FUZZY STATE SPACE MODELING FOR SOLVING INVERSE PROBLEMS IN MULTIVARIABLE DYNAMIC SYSTEMS

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IN MULTIVARIABLE DYNAMIC SYSTEMS

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To my husband
Mohd Aminudin bin Md Amin.

To my children
Mohd Hanif, Mohd Zharif, Nur Hanis,

Mohd 'Afif, Nur Husna, Nurul Huda.

In memory of my parents

Hajah Rokiah binti Abdullah
(deceased 10 August 2001)
Haji Ismail bin Zainal
(deceased 4 January 2002)

AL-FATIHAH

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In the Name of ALLAH s.w.t, The Most Beneficient, The Most Merciful. All praise is due only to ALLAH s.w.t, the Lord of the universe. Ultimately, only ALLAH s.w.t has given us the strength and courage to proceed with life in its entirety. His works are truly splendid and wholesome. His knowledge is truly complete with due perfection.

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ABSTRACT

The main objective of this study is to develop a novel inverse modeling technique, known as Fuzzy State Space Model (FSSM). This model is used for optimization of input parameters in multivariable dynamic systems. In this approach, the flexibility of fuzzy modeling is incorporated with the crisp state space models proposed in the modern control theory. The vagueness and uncertainty of the parameters are represented in the model construction, as a way of increasing the available information in order to achieve a more precise model of reality. Subsequently, the inverse Fuzzy State Space algorithm is formulated for a multiple-input single-output system, which leads to the derivation of Modified Optimized Defuzzified Value Theorem. This algorithm is enhanced to address the optimization of parameters for a multiple-input multiple-output system, which leads to the derivation of an Extension of Optimized Defuzzified Value Theorem. The proofs of these theorems are presented. To facilitate the implementation of these algorithms, a semi-automated computational tool using Matlab® programming facilities is developed. The effectiveness of this modeling approach is illustrated by implementing it to the state space model of a furnace system of a combined cycle power plant. The results obtained in this application demonstrate that the proposed new modeling approach is reasonable and provides an innovative tool for decision-makers. In addition, the investigations on the properties and characteristics of FSSM have resulted in the derivation of some lemma and theorems related to convexity and normality of the induced solution of the model, and bounded stability of the Fuzzy State Space system. Finally, the properties of the induced solution of a single FSSM are generalized to the multi-connected systems of FSSM. Some algebraic views related to the systems of FSSM are also discussed.

ABSTRAK

Objektif utama kajian ini adalah untuk membangunkan satu teknik pemodelan songsang yang baru, dikenali sebagai Model Keadaan Ruang Kabur (MKRK). Model ini di gunakan untuk pengoptimuman parameter masukan dalam sistem dinamik multi-pembolehubah. Dalam pendekatan ini, pemodelan kabur yang fleksibel digabungkan dengan model keadaan ruang rangup dari teori kawalan moden. Kesamaran dan ketidakpastian bagi parameter di wakili dalam pembentukan model, sebagai satu cara menambah maklumat supaya menghasilkan model yang lebih tepat. Seterusnya, algoritma Keadaan Ruang Kabur dibentuk untuk sistem pelbagai-masukan keluaran-tunggal, yang mana Teorem Nilai Penyahkaburan Optimum Ubahsuaian diterbitkan. Algoritma ini ditambahbaik untuk pengoptimuman parameter masukan bagi sistem pelbagai-masukan pelbagai-keluaran, yang mana Teorem Nilai Penyahkaburan Optimum Lanjutan diterbitkan. Bukti teorem-teorem tersebut ditunjukkan. Untuk memudahkan perlaksanaan algoritma ini, alat pengkomputeran separa-automotik menggunakan kemudahan pengaturcaraan Matlab® disediakan. Keberkesanan pendekatan pemodelan ini ditunjukkan dengan perlaksanaan keatas model keadaan ruang sistem relau bagi sebuah loji janakuasa kitar padu. Keputusan yang dihasilkan menunjukkan bahawa pendekatan pemodelan baru yang dicadangkan adalah berpatutan dan boleh menjadi satu alat yang innovatif kepada pembuat kataputus. Selain daripada itu, penyelidikan mengenai sifat dan ciri-ciri MKRK telah menghasilkan beberapa lemma dan teorem yang berkaitan dengan kecembungan dan normal bagi penyelesaian teraruh model , dan kestabilan terbatas bagi sistem Keadaan Ruang Kabur. Akhir sekali, sifat-sifat penyelesaian teraruh bagi sebuah MKRK dilanjutkan kepada beberapa sistem MKRK yang berhubung. Beberapa pandangan dari sudut algebra berkaitan dengan sistem MKRK juga dibincangkan.

