

**THERMAL ANALYSIS OF A VERTICALLY BASED FIN ARRAY IN A
BOUNDED REGION - A NUMERICAL EXPERIMENT**

LEE LAI KIN

**Laporan projek ini dikemukakan
sebagai memenuhi sebahagian daripada syarat
penganugerahan ijazah Sarjana
Kejuruteraan Mekanikal (Mekanikal)**

**Fakulti Kejuruteraan Mekanikal
Universiti Teknologi Malaysia**

MARCH, 2003

To my beloved father, mother, Li Yun, sisters and younger brother.....

ACKNOWLEDGMENTS

I would like to take this opportunity to express my sincere gratitude to my thesis supervisor Professor Amer Nurdin Darus for his support and guidance throughout the project. This project would not be possible without his contribution.

I would also like to express my gratitude to those who have contributed to this project, especially my beloved wife Li Yun, family and friends who give valuable support and encouragement throughout this project.

Last but not less, special thanks are given to Professor Mohamad Afifi for his understanding during the project is carried out.

ABSTRACT

This study uses finite volume method (QUICK) to simulate the natural convection heat transfer of a vertically based fin array where each of the surfaces are isothermal for different aspect ratios and different angles of the base surface. A computer programme is developed to solve the problem by applying continuity equation, momentum equation and energy equation. The velocity contour and the temperature contour for different aspect ratios and different angles of the base surface will be observed. Based on the Nusselt number and Rayleigh Number that will be calculated, a natural convection correlation equation of a vertically based fin array is derived. The accuracy of the developed correlation will be compared with the correlation obtained from literature.

ABSTRAK

Kajian ini menggunakan kaedah isipadu terhingga (QUICK) untuk melaksana simulasi perpindahan haba secara olakan semulajadi ke atas sirip-sirip yang bertapak tegak dengan nisbah bidang dan sudut tapak yang berlainan. Satu komputer program telah dibina untuk menyelesaikan masalah ini dengan menggunakan persamaan keterusan, persamaan momentum dan persamaan tenaga. Kontur-kontur bagi halaju dan suhu dengan nisbah bidang dan sudut tapak yang berlainan telah diperhatikan. Satu sekaitan persamaan olakan semulajadi telah dihasilkan melalui nombor Nusselt and nombor Rayleigh yang dihitung dengan komputer program. Kejituan persamaan sekaitan yang dihasilkan telah dibandingkan dengan persamaan sekaitan yang lain diperolehi dari kajian latar belakang.

CONTENTS

CHAPTER	SUBJECT	PAGE
	TITLE	i
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	CONTENTS	vii
	LIST OF TABLES	ix
	LIST OF FIGURES	x
	NOMENCLATURE	xiii
	LIST OF APPENDIX	xv
1	INTRODUCTION	1
	1.1 Introduction	1
	1.2 Problem Statement	2
	1.3 Objective	4
	1.4 Scope	4
	1.4 Research Methodology	4

2	LITERATURE REVIEW	6
	2.1 Literature Review	6
3	MATHEMATICAL FORMULATION	13
	3.1 Governing Equation	13
	3.2 Finite Volume Method	16
	3.2.1 Step 1: Grid Generation	17
	3.2.2 Step 2: Discretisation	17
	3.2.3 Step 3: Iteration	20
	3.3 Natural Convection Correlations	21
	3.3.1 The Elenbaas Correlation	22
	3.3.2 Modified Elenbaas Correlation	25
4	RESULTS	28
	4.1 Initial Input	28
	4.2 Computer Programme Flow Chart	30
	4.3 Velocity Contour and Temperature Contour	32
	4.4 Calculated Nusselt Number	57
5	DISCUSSION	59
	5.1 Validation	59
	5.2 Discussion	60
6	CONCLUSION	62
	6.1 Future Work And Development	62
	6.2 Conclusion	63
	REFERENCES	64

LIST OF TABLES

TABLE NO.	TITLE	PAGE
3.1	Nusselt Nombres for fully developed and isolated plate natural convection in two-dimensional channel (Elenbaas, 1942)	25
4.1	Initial input data	28
4.2	Summary of number of numerical calculations.	30
4.3	Summary of Nusselt and Rayleigh number calculation	57
5.1	The percentage error between computer calculated Nu and derived correlation	61

