LATTICE GAS AUTOMATA SIMULATIONS OF A SINGLE-PHASE AND TWO-PHASE FLOW IN HETEROGENEOUS POROUS MEDIA

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This thesis is dedicated to
my beloved mother and father,
my wife Nita, my daughters Nesha and Cloudy
(*Dalam kasih Tuhan jerih payahmu tidak akan sia-sia*, *sebab*
kasih mengindahkan orang lain dan mendatangkan damai sejahtera)

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ABSTRACT

Modeling of fluid flow in porous media and predicting its performance is one of the important subjects in petroleum engineering. This research reports the development of a lattice gas automata method to study and simulate a single-phase and two-phase flow in heterogeneous porous media as an alternative to conventional methods (finite difference and finite element). In this work, the FHP-II (Frisch, Hasslacher and Pomeau, FHP) model of lattice gas automata was developed to simulate microscopic fluid flow and estimate the macroscopic properties of heterogeneous porous media. Heterogeneity of the porous media was constructed by placing solid obstacles randomly in a two-dimensional test volume. Effects of grain shape and size geometry, and their distribution in the porous media were taken into account. In addition, macroscopic properties of the heterogeneous porous media were estimated in terms of the shape, size, number of the solid obstacles and by the distribution of the solid obstacles within the volume.

In the single-phase flow simulation part, a heterogeneous porous media was constructed, and correlations between various macroscopic properties, i.e., tortuosity, specific surface area, effective porosity and permeability were obtained. In the two-phase flow simulation part, the phase separation of the two immiscible fluids was described. Furthermore, the surface tension and capillary pressure were also estimated. The displacement mechanisms of carbon dioxide to displace oil and displacement efficiency of the process in the heterogeneous porous media were also predicted. Generally, the lattice gas automata simulation produced similar results with previous researchers and experiments. Errors of between 10% and 25% were associated with the computed results from the single-phase flow simulation part for the permeability prediction, compared with the laboratory experiments, while for the immiscible fluids displacement process it was less than 5%. Based on the results, it is obvious that the lattice gas automata method was indeed capable of being applied in petroleum engineering for simulation of a single-phase and two-phase flow in heterogeneous porous media.

ABSTRAK

Permodelan dan kesan peramalan aliran bendalir di dalam media berliang merupakan suatu subjek yang penting di dalam kejuruteraan petroleum. Kajian ini melaporkan tentang pembangunan model kekisi gas automata untuk melakukan pembelajaran dan penyelakuan aliran bendalir satu fasa dan dua fasa melalui media berliang yang heterogen sebagai pilihan lain daripada penggunaan kaedah konvensional (perbezaan terhad dan perbezaan elemen). Di dalam kajian ini, model FHP-II (Frisch, Hasslacher and Pomeau, FHP) dari kekisi gas automata dibangunkan untuk menyelakukan aliran mikroskopik dan pengiraan sifat makroskopik di dalam media berliang yang heterogen. Keheterogenan media berliang dilukiskan oleh penempatan pepejal yang acak di dalam pengujian dua dimensi. Kesan daripada geometri bentuk butiran dan saiz, dan agihannya di dalam media berliang diambil perhatian. Selanjutnya, sifat makroskopik di dalam media berliang yang heterogen diperkirakan berazaskan bentuk, saiz, bilangan daripada pepejal dan agihan daripada pepejal tersebut di dalam merintangi aliran bendalir terhadap volume.