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General symbols

$\{x_1, x_2, x_3, \ldots\}$	-	Set of elements $x_1, x_2, x_3, \ldots$
$\{x \mid p(x)\}$	-	Set determined by property p
$(x_1, x_2, x_3, \ldots, x_n)$	-	n-tuple
$[x_{ij}]$	-	Matrix
$[x_1, x_2, x_3, \ldots, x_n]$	-	Vector
$\lVert \cdot\cdot \rVert$	-	Euclidean norm
min $[x_1, x_2, x_3, \ldots, x_n]$	-	Minimum of $x_1, x_2, x_3, \ldots, x_n$
max $[x_1, x_2, x_3, \ldots, x_n]$	-	Maximum of $x_1, x_2, x_3, \ldots, x_n$
i, j, k	-	Arbitrary identifier (indices)
$[a, b]$	-	Closed interval of real numbers between a and b
$[a, b)$	-	Interval of real numbers closed in a and open in b
$[a, \infty)$	-	Sets of real numbers greater than or equal to a
(a, b)	-	Open interval of real numbers between a and b
$(a, b]$	-	Interval of real numbers open in a and close in b
$\Re$	-	Set of real numbers
$\Re^+$	-	Set of positive real numbers
$\Re^n$	-	Set of n-tuple of real numbers
$\aleph$	-	Set of positive integers (natural numbers)
U or X	-	Universal set
$A, B, C, \ldots$	-	Arbitrary sets (crisp or fuzzy)
$A = B$	-	Set equality
$A \neq B$	-	Set inequality
$A \subset B$	-	Proper set inclusion
$A \subseteq B$	-	Set inclusion
$A \cap B$	-	Set intersection

$A \cup B$	-	Set union
$A \times B$	-	Cartesian product of sets A and B
$\varnothing$	-	Empty set
$f\colon X \to Y$	-	Function of f from X into Y
$f^{1}\colon X \to Y$	-	Inverse function of f from Y to X
$R\,(X, Y)$	-	Relation on $X \times Y$
$\Rightarrow$	-	Implies
$\Leftrightarrow$	-	If and only if
$\in$	-	Element of
$\notin$	-	Not an element of
$\ni$	-	Such that
$\exists$	-	There exist (at least one)
$\forall$	-	For all
$\mu_A(x)$	-	Membership grade of x in fuzzy set A
${}^{\alpha}A$	-	α-cut of fuzzy set A
${}^{\alpha+}A$	-	Strong α-cut of fuzzy set A
sup A	-	Supremum of A
supp A	-	Support of fuzzy set A
h(A)	-	Height of fuzzy set A
core(A)	-	Core of fuzzy set A
$a\vert b$	-	a divides b
gcd(a, b)	-	Greatest common divisor of a and b
gcf(a, b)	-	Greatest common feeder of a and b
S_{gF}	-	Fuzzy State Space Model
$[u_1, u_2, \ldots, u_n]^{\mathrm{T}}$	-	Input vector
$[y_1, y_2, \ldots, y_m]^{\mathrm{T}}$	-	Output vector
$F_{Ii}\,(x)$	-	Degree of desirability of using a particular value x
F_{ind}	-	Fuzzy induced performance parameter
$F_{SgF.}$	-	Desired performance parameter
S_{GF}	-	A collection of interconnected FSSM systems
S_{gF}	-	Common feeder
S_{FF}	-	Greatest common feeder