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
1.1	A typical vertically based fin arrays with isothermal surfaces.	3
1.2	Boundary condition and calculation domain of free convection for isothermal surfaces.	3
1.3	Research methodology flow chart.	5
3.1	A general nodal point, P and its neighbor nodes.	17
3.2	A flow chart for typical subroutine of transport properties, ϕ .	20
3.3	Developing temperature and velocity profiles for natural convection in a vertical channel (Elenbaas, 1942).	23
4.1	Typical dimension for the heat sink with APR = 1.	29
4.2	The main flow chart for obtaining results.	31
4.3(a)	Temperature and x direction velocity ($\times 10^{-16}$) contour with APR = 0.5 and $\theta = 0^\circ$.	32
4.3(b)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 0.5 and $\theta = 30^\circ$.	33
4.3(c)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 0.5 and $\theta = 45^\circ$.	34
4.3(d)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 0.5 and $\theta = 60^\circ$.	35
4.3(e)	Temperature and y direction velocity ($\times 10^{-16}$) contour with APR = 0.5 and $\theta = 90^\circ$.	36

4.4(a)	Temperature and x direction velocity ($\times 10^{-16}$) contour with APR = 1.0 and $\theta = 0^\circ$.	37
4.4(b)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 1.0 and $\theta = 30^\circ$.	38
4.4(c)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 1.0 and $\theta = 45^\circ$.	39
4.4(d)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 1.0 and $\theta = 60^\circ$.	40
4.4(e)	Temperature and y direction velocity ($\times 10^{-16}$) contour with APR = 1.0 and $\theta = 90^\circ$.	41
4.5(a)	Temperature and x direction velocity ($\times 10^{-16}$) contour with APR = 1.5 and $\theta = 0^\circ$.	42
4.5(b)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 1.5 and $\theta = 30^\circ$.	43
4.5(c)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 1.5 and $\theta = 45^\circ$.	44
4.5(d)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 1.5 and $\theta = 60^\circ$.	45
4.5(e)	Temperature and y direction velocity ($\times 10^{-16}$) contour with APR = 1.5 and $\theta = 90^\circ$.	46
4.6(a)	Temperature and x direction velocity ($\times 10^{-16}$) contour with APR = 2.0 and $\theta = 0^\circ$.	47
4.6(b)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 2.0 and $\theta = 30^\circ$.	48
4.6(c)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 2.0 and $\theta = 45^\circ$.	49
4.6(d)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 2.0 and $\theta = 60^\circ$.	50
4.6(e)	Temperature and y direction velocity ($\times 10^{-16}$) contour with APR = 2.0 and $\theta = 90^\circ$.	51
4.7(a)	Temperature and x direction velocity ($\times 10^{-16}$) contour with APR = 2.5 and $\theta = 0^\circ$.	52

4.7(b)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 2.5 and $\theta = 30^\circ$.	53
4.7(c)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 2.5 and $\theta = 45^\circ$.	54
4.7(d)	Temperature, x and y direction velocity ($\times 10^{-16}$) contour with APR = 2.5 and $\theta = 60^\circ$.	55
4.7(e)	Temperature and y direction velocity ($\times 10^{-16}$) contour with APR = 2.5 and $\theta = 90^\circ$.	56
4.8	Graph of Nusselt number vs Rayleigh number for different aspect ratios (APR).	58
5.1	Temperature contour obtained from Fluent with APR = 1 and $\theta = 0^\circ$.	59

NOMENCLATURE

A	-	area, m^2
APR	-	aspect ratio
c_p	-	specific heat at constant pressure, $J/kg\ K$
D	-	fin spacing, m, diffusion conductance at cell faces
F	-	convective mass flux per unit area at cell faces
g	-	gravitational acceleration, m/s^2
H	-	height of fin, m
h	-	convection heat transfer coefficient, $W/m^2\ K$
k	-	thermal conductivity, $W/m\ K$
L	-	length of fin, m
Nu	-	Nusselt number
p	-	pressure, N/m^2
Pr	-	Prandtl number
q''	-	Heat flux, W/m^2
r	-	hydraulic radius
Ra	-	Rayleigh number
Re	-	Reynolds number
S	-	source term
T	-	temperature, K
T_w, t_w	-	wall temperature, K
T_∞, t_∞	-	ambient temperature, K
u, v	-	mass average fluid velocity components, m/s
V	-	volume, m^3
x, y	-	rectangular coordinates, m
β	-	volumetric thermal expansion coefficient, K^{-1}

