water resources for domestic and industrial uses to a large part of the human population. Groundwater refers to the subsurface water that occur beneath the water table in soil and geologic formations that are fully saturated. It is sometimes known as phreatic or plerotic water although these words are less often found in literature to groundwater. In some localities, the demand for groundwater has increased faster than for surface water. As the result, the rate of groundwater withdrawal has exceeded the rate of natural replenishment in these areas (Thomas, 1951). Concern of groundwater environment has greatly stimulated research of solute transport phenomena which is generally associated with the advection-diffusion equation (Leij et al., 1991). As such, advection-diffusion is sometimes referred to as advection-dispersion, convection-diffusion or convection-dispersion. It has also been referred to as hydrodynamic dispersion or drift dispersion. In this research, the advection-dispersion equation and its aliases are abbreviated as ADE.

1.3 Statement of Problem

To help better understand our world, we often describe a particular phenomenon mathematically. Such mathematical model is an idealization of the real world phenomenon and never a completely accurate representation. In modeling world, we often are interested in predicting the value of a variable at some time in the future.

In this research, in order to study the groundwater flow with the effect of dispersion and advection in a medium, we need to study a suitable mathematical model that represents the physical phenomenon. In this purpose, we are determining one-dimensional (1D) mathematical model of groundwater flow with the effect of dispersion and advection.

1.4 Objectives of the Study

The objectives of this research are:

1. To formulate 1D mathematical modeling of groundwater flow with the effect of dispersion and advection in a porous medium.
2. To solve 1D advection diffusion equation by analytical solution of continuous injection and instantaneous injection condition.
3. To solve 1D advection diffusion equation by numerical method of continuous injection.

1.5 Significance of the Study

By studying the mathematical modeling of groundwater flow with the effect of dispersion and advection in a medium we will get better understanding of transportation of water in underground and pipe.

1.6 Scope of the Study

Our main project is to incorporate imprecise information into solute transport process modeling using interval arithmetic. Where, in this project we focus only on 1D advection-dispersion equation involving internal parameters of longitudinal dispersion in porous media.

1.7 Dissertation Organization

This thesis is organized into six chapters. First chapter contains the study framework. An introductory chapter where we briefly described background of the problem, statement of the problem, objectives, scope, and significance of the study are discussed in this chapter.

In Chapter two, present the literature review of this dissertation. This chapter explained briefly the one dimensional advection-dispersion equation, the definition, the background and also the previous research on ADE solve analytically and numerical method.

In Chapter three, the methodology of the study will be discussed. This chapter will focus on Laplace transform method as analytical solution, while finite difference method as numerical solution of ADE on this dissertation.

In Chapter four, we will show the complete analytical solution of ADE for both boundary condition of groundwater flow. Also, the numerical solution for ADE solve by Mathematica software. Where, in this chapter result analysis of data will be considered.

In Chapter five, our focus is to discuss about the results and discussion of the problem. Finally, some suggestion and conclusion of the study will be discussed in the chapter six.

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