Furnace System

C_F	-	Fuel calorific value	J/kg
c_{pg}	-	Specific heat of exhaust gases at constant pressure	Js/kg^{o}K
c_{gs}	-	Combustion gas specific heat capacity	Js/kg^{o}K
w_F	-	Fuel flow	kg/s
w_A	-	Air flow	kg/s
w_{EG}	-	Gas mass flow through the boiler	kg/s
w_G	-	Exhaust gas (from the gas turbine) flow	kg/s
w_{in}	-	Inlet mass flow	kg/s
w_{ou}	-	Outlet mass flow	kg/s
w_g	-	Flow of substances entering combustion	kg/s
h_G	-	Exhaust gas (from the gas turbine) specific enthalpy	J/kg
h_A	-	Air specific enthalpy	J/kg
h_{EG}	-	Gas specific enthalpy	J/kg
h_{ref}	-	Reference exhaust gases enthalpy	J/kg
h_{in}	-	Inlet specific enthalpy	J/kg
h_{ou}	-	Outlet specific enthalpy	J/kg
Q_{ir}	-	Heat transferred by radiation to risers	J/s
Q_{is}	-	Heat transferred by radiation to the superheater	J/s
Q_{rs}	-	Heat transferred to reheater	J/s
Q_{gs}	-	Heat transferred to superheater	J/s
Q_{es}	-	Heat transferred to the economiser	J/s
Q_{in}	-	Incoming heat flow	kg/s
Q	-	Heat flow	J/s
T_g	-	Gas temperature	^{o}K
T_{gs}	-	Gas temperature at the superheater	^{o}K
T_{st}	-	Superheater metal tube temperature	^{o}K
T_{gr}	-	Combustion gas temperature at the reheater	^{o}K
T_{rh}	-	Reheater metal tube temperature	^{o}K
T_{ge}	-	Combustion gas temperature at the economiser	^{o}K
T_{et}	-	Economiser metal tube temperature	^{o}K
T_{gl}	-	Boiler exhaust gas temperature	^{o}K
T_{ref}	-	Reference exhaust gases temperature condition	^{o}K

T_m	-	Metal temperature	^oK
T_s	-	Steam temperature	^oK
R_s	-	Stoichiometric air/fuel volume ratio	(-)
R_{EG}	-	Ideal gas constant for exhaust gases	
y	-	The percentage excess air level	%
V	-	Volume	m^3
V_F	-	Combustion chamber volume	m^3
σ	-	Stefan-Boltzman constant	
θ	-	Burner tilt angle $0< \theta <1$	(-)
γ_A	-	Content of fresh air in exhaust from gas turbine	(-)
γ	-	Friction coefficient	(-)
ρ_{EG}	-	Gas density	kg/m^3
ρ	-	Specific density	kg/m^3
p_G	-	Gas pressure in furnace	Pa
p_{in}	-	Inlet pressure	Pa
p_{ou}	-	Outlet pressure	Pa
k	-	An attenuation coefficient	(-)
k_{gs}	-	An experimental coefficient	J/kg^oK
k_{rs}	-	An experimental coefficient	J/kg^oK
k_{es}	-	An experimental coefficient	J/kg^oK
k_f	-	A friction coefficient	
K	-	A coefficient	(-)

Abbreviations

FSSM	-	Fuzzy State Space Model
MISO	-	Multiple-input Single-output
MIMO	-	Multiple-input Multiple-output
BIBO	-	Bounded-input Bounded-output
TFN	-	Triangular Fuzzy Number
FWA	-	Fuzzy Weighted Average
LIA	-	Level Interval Algorithm

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CHAPTER 1

INTRODUCTION

1.1 Background and Rationale

The design of mathematical models of complex real-world systems is essential in many fields of science and engineering. The need for new approaches and philosophies in modeling and control of complex industrial systems is much influenced by recent advances in information technology, increased market competition, the demand for low cost operation and energy efficiency. In the electricity industry for example, power generation plants need to operate optimally in order to stay competitive, as even a small improvement in energy efficiency would involve substantial cost savings. For a large complex system such as power generation plants, it is useful to decompose the system into subsystems or components that can be analyzed and understood separately. The physical structure of the system often suggests a suitable subdivision. The inverse concept, composition, is naturally applied to construct large systems from simple components or subsystems. These subsystems can be interconnected in a flat or hierarchical structure such that an output of one subsystem becomes an input to another subsystem.