ϕ	-	transport properties
μ	-	viscosity, kg/s m
ν	-	kinematic viscosity, m ² /s
θ	-	zenith angle, degree
ψ	-	geometric parameter
Λ	-	combination of modified Bessel function
ρ	-	mass density, kg/m ³

LIST OF APPENDIX

APPENDIX	TITLE	PAGE
A	PROGRAMMING CODE	69

CHAPTER 1

INTRODUCTION

1.1 Introduction

Extended surface assemblies comprising a primary surface and attached fins are used throughout the process industries, in all vehicles for radiators and internal heat exchangers, and in domestic applications such as radiators in rooms, condensing central heating exchangers, air-conditioning units and internal and external surfaces in refrigeration units, electronic board such as computer processor cooling units and other electronic chips.

In the present day competitive environment, superior thermal performance must be attained simultaneously with lower manufacturing costs, minimum added mass, and shortened product development cycles. Thus, the thermal analysis for the primary surface with attached fins is needed to produce more cost-effective heat exchangers.

Frequently, one of the heat resistances to the flow of heat dominates the other resistances and so controls the size of the equipment to transfer the heat duty. The dominance of such a resistance can be broken by replacing the primary surface with an extended surface. The lower resistance due to the extended is caused by connection to the surrounding fluid from all exposed surfaces as well as conduction through the fins. Therefore it is very important to have a detailed and thorough analysis of these combined conduction-convection systems so that adequately sized equipment can be designed to meet the specified heat duties.

The present work concerns on a numerical investigation of a vertically based fin arrays with different set of fin arrays geometry and different orientation. The temperature field and flow field of a vertically based fin arrays in a bounded region are determined numerically. Thermal analysis for the vertically based fin arrays is limited to natural convection and the flow passing through the fins is assumed incompressible constant property flow. The base and fins is assumed isothermal. The medium around the fins is assumed non-participating gas and the steady stage only will be observed.

1.2 Problem Statement

The current study develops a numerical finite volume programme to simulate the natural convection by a vertically based rectangular fin array with different sets of fin array dimension and different orientation. The physical model of the problem studied is sketched in Figure 1.1, which is a heat sink with rectangular fins. In this study, we are only interested in two rectangular fins with base plate as shown in Figure 1.2. The height of the rectangular fins is H , which are placed parallel to each other, separated by a distance D . The three surfaces are assumed to be isothermal. The medium around the fins is assumed to be non-participating gas (air) and the steady stage will only be observed. The flow passing through the fins is also

assumed to be incompressible constant property flow and fully developed. The thickness of the fins is assumed to be thin enough to negligible. The length of the fins, L is long enough so that the problem can be considered as a two-dimensional problem.

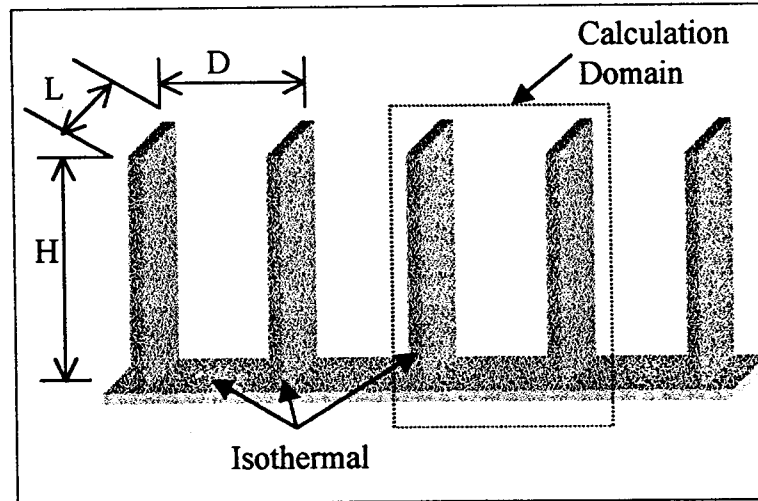


Figure 1.1: A typical vertically based fin arrays with isothermal surfaces.