Pada bahagian penyelakuan aliran satu fasa, sifat heterogen daripada media berliang boleh dibangunkan, dan korelasi-korelasi diantara pelbagai parameter makroskopik, iaitu tortuositi, luas permukaan spesifik, keliangan efektif dan ketertelapan telahpun diperolehi. Sementara itu dalam penyelakuan aliran dua fasa, pemisahan fasa daripada dua bendalir yang tak larut campur boleh dilukisan, dan tegangan permukaan serta tekanan rerambut boleh diperkirakan. Mekanisma penyesaran karbon dioksida yang menyesarkan minyak dan kecekapan penyesaran di dalam media berliang telahpun diramalkan. Pada amnya, hasil yang memuaskan dari penyelakuan kekisi gas automata dengan hasil penyelidik terdahulu dan keputusan kajian makmal telahpun diperolehi. Keputusan penyelakuan untuk peramalan ketertelapan pada model satu fasa terdapat kesalahan perhitungan yang berjulat antara 10% sehingga 25%, jika diperbandingkan dengan kajian makmal, seterusnya untuk proses penyesaran tak larut campur kurang dari 5 %. Berdasarkan keputusan tersebut, kaedah kekisi gas automata adalah benar kapabel untuk diaplikasikan di dalam kejuruteran petroleum guna menyelakukan aliran satu dan dua fasa di dalam media berliang yang heterogen.

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A_0 - coefficient of the equilibrium distribution function

a - the unit bond length in interface model

b - the distance between the two parallel plates

b' - the post-collision states of blue particle

C - describes the collision operator (only particles at the same node are involved)

$C(r,t)$ - color density

C_i' - constant

$c^{(i)}$ - undetermined coefficients (constant)

c - particle velocity

c_s - speed of sound

c_0 - coefficient of the equilibrium distribution function

$c_{\alpha i}$ - the unit length velocity vector at site i, pointing into direction α ($\alpha = 1, 2, \ldots, 6$)

D - diffusion coefficient

dp/dx - pressure gradient

d - the mean density per link given by $\rho/6$ for FHP-I and $\rho/7$ for FHP-II and FHP-III models

e_i - unit vector in the direction of link i

$e^{-r/\lambda}/r$ - mean color attractive potentials

F - a local flux for holes

$F(r,t)$ - body force per unit mass

$\overline{F}_{14}$ - the joint two-particle distribution function

$\overline{f}$ - equilibrium distribution function

f^* - non-equilibrium component of the distribution function

$f_\alpha(x,t)$ - distribution function gives the probability of finding a particle with velocity c_α at position x and time t

$f(\phi,\tau)$ - a dimensionless function of ϕ and τ
$f^{(r)}$ - distribution function of red fluid
$f^{(b)}$ - distribution function of blue fluid
G - local flux for the colored particles
$g(\rho)$ - Galilean invariance term
g - the acceleration due to gravity
$J(t,x)$ - mean mass current
k - permeability of the porous medium
L - length scales of the flow
M_i - number of particles on link i in an averaging cell
m - particle mass
N_i - mean population of link i
$\overline{N_i}$ - equilibrium mean population of link i
$N_{\alpha i}$ - the number of particles at site i, moving into direction α
N_s - the number of scatterers at given site
$n_i(x_{ij},t_n)$ - (n_1, n_2, … , n_6) are Boolean variables that indicate the presence (n_i-1) or absence (n_i-0) or particles moving from a lattice site situated at position x to the neighboring site situated at position $x + c_i$
$n_i^{(r)}$ - the occupation numbers for red particles on link c_i
$n_i^{(b)}$ - the occupation numbers for blue particles on link c_i
P - momentum
p - pressure
$P_{\alpha\beta}$ - pressure tensor
p_r - pressure of red particles inside the drop
p_b - pressure of blue particles outside the drop
P^s - the probability that a lattice site will act as scatterers
q - the volumetric rate of flow per unit area
q_c - the critical rate
Re - Reynolds number
R - radius of the drop
R_0 - the hydraulic radius of the obstacles
r - position vector of any site
r' - the post-collision states of red particle
S - streaming operator

s - $s_1, s_2,\ldots,s_b$ an in-state at time t^- just before the collision

s' - $s_1', s_2',\ldots,s_b'$ an out-state at t^{-+} just after the collision

s_i - i-th component of in-state

s_i' - i-th component of out-state

$T_{i\alpha\beta\gamma\delta}$ - four-dimensional tensor

T - the total change per time step

t - integer-valued and the duration of a time step is taken to be unity

t_n^- - the time of pre-collision

t_n^+ - the time of post-collision

U - rotational velocity

u - the initial average velocity

u_0 - the initial flat velocity distribution

V - the volume in two-dimensional space

V_0 - the volume of the obstacles

W - the channel width

x_i - the normal velocity of the interface

x_q - the linear stability displacement

Greek Symbols

$\beta_{\alpha b}$ - gives the probability that a particle that arrives at a particular site from direction c_b leaves in direction c_α with $\alpha \neq b$