The traditional “mechanistic” approach to modeling is based on a thorough understanding of the nature and behavior of the actual system, and on a suitable mathematical treatment that leads to the development of a model (Babuska, 1998). This approach is also termed as first-principle, physical or white-box modeling. However, the requirement for a good understanding of the physical background of the system has proven to be a severe limiting factor in practice, when complex and poorly understood systems are considered. Another approach for building mathematical models is called black-box modeling (Ljung, 1987). In this approach, the modeling problem reduces to postulating an appropriate structure of the approximator, in order to correctly capture the dynamics and nonlinearity of the system. A severe drawback of this approach is that the structure and parameters of these models usually do not have any physical significance, and therefore are less useful for industrial practice. The limitations of these two approaches have led to the development of semi-mechanistic or grey-box modeling approaches (Lindskog, 1997; Babuska, 1998). This modeling technique combines the advantages of the white-box and black-box approaches. The known parts of the system are modeled using physical knowledge, and the unknown or less certain parts are approximated using process data and black-box modeling structures with suitable approximation properties.

Traditional grey-box approaches assume that the structure of the model is given directly as a parameterized mathematical function, which is based on physical principles. However, for many real-world systems a great deal of information is provided by human experts, who describe the system verbally through vague, uncertain or imprecise statements. The fact that humans are often able to manage complex tasks under significant uncertainty has stimulated the search for alternative modeling and control paradigms.

The most relevant information about any system comes in one of three ways, that is: a mathematical model, sensory input and output data or measurement, and human expert knowledge. The common factor in all these three sources is knowledge. For many years, classical control designers began their effort with a mathematical model and did not go any further in acquiring more knowledge about the system (Jamshidi, 1997). Today, control engineers can use all of the above sources of information. Apart from a mathematical model whose utilization is clear, numerical input-output data can be used to develop an approximate model as well as a controller, based on the best available knowledge to treat uncertainties in the system. A typical example of techniques that make use of human knowledge and deductive processes is fuzzy modeling. Furthermore, fuzzy sets also provide a tool for handling ill-conditioned or ill-posed problems, which exist as a result of combining measurements with engineering models. The inverse problem (Hensel, 1991), or more precisely the inverse modeling, is one type of ill-conditioned or ill-posed problems. In inverse modeling, the desired responses are given and a model is used to estimate the input parameters. Thus, fuzzy models can be seen as logical models which use logical operators to establish qualitative relationships among the variables in the model. At the same time, at the computational level, fuzzy models can be regarded as flexible mathematical structures, which can approximate a large class of nonlinear systems to a desired degree of accuracy (Zeng and Singh, 1995). This duality allows qualitative knowledge to be combined with quantitative data in a complementary way.

Fuzzy sets were first proposed in the early 1960s by L. A. Zadeh as a general model of uncertainty encountered in engineering systems (Zadeh, 1965). He wanted to generalize the traditional notion of set and statement to allow grades of memberships and truth values, respectively. His efforts were prompted by the two main complications that arise during physical modeling in the real world (Zimmerman, 1985). Firstly, many real situations are not crisp and resolute; hence they cannot be described precisely. Secondly, the complete description of a real system often requires far more detailed data than a human being could ever recognize simultaneously, process and comprehend. Thus, his approach emphasized modeling uncertainties that arise commonly in human thought processes. According to him, a fundamental limitation of our ability to characterize complex dynamic systems is best captured by the “principle of incompatibility” (Zadeh, 1973). Roughly, this principle states that the more complex a system is, the less is our ability to make precise yet relevant characterization about it. The implication of this principle is two fold. First, we may not be able to acquire a precise model at all in the conventional quantitative sense for a complex dynamic system. In this case, we need to settle with a less precise alternative model formulism suitable to represent our knowledge about the system. Second, even if we are able to obtain a precise quantitative model for a complex system, the precision offered by the model often reveals too many details of the system. This tends to blur its critical characteristics which are useful for some specific purpose. Therefore, it is essential that a complex system is modeled just at the right “resolution” without going into unnecessary details.