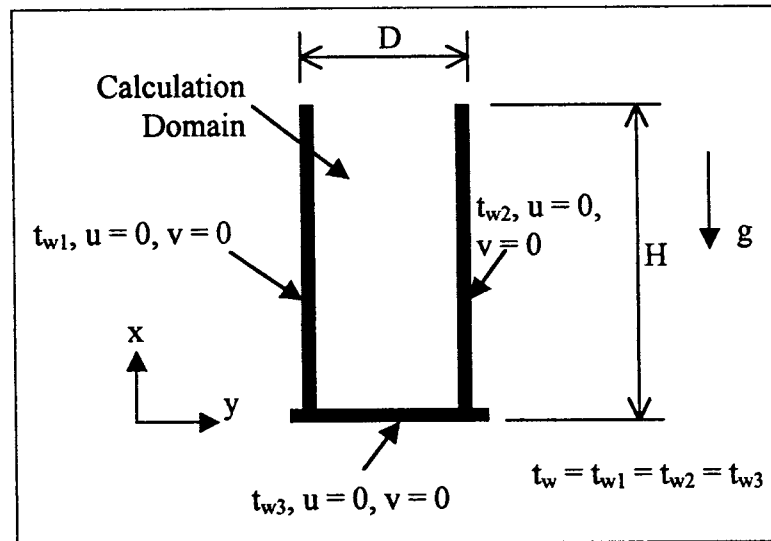


Figure 1.2: Boundary condition and calculation domain of free convection for isothermal surfaces.

1.3 Objective

The objectives of this project are

- (a) To develop a computer programme that is capable of calculating both temperature and velocity of a vertically based fin array.
- (b) To study temperature field and flow field of a vertically based fin array with various dimension when the based surface orientation is changing.
- (c) To find the natural convection correlation equation of a vertically based fin array.

1.4 Scope

The study is concentrated in numerical investigation. Thermal analysis of a vertically based fin array is limited to natural convection. The flow which passes through the fins is assumed to be incompressible constant property flow and fully developed. The primary surface and attached fins are assumed to be isothermal. The medium around the fins is assumed to be non-participating gas and steady stage will only be observed.

1.5 Research Methodology

- (a) Semester I

In semester I, we will concentrate on literature review of the natural convection heat transfer of fin array. From the literature, the experimental result with numerical solution (if any) will be compared. Then, the governing equation of

the fin array natural convection heat transfer problem will be formulated with different sets of fin array geometry and orientation. The solving strategy will be proposed and the numerical methods will be determined.

(b) Semester II

In semester II, the work of solving strategy will be continued. A solution flow chart will be drawn to simulate the solving strategy. Then, the numerical scheme and the implementation of frame will be developed in computer language. The simulation programme will be run to check the effectiveness of the scheme. When the programme is developed, a real run will be conducted to compare the results obtained from experiment (if any) for checking the validity of the programme. Finally, the next run will be conducted with different sets of fin array geometry and orientation. The flow chart of the research methodology is shown Figure 1.3..