Δ_i - the collision operator (function), which describes the change in $n_i(t,x)$ during a collision at time t at site x

σ - surface tension coefficient

θ - the contact angle

ε - convolution (the composition of collision and streaming)

ξ - the random variable ($\xi = 1$)

ζ - kinematic bulk viscosity

ν - kinematic shear viscosity

μ - fluid viscosity

χ - interfacial energy

$\Pi_{\alpha\beta}$ - momentum flux tensor

$\Pi^{(0)}$ - the zeroth-order approximation of the momentum tensor Π
$\Pi^{(1)}$ - the first-order approximation of the momentum tensor Π
ρ - density
ρ_0 - initial density and/or constant density
τ - tortuosity of porous medium
ϕ - porosity of porous medium
ϕ_c - critical porosity or percolation threshold of porous medium
λ - the lattice mean free path
η - the growth rate of displacement process
Ψ - free energy
Ω_α - the collision term (function)
$\Im$ - implementation operator for the long range interaction

Superscript

(b) - blue particles
(r) - red particles
(0) - zeroth-order terms
(1) - first-order terms
$-$ - pre-collision
$+$ or $'$ - post-collision

Subscript

α,β - label the two spatial components of the velocity vectors
b - blue particles
f - fluid
i - i-th component
j - j-th component
r - red particles

Abbreviations

BC	-	Boundary Condition
CFD	-	Computational Fluid Dynamic
FHP	-	Frisch, Hasslacher and Pomeau (FHP Model)
FCHC	-	Face Centered Hyper Cubic
HPP	-	Hardy, de Pazzis and Pomeau (HPP Model)
ILG	-	Immiscible Lattice Gas
LGA	-	Lattice Gas Automata
LGCA	-	Lattice Gas Cellular Automata

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CHAPTER 1

INTRODUCTION

This chapter describes the background of the lattice gas automata methods as a numerical method for simulation of fluid flow in porous media. Simulation studies conducted by previous researchers are also reviewed. Objectives and scope of work based on the statement problem of the research are also described. Finally, the chapter provides an overview of the content of this thesis.

1.1 Background

Modeling of fluid flow in porous media for both single-phase and two-phase flows is of importance in petroleum engineering. Most models for reservoir simulations are on the scale of centimeter to hundred of meters. Usually, increasing resolution in geological measurements result in finer geological models. Many numerical methods have been developed to simulate fluid flow in porous media. Numerical models of fluid flow in porous media can be developed from either microscopic or macroscopic properties. Attention is then typically focused on the determination of the petrophysical properties of the porous media and its performance based on the microscopic pore-space geometry. Due to the intrinsic inhomogeneity of porous media makes the application of proper boundary conditions difficult. Hence, microscopic flow calculations have typically been achieved with idealized arrays of geometrically simple pores and throats.

Because fluid flow in porous media is an important subject in petroleum engineering, numerous theoretical and experimental studies have attempted to investigate its performance. Rothman (1988), reported that although these investigations are diverse in approach, they can be classified broadly into three categories based on their use of microscopic data. First, some studies employ no microscopic data at all; these studies attempt instead to relate macroscopic rock properties, such as relating permeability to resistivity and porosity (e.g., Walsh and Brace, 1984; Paterson, 1983). In the second category are studies that collect microscopic data on pore-space geometry, usually via microscopic and digital image analysis (e.g., Lin and Cohen, 1982), and then compute macroscopic statistics from these microscopic data in attempt to relate their macroscopic rock properties to the statistical properties (e.g., Berryman and Blair, 1986; Lin *et al.*, 1986). The third category is based entirely on microscopic rock geometry (e.g., Koplik *et al.*, 1986).