Based on this background discussion, this research presents a new inverse modeling technique for multivariable dynamic systems, called the Fuzzy State Space Model (FSSM). The philosophy in the construction of this model is discussed in the next section. In this model, the flexibility of fuzzy modeling is incorporated with the crisp state space models proposed in the modern control theory. There are two important facts that make this modeling approach intuitively appealing. Firstly, there are always uncertain factors affecting the system in a real-world modeling situation. This indicates that a complete physical model can hardly ever be constructed. However, uncertain factors can be taken care of by employing a sufficiently flexible model. Secondly, the restriction on the flexibility to comply with prior knowledge is allowed in the modeling procedure.

1.1.1 Approaches in Model Construction

In the knowledge-based construction of the FSSM, three different kinds of models are considered. The relations between these models are shown in Figure 1.1. From experience, intuition and expert knowledge, we build a mental model in our mind. The verbal model is then formulated using “If…Then…” rules, which is a very common means of description in everyday life. The verbal model can also be formulated based on fuzzy or uncertain descriptions such as “about 15”, “almost 40”, “around 600”. To take account of these uncertainties in the model, the uncertain value parameters of the system are represented by fuzzy numbers (Kaufmann and Gupta, 1985) with their membership function derived from expert knowledge. Fuzzy numbers, which are based on the concept of fuzzy sets (Zadeh, 1965), are used to analyze and manipulate approximate numeric values. Thus, fuzzy sets serve as a smooth interface between qualitative variables and numerical domains of the inputs and outputs of the model.