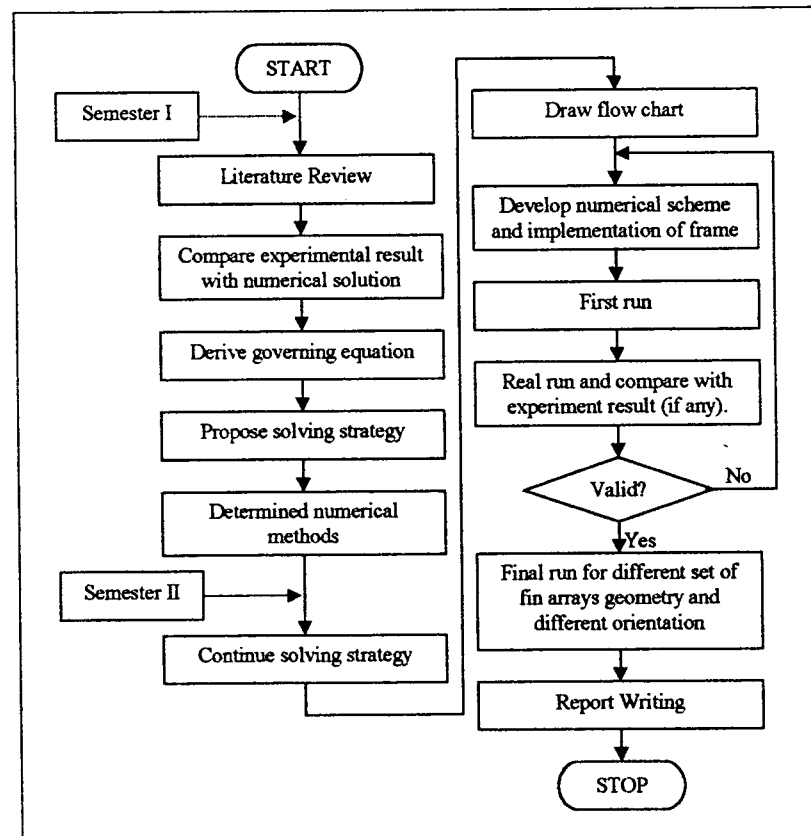


Figure 1.3: Research methodology flow chart.

REFERENCES

- Acharya, S. and Goldstein, R. J. (1983). *Natural convection in an externally heated vertical or inclined square box containing internal energy sources*. ASME Proceedings of the 1983 National Heat Transfer Conference, HTD-Vol 26, pp 17-26
- Arco, E. C., Bontour, P., Sani, R. L. Hardin, G., Extermet, G. P. and Chikhaoui, A. (1988). *Finite Difference Solutions For Three-Dimensional Steady and Oscillatory Convection In Vertical Cylinders-Effect of Aspect Ratio*. Proceeding of ASME Annual Meeting, Heat Transfer Division.
- Bergles, A. E. (1986). *The Evaluation of Cooling Technology for Electrical, Electronic, and Microelectronic Equipment*. ASME Heat Transfer in Electronic Equipment.
- Bergman, T. L. and Petri, J. G. (1988). *Natural convection Cooling of Discrete Heated Elements In An Enclose Using Pressure Helium-xenon gas mixtures*. Proceeding of ASME Annual Meeting, Heat Transfer Division.
- Bilitsky, A. (1986). *The Effect of Geometry On Heat Transfer By Free Convection From A Fin Array*. MS Thesis, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer Sheva, Israel.
- Bhavnani, S. H. and Bergles, A. E. (1988). *An experimental study of laminar natural convection heat transfer from wavy surface*. ASME Proceedings of the 1988 National Heat Transfer Conference, HTD-Vol 96, Vol 2.

- Cha, Wei (1989). *Numerical Study of Natural Convection Between Two Vertical Parallel Plates With One Oscillating Surface Temperature*. MS. Thesis, Department of Mechanical Engineering, Michigan State University, USA.
- Elenbass, W. (1942). *Heat Dissipation of Parallel Plates by Free Convection*. *Physica*, 9, 1-28.
- Hamady, F. (1987). *Experimental study of local natural convection heat transfer in Inclined and rotating enclosures*. Dissertation for Degree of Ph.D, Michigan State University.
- Hornbeck, W. R. (1973). *Numerical Marching Techniques for Fluid Flows With Heat Transfer*. Scientific and Technical Information Office, National Aeronautics and Space Administration, Washington, USA.
- Incropera, P. F. and Dewitt, P. D. (1985). *Introduction to Heat Transfer*. John Wiley & Sons, New York.
- Jones, C. D. and L. F. Smith (1970). *Optimum Arrangement of Rectangular Fins on Horizontal Surface for free Convection Heat Transfer*. *Journal of Heat Transfer*, 92, 6.
- Kao, T. T.(1976). Locally non-similarity solution for laminar free convection adjacent to a vertical wall. *ASME Journal of Heat Transfer*, pp 321-322.
- Kraus, D. A. and Avram Bar-Cohen (1995). *Design and Analysis of Heat Sinks*. John Wiley & Sons, Inc., New York.
- Lee, S and Yovanovich, M. M. (1988). *Laminar natural convection from a vertical plate with variation in surface heat flux*. Proceedings of the 1988 National Heat Transfer Conference, HTD-Vol 96, Vol 2.