The finite difference and finite element methods have been useful for simulating single-phase and two-phase flow in porous media, and have been used extensively. Numerical methods based on the finite difference approximation of the governing equations are probably the most commonly used tools for simulating the single-phase and two-phase flow process, and predicting their performance. In practice, the porous media are usually represented by discrete grid block, and transfer of each constituent being tracked is computed across each block face for a succession of small time increments. Finite difference or finite element methods use floating-point numbers to describe properties, a large number of grid blocks are often required, and appropriate boundary conditions are difficult to be applied. As a result, they may not be the most efficient numerical method for this problem.

Despite this extensive study, Dullien (1979), shown that theoretical estimates of macroscopic rock properties are often in error by as much as an order of magnitude or more. The lack of success of these theoretical models, however, could be the result of faulty flow models, inadequate representations of pore space, or both. In this research, a different approach is used to model the fluid flow in heterogeneous porous media. The difference between this work and the existing theoretical literature (finite element and/or finite difference methods) on fluid flow in porous media lies in the numerical method used to model the fluid flow. The numerical method used is Lattice Gas Automata. This alternative method was first introduced by Hardy *et al.* (1973; 1976) and was extended by Frisch *et al.* (1986; 1987), for the computational of fluid dynamic. The method is based on the knowledge of microscopic rock geometry, which falls in the third category detailed above. This is due to the microgeometric space as the Navier-Stokes equations are solved numerically with appropriate boundary conditions. Therefore, the results obtained with the lattice gas automata should agree well with the experimental results insofar as the microscopic model adequately represents the real porous media. Hence, lattice gas automata methods are applicable to the study of fluid flow in porous media.

Dullien (1979), also reported that previous numerical techniques have typically employed an array of geometrically simple pores and throats; the results have been approximate models of the microscopic flow. Recent advances in fluid mechanics (Frisch *et al.*, 1986) and computer science (Margolus *et al.*, 1986), suggest that accurate calculations of microscopic flow are practicable in arbitrarily complex pore-space geometry. Rothman (1988), also reported that the relevant advance in fluid mechanics is the advent of the discrete lattice gas automata. Although lattice gas automata can be implemented on any computer, massively parallel machines and certain special-purpose computer, perform these flow computations considerably more efficiently than conventional methods (finite difference and finite element). The utility of lattice gas automata for computations of fluid flow in porous media, stems from the ease with which computations are made in grossly irregular geometries, where no special grids blocks are required, and appropriate boundary conditions are easily applied at all solid-fluid boundaries.

Lee *et al.* (1993), used the lattice gas automata method for hydrodynamic calculations. The lattice gas automata method employs interactions of discrete fluids on a regular lattice analogous to microscopic molecular dynamics. Therefore, a complex system can be simulated by simple rules of particle interactions at a lattice. Macroscopic variables are then recovered by averaging over a spatial and temporal space. Computationally this method has two main advantages over conventional methods. Firstly, the mathematical operations are mainly bit manipulation, which provides memory efficiency, thereby easily simulating a very large system. Secondly, the algorithm is inherently parallel.

The microscopic and macroscopic nature and bit basis of lattice gas automata means it is also endowed with several other attractive features, as follows (Biggs *et al.*, 1998): (a) it replaces costly floating-point calculations with fewer Boolean and/or table look-up operations; (b) each point in space demands significantly less memory; and (c) boundary conditions are easily and simply applied even for complex geometries such as those found within porous solid.

The lattice gas automata model for the numerical solution of the Navier-Stokes equations provide the lattice gas model with sufficient symmetry and the local rules for collisions between particles obey the conservation law as presented by Frisch *et al.* (1986), and d’Humieres and Lallemand (1986). In their model, time, space, mass and velocity of microscopic fluid particles are all discrete. Macroscopic properties can be obtained from averaging microscopic properties over time and space domains.