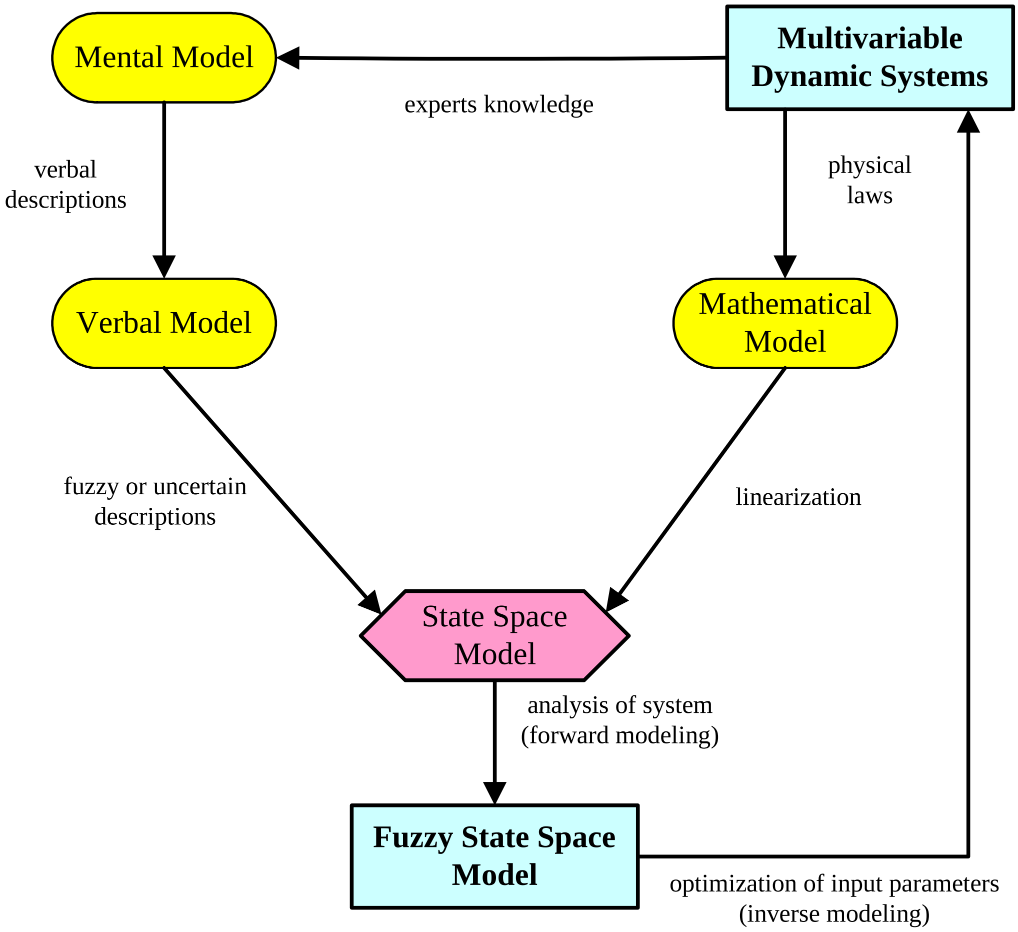

Figure 1.1 Construction of the Fuzzy State Space Model

For system analysis and engineering purposes, mathematical models are often constructed, for instance, based on algebraic and differential or difference equations which are derived from physical laws. For well-defined systems, these standard mathematical tools lead to good models, even though the modeling process is often very tedious. However, most of the real-world systems are complex and nonlinear. An analytical approach for such systems is available only to a very limited extent (Bossel, 1994). On the other hand, a well-developed set of analytical tools is readily available for linear systems. Thus, linearization of nonlinear systems into linear systems plays an important role in this study. The transformation of a nonlinear dynamic system into a linear state space model is presented in Figure 1.2.

A **nonlinear dynamic system** with n inputs $u_1(t), u_2(t), \ldots, u_n(t)$, m outputs $y_1(t), y_2(t), \ldots, y_m(t)$ and p state variables $x_1(t), x_2(t), \ldots, x_p(t)$ may be described by vector functions

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \quad \text{for} \quad 0 \le t < \infty$$

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \quad \text{for} \quad 0 \le t < \infty$$

⇓ LINEARIZATION

A **linear state space model**

$$\dot{\boldsymbol{x}}(t) = A\,\boldsymbol{x}(t) + B\,\boldsymbol{u}(t) \quad .$$

$$\boldsymbol{y}(t) = C\,\boldsymbol{x}(t) + D\,\boldsymbol{u}(t)$$

where A is the state matrix of order p, B is the $p \times n$ input matrix, C is an $m \times p$ output matrix and D is an $m \times n$ direct transmission matrix.

Figure 1.2 Linearization of a nonlinear dynamic system

The most important advantage of the crisp state space model is that the system dynamic properties are condensed in the model (Cao and Rees, 1995). For example, stability of the system is examined from the state matrix A, and a controller can be designed based on the system model (A, B). The system model gives both the external and internal behavior of the system. Furthermore, the state space representation is suitable when the prior available knowledge allows to determine the structure of the system under study and to identify the state variables (Babuska and Verbruggen, 1996). Therefore, FSSM can be seen as a modeling framework for blending information of different kinds, qualitative as well as quantitative. It can adequately process not only the given data, but also the associated uncertainty.

The developed FSSM can be used to observe the influence of parameters on system behavior, and apply that knowledge to achieve a desired outcome. The desired outcome is the "known" information, whereas the input parameters to achieve that outcome need to be deduced. This is analogous to an inverse problem, where the measured response is given and a model is used to estimate input parameters. Traditionally, such inverse problems have been addressed by repeated simulation of forward problems, for example Ordys *et al*. (1994), Ram and Patel (1998), which requires excessive computer time. Thus, an inverse methodology using Fuzzy State Space algorithms is developed, whereby the manipulation of imprecise, uncertain quantities is considered. This novel model provides algorithms that address inverse problem in multivariable systems directly. To facilitate the implementation of this model, the algorithms are coded using Matlab® software packages to form a semi-automated computational tool. Using this computational tool, the users can evaluate more alternatives in less time, and at the same time, the users can obtain more information on the performance of each of those alternatives.

The effectiveness of this modeling approach is illustrated by implementing it in a furnace system of a combined cycle power plant, which is regarded as an important constituent in the heat treatment system of a boiler. The objective of optimization of a furnace system is to minimize energy losses and, at the same time, to keep variables within constraints. The theoretical basis that leads to the formulation of the mathematical model for the furnace system can be found in Ordys *et al*. (1994). Based on this mathematical model, the state space model of the furnace system is developed. Subsequently, the state space model is used in the implementation of the Fuzzy State Space algorithm of the furnace system by considering it as a multiple-input single-output (MISO) system. The Fuzzy State Space algorithm is further enhanced and implemented in the furnace system by considering is as a multiple-input multiple-output (MIMO) system. The results obtained in these implementations demonstrate that the proposed new modeling approach is promising, reasonable and effective (Razidah *et al*., 2002a, 2002b, 2004).