- Lin, Y. S. and Akins, R. G. (1983). *An experimental study of flow patterns and heat transfer by natural convection inside cubical enclosures*. ASME Proceedings of the 1983 National Heat Transfer Conference, HTD-Vol 26, pp 27-34
- O'Meara, M. and Poulikakos, D. (1987). *Experiments on the cooling by natural convection of an array of vertical heated plates with constant heat flux*. Heat and Fluid-Flow, Vol 8, No 4.
- Ostrach, S., Loka, R. R. and Kumar, A. (1980). *Natural convection in low aspect-ratio rectangular enclosures*. ASME Proceedings of the 1980 National Heat Transfer Conference, HTD-Vol 8, pp 1-10.
- Park, K. - A. and Bergles, A. E. (1987). *Natural Convection Heat Transfer Characteristics of Simulated Microelectronics Chips*. ASME Journal of Heat Transfer 109, 90-96
- Penny, J. Dr. (1995). *Numerical Method Using MATLAB*. Ellis Horwood, New York.
- Ramanathan, S. and Kumar, R. (1988). *Correlations for Natural Convection Between Heated Vertical Plates*. Proceeding of ASME Annual Meeting, Natural and Mixed Convection in Electronic Equipment Cooling.
- Schilling J. R. and Harris L. S. (2000). *Applied Numerical Methods for Engineers – Using MATLAB and C*. Brooks/Cole, Pacific Grove.
- Sparrow, E. M., Quack, H. and Boerner, C. J. (1936). *Local non-similarity boundary-layer solution*. AiAA Journal, Vol. 8, No. 11.
- Sparrow, E. M., and Gregg, J. L. (1956). *Laminar free convection from a vertical plate with uniform surface heat flux*. Journal of Heat Transfer, pp 435-440.

- Sparrow, E. M., and Yu, H. S.(1971). *Local non-similarity thermal boundary-layer solution*. ASME Journal of Heat Transfer, pp 328-334.
- Starner, K. E. and H. N. McManus (1963). *An Experimental Investigation of Free Convection Heat Transfer from Rectangular Fin Arrays*. Journal of Heat Transfer, 85, 273-278.
- Steinberg, D. S. (1980). *Cooling Techniques for Electronic Equipment*. Wiley-Interscience Publication.
- Torikoshi, K., Kawazoe, M. and Kurihara, T (1988). *Convective Heat Transfer Characteristics of Arrays of Rectangular Books Affixed to On Wall of A Channel*. Proceeding of ASME Annual Meeting, Natural and Mixed Convection in Electronic Equipment Cooling.
- Van de Pol, D. W. and J. K. Tierney (1973). *Free Convection Nusselt Number for Vertical U-Shaped Channel*. Journal of Heat Transfer, 95, 542-543.
- Versteeg, H. K. and Malalasekera, W. (1995). *An Introduction to Computational Fluid Dynamic The Finite Volume Method*. Longman Scientific & Technical, England.
- Wang, Y. Z. and Bau, H. H. (1988). *Low Rayleigh Number Convection in Horizontal, Eccentric Annuli*. Proceeding of ASME Annual Meeting, Natural and Mixed Convection in Electronic Equipment Cooling.
- Welling, J. R. and C. B. Wooldridge (1965). *Free Convection Heat Transfer Coefficients From Rectangular Vertical Fins*. Journal of Heat Transfer, 87, 439-444.
- Wirtz, R. A and Stutzman, R. J. (1982). *Experiments on free convection between vertical plates with symmetric heating*. ASME Journal of Heat Transfer, Vol 104, Pp 501-507.

Wirtz, R. A. and Tseng, W. F. (1980). *Natural convection across tilted, rectangular enclosures of small aspect ratio*. ASME Proceedings of the 1980 National Heat Transfer Conference., HTD-Vol 8, pp 47-54.