Wolfram (1986), showed that the macroscopic behaviour of certain cellular automata correspond to the Navier-Stokes equations for fluid flow. He derived the kinetic and hydrodynamic equations for a particular cellular automata field. Slightly modified Navier-Stokes equations were obtained in two and three dimensions with certain lattices. Viscosity and other transport coefficients were calculated using the Boltzmann transport equation approximation. He showed that the cellular automata method could potentially be applied to a wide variety of processes conventionally described at a macroscopic scale by partial differential equations.

There are numerous research and publications on the applications of lattice gas automata in hydrodynamics, but there are only a handful of studies related to applications in porous media. The first work was that of Balasubramanian *et al.* (1987). They created the FHP-I model of lattice gas automata to study fluid flow in porous media. Balasubramanian *et al.* (1987) and Hayot (1987), introduced into the lattice gas model a random distribution of fixed points scatterers. The Navier-Stokes equation was modified by introducing a damping terms which is proportional to velocity. Then, Darcy’s law obtained and permeability was related to the scatterers of particles density. Even though the permeability of porous media can be obtained, the effects of grain sizes distribution in porous media was not taken into account.

Rothman (1988), showed that lattice gas automata fluids could be applied to the study of flow in porous media. The complex geometry of the porous media simulated by placing solid obstacles of specific shapes in the fluid and imposing a no-slip boundary condition on all nodes within this boundary. The loss in momentum was related to the pressure gradient. Rothman's model, in principle, allows for the incorporation of pore and grain shapes of arbitrary complexity. This is very useful in the calculations of permeability. Where, the permeability calculated by Rothman is based on Darcy's equation. However, other approach using other equations such as the Carman-Kozeny equation is still required in predicting of permeability.

Zhang (1989), illustrated the linear and nonlinear behaviour of lattice gas automata for simulating fluid flow in porous media, as derived from relevant boundary conditions and creating a pressure gradient from Darcy's law. Stauffer (1991), applied the lattice gas automata model in fluid mechanics and summarized the results for applications in flow through porous medium. Chen *et al.* (1991b), used the lattice gas automata model to study the variation of the Forcheimer equation parameters as a function of Reynolds number for a two-dimensions porous solid model. While Knackstedt *et al.* (1993), used the lattice gas automata model on two-dimension porous solid to confirm the existence of a scaling law for the dynamic permeability. Gao (1994), also used lattice gas automata model to study the effect of structure on the petrophysical properties of porous media and dispersion within a porous solid or pores. Based on these studies, it is shown that lattice gas automata can be applied to simulate fluid flow in porous media. However, studies have been conducted mostly in the area of homogeneous porous media. Hence, extensive work is needed to apply the lattice gas automata model in heterogeneous porous media.

Furthermore, the first lattice gas automata model on the study of immiscible fluids was that of Rothman and Keller (1988), called immiscible lattice gas (ILG) model. The model was originally developed for two immiscible species in two-dimensions. The two-dimension immiscible lattice gas model builds upon the original FHP models (Frisch *et al.*, 1986; d'Humieres *et al.*, 1987), with additional requirement of species conservation during collision. The separation of two species into separate phase was induced by biasing collision outcomes in such a way that black particles head towards regions of highest black particle concentration, and vice versa for red particles. Rem and Somers (1989), developed the immiscible lattice gas algorithm to calculate the colour gradients at each site using concentration from adjacent site for flow involving solid surfaces such as those within porous media.