In this study, the furnace system of the combined cycle power plant is used as an illustration, but in general, the FSSM is applicable to any multivariable dynamic systems as long as the governing equations of the systems can be transformed into linear state space representations.

1.1.2 Related Studies

The current literature on fuzzy modeling clearly exhibits a wealth of diverse approaches to fuzzy modeling, supporting various methodological points of view and embracing distinct classes of models (Pedrycz, 1995). The ideas of fuzzy models and fuzzy modeling have been formulated and analyzed from both the methodological and experimental standpoints. There is no doubt that the methodology of fuzzy modeling is vital to any application of fuzzy sets.

The terminology "Fuzzy State Space Models" was first cited in a publication by Won *et al*. (1995), where stability analysis and stabilization of the linguistic models using the "If…Then…" rules are discussed. They deal with the fuzzy model proposed by Tanaka and Sugeno (1992) except that the fuzzy model is treated as a linear system having modeling uncertainties and the consequent part of each rule is represented by a state equation. Won *et al*. (1995) used the term "linguistic fuzzy state space models" to be synonymous to linguistic fuzzy dynamic models.

A few years later, Cao *et al*. (1999) proposed a new kind of dynamic fuzzy state space model based on dynamic fuzzy models (Cao and Rees, 1995), which are an extended structure of the fuzzy model used in Sugeno and Yasukawa (1993). They defined a dynamic fuzzy model for a single-input single-output system as follows:

$$\begin{aligned} R^l: \quad & \text{IF } z_1 \text{ is } F_1^l \text{ AND } \ldots z_{2n} \text{ is } F_{2n}^l \\ & \text{THEN } \quad x(t+1) = A_l\, x(t) + B_l\, u(t) \\ & \qquad\qquad y(t) = C_l\, x(t) + d^l \\ & \qquad\qquad l = 1, 2, \ldots, m \end{aligned} \tag{1.1}$$

where R^l denotes the l th approximation inference rule. m is the number of approximation inference rules, $y(t)$ and $u(t)$ are the output and input variables of the system, $x(t)$ are the state variables, (A_l, B_l, C_l, d^l) represent the crisp input and output relationship or dynamic properties of the system in the local region F^l where

$$F^l = \prod_{i=1}^{2n} F_i^l \text{ , and} \qquad z(t) = [y(t),\ldots, y(t-n+1), u(t),\ldots, u(t-n+1)] \qquad (1.2)$$

$$= (z_1, z_2,\ldots, z_{2n})$$

By using the fuzzy-inference methods described in Cao and Rees (1995), the dynamic fuzzy model (1.1) is expressed by the dynamic fuzzy state space model as follows:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + D \end{aligned} \qquad (1.3)$$

where $A = \sum_{l=1}^{m} \mu_l A_l$ $\qquad B = \sum_{l=1}^{m} \mu_l B_l$

$C = [0,0,...,0,1]$ $\qquad D = \sum_{l=1}^{m} \mu_l d^l$

In other words, the above dynamic fuzzy state space model (1.3) represents the evolution of the dynamic fuzzy model (1.1). Subsequently, a parameter estimator is developed for a plant that can be represented by a dynamic fuzzy state space model (Cho *et al*., 2001). The fuzzy controller is constructed from a fuzzy feedback linearization controller whose parameters are adjusted indirectly from the estimates of plant parameters. Thus far and to the best of our knowledge, work reported in the literature related to fuzzy state space models has focused on the direct or forward modeling techniques for designs of controllers.