Zaleski *et al.* (1990), developed a theoretical basis of the phase separation algorithm based on Boltzmann assumption. Based on this assumption, the immiscible lattice gas reproduced Laplace's equation for droplets within a stationary mixture (Zaleski *et al.*, 1990; Somers *et al.*, 1991). Furthermore, the immiscible lattice gas model was used by Flekkoy and Rothman (1995), to study the interface between two immiscible fluids, while Boghosian *et al.* (1996), extended the method to the study of emulsions. All these studies essentially used for phase separation simulation and to estimate surface tension of two immiscible fluids. However, the extension study to estimate specifically the interface width and capillary pressure has not been discussed yet. Capillary pressure is one of the important parameters which needs to be identified in order to displace oil in the reservoir by the other fluids when two immiscible fluids are in contact.

Based on the review of the simulation studies conducted by previous researchers, it is shown that majority of previous studies concentrated on the development of lattice gas automata as a tool for permeability prediction on homogeneous porous media based on Darcy's equation and validation of scaling laws for this property. Further work is required to conduct research on the application of lattice gas automata for simulation of a single-phase and two-phase fluid flow in heterogeneous porous media. These simulation works are motivated by the fact that laboratory experimental determination of macroscopic properties and fluid flow behaviour of heterogeneous porous media can be expensive and time consuming, and limited to relatively small samples compared to the applications on real porous media. Besides that, the laboratory displacement experiments are usually conducted on sandpack and cores. Unfortunately these experiments do not allow us to observe directly the physics of the displacement on the microscopic pore level, such as displacement mechanisms and mechanism of residual oil saturation.

1.2 Statement of Problem

Numerous simulation studies have been established regarding fluid flow in porous media. A great many researches had employed different numerical methods to reach different objectives. Among the numerical methods used is the lattice gas automata method that shows a promising future for use in simulation systems with potential capability of predictions. Unfortunately, most of these studies have not considered some aspects of fluid flow in porous media. For instance, most of the researches of lattice gas automata method have focused on homogeneous porous media.

In such studies, aspects of the disordered morphology such as pore size geometry and distribution on the pore space have not been taken into account with only general assumption made on the homogeneous porous media. Although, such assumptions seem quite ideal, they lead to limited applicability of these studies since almost all porous media considered in petroleum engineering are of that type which composes a heterogeneous structure. A comprehensive study is thus required to take these aspects into consideration. Such a study would be essential to enhance the applicability of the lattice gas automata method. In addition, the inclusion of such aspects are expected to improve the understanding of the effects of heterogeneity on the fluid flow process through porous media in both single-phase and two-phase fluid flow, which is of importance in reservoir engineering. Furthermore, it is also important for other relevant processes, such as phase separation of two immiscible fluids and immiscible fluids displacement.

Assumption on heterogeneous porous media requires focusing on the effects of grain shape and size geometry and its distribution on the pore space both for single-phase and two-phase fluid flow. These specific problems call for studies of the macroscopic properties of heterogeneous porous media, the fluid flow behaviour, and performance of lattice gas automata method to predict these properties. All these prescribed lines will bring the whole system to the main question, i.e., is the lattice gas automata method capable of describing the single-phase and two-phase fluid flow in heterogeneous porous media.

In summary, the following problems are addressed:

a. Most corresponding studies utilizing lattice gas automata method have assumed a homogeneous porous media, which is not the case in most cases of real porous media.
b. A heterogeneous porous media is always the real media as it allows the study of the effects of grain size geometry and distribution on the pore space on fluid flow and immiscible displacement process.
c. Lack of studies dealing with these aspects for both single-phase and two-phase fluid flow in heterogeneous porous media.
d. Comprehensive studies of the macroscopic properties, displacement mechanisms of the immiscible displacement process in the heterogeneous porous media, and phase separation of two immiscible fluids are essential in order to reach an appreciable solution for the problems in question.

1.3 Objectives of the Research

In order to solve the problems above, the objectives of the research are as follows:

a. To formulate a micro level model that can describe a single-phase and two-phase fluid flow through heterogeneous porous media in two-dimensions.
b. To determine the macroscopic properties of heterogeneous porous media, i.e., tortuosity, specific surface area, effective porosity and permeability, and to correlate between those properties.
c. To determine the effects of grain shape and size geometry and their distribution in the pore space on fluid flow and immiscible displacement processes.
d. To study the phase separation, and estimate the surface tension and capillary pressure of two immiscible fluids in porous media.
e. To determine the displacement mechanisms of displacing and displaced fluids, and to predict the displacement efficiency (E_D) of the process in heterogeneous porous media.

1.4 Scope of Work

Scope of work of the research, are as follows:

1. To build a FHP-II model of lattice gas automata that can be used to simulate and study a single-phase flow in heterogeneous porous media. Two sizes of models with 400x300 lattice units and 800x600 lattice units will be built.

2. To build a FHP-II model of lattice gas automata that can be used to simulate and study a two-phase fluid separation (immiscible fluids) in porous media. Three sizes of models with 200x200 lattice units, 500x500 lattice units and 700x700 lattice units will be built.

3. To build a FHP-II model of lattice gas automata that can be used to simulate and study the immiscible displacement process in heterogeneous porous media. A model size of 800x600 lattice units will be built.

4. To develop a two-dimensional simulator of a single-phase and two-phase fluid flow in heterogeneous porous media. A computer programme will be written in Borland Delphi-5.

5. To use a simulator to study:
   a. Effects of grain shape and size geometry of obstacles constructed representing the heterogeneous porous media to estimate the quantities of macroscopic reservoir rock properties, i.e., tortuosity, specific surface area, effective porosity and permeability. Two types of shapes (spherical and rectangular) for three grains sizes (10, 20, and 30 lattice units) will be studied.
   b. Phase separation, surface tension and capillary pressure of two immiscible fluids.
   c. The displacement mechanisms of displacing and displaced fluids, and to predict the displacement efficiency (E_D) of the process in heterogeneous porous media.

1.5 Outline of the Thesis

In Chapter 2, the properties of reservoir rock and fluid flow in porous media are discussed. The properties of reservoir rocks are discussed in terms of porosity, permeability, tortuosity, fluid saturations, specific surface area, surface force and capillary pressure. Reservoir heterogeneity, horizontal flow and the immiscible fluids displacement are also discussed.

In Chapter 3, the lattice gas automata methods are described in detail. This chapter covers types of lattice gas models and evolution of particles on the lattice. In addition, initial conditions, boundary conditions, the Navier-Stokes equation of lattice gas automata, microscopic and macroscopic equations for the lattice gas model, fluid flow in porous media by lattice gas automata, and lattice gas model for immiscible fluids are also looked at.

In Chapter 4, the lattice gas automata model is applied for simulation of a single-phase flow in heterogeneous porous media. The model used is the FHP-II model of lattice gas automata. Correlations between macroscopic parameters of transport phenomena in porous media, i.e., tortuosity, specific surface area, effective porosity and permeability are determined. Comparison of the simulation results with results from previous research and laboratory experiments are also presented.

In Chapter 5, the lattice gas automata model is applied to simulate a phase separation of two immiscible fluids. The FHP-II model of lattice gas automata is used as the basis of the model. The study focuses on the simulation of phase separation mechanisms of two immiscible fluids, estimation of surface tension based on Laplace’s equation, and estimation of the capillary pressure.

In Chapter 6, the FHP-II model of lattice gas automata is applied to simulate the immiscible displacement process in heterogeneous porous media. The displacement mechanisms and relative permeability curves are determined. Displacement efficiency (E_D) of the process of displacing and displaced fluids is predicted. Furthermore, comparison of the simulation results with prediction from previous research and laboratory experiments are also presented.

Chapter 7 consists of the conclusions based on the findings of this research and the recommendations for future studies.

1.6 Summary

Numerical simulations are increasingly used in the study of fluid flow in porous media. They are often very useful in connecting theory with experiments, and they can also be used to reduce the number of experiments. The use of lattice gas automata models is possible as an alternative method to the finite difference and finite element methods. The next chapter will discuss the properties of reservoir rocks and fluid flow in porous media.

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