On the contrary, the present study considers a new approach in the formulation of the Fuzzy State Space Model for solving inverse problems in multivariable dynamic systems, for both MISO and MIMO systems. Fuzzy arithmetic and fuzzy number are used in the computation and evaluation of the influences of uncertain or vague parameters, which is different from the fuzzy rule-based model used in the earlier studies. In the fuzzy rule-based model, the number of rules needed to describe the multivariable system increases exponentially. This is a drawback for real-world applications. Fuzzy arithmetic and its operations have been the subject of many publications and several textbooks, for example Dubois and

Prade (1980), Kaufmann and Gupta (1985), Zimmerman (1985), Terano *et al.* (1987), Klir and Yuan (1995), Wang (1997). Various applications of fuzzy arithmetic were reported in the literature, for example Dong and Wong (1987), Wood and Antonsson (1989), Giachetti and Young (1997), Hanss *et al.* (1998), Ahmad *et al.* (1997). In this study, the approach of using fuzzy numbers in developing a fuzzy algorithm for MISO system, which was introduced by Ahmad (1998) and published in Ahmad *et al.* (2004) is of special interest. He adopted the Level Interval Algorithm (Wood and Antonsson, 1989) and used algebraic equations for optimizing parameters of microstrip lines with the aim of minimizing crosstalk.

In this study, we defined the Fuzzy State Space Model of a multivariable dynamic system as

$$\begin{aligned} S_{gF}: \quad \dot{\boldsymbol{x}}(t) &= A\,\boldsymbol{x}(t) + B\,\tilde{\boldsymbol{u}}(t) \quad . \\ \tilde{\boldsymbol{y}}(t) &= C\,\boldsymbol{x}(t) \end{aligned} \tag{1.4}$$

where $\tilde{\boldsymbol{u}}(t)$ denotes the fuzzified input vector $[u_1, u_2,\ldots, u_n]^{\mathrm{T}}$ and $\tilde{\boldsymbol{y}}(t)$ denotes the fuzzified output vector $[y_1, y_2,\ldots, y_m]^{\mathrm{T}}$ with initial conditions as $t_0 = 0$ and $\boldsymbol{x}_0 = \boldsymbol{x}(t_0) = 0$. T is the vector or matrix transposition. The elements of state matrix $A_{p\times p}$, input matrix $B_{p\times n}$, and output matrix $C_{m\times p}$ are known to a specified accuracy. The development of this new FSSM is elaborated in Chapter 4, where inverse Fuzzy State Space algorithms are formulated. The uncertain value parameters of the system are represented by fuzzy numbers with their membership function derived from expert knowledge. In many respects, fuzzy numbers depict the physical world more realistically than single-valued numbers, as the concept takes into account the fact that all phenomena have a degree of uncertainty. The ability of this novel method to address inverse problems in multivariable systems directly, is an outstanding advantage especially in reducing computation time and cost. At the same time, some valuable properties and characteristics of the induced solution of the FSSM are also established. These properties are generalized to a multi-connected system of FSSM. In addition, we have laid some possible theory for systems of FSSM from the algebraic point of view.

1.2 Objectives of the Study

The objectives of this study are as follows:

(i) To develop a Fuzzy State Space Model for optimal input parameters estimation in a multivariable dynamic system.

(ii) To derive and prove some important properties of the induced solution of the Fuzzy State Space Model.

(iii) To implement the Fuzzy State Space Model in the furnace system of a combined cycle power plant.

(iv) To generalize the properties of Fuzzy State Space Model to multi-connected systems.

1.3 Implications of the Study

The most significant advantage of this study is the ability to assess the condition of the multivariable dynamic systems at the preliminary design stage. The Fuzzy State Space Model developed in this study can assist plant operators or designers to recommend a rational and systematic measure for optimizing the input parameters of any multivariable dynamic system. The resulting model finally can be embedded into a special conception of fuzzy-model-based control. At the same time, the outcomes of this study will be disseminated at the national and international level through journals, seminars and conferences.

1.4 Scope of the Study

The study involves developing a new inverse modeling approach for optimal input parameters estimation in multivariable dynamic systems. In order to achieve the stated objectives, the scope of the study is divided into several major areas.

S1. Development of the Fuzzy State Space Model.

A new technique for optimal input parameters estimation for multivariable dynamic systems is developed, where the flexibility of fuzzy modeling is incorporated with the crisp state space models proposed in the modern control theory. This leads to the development of the inverse Fuzzy State Space algorithm for a MISO system, which requires a derivation of a theorem for optimization. A Modified Optimized Defuzzified Value Theorem is derived and proven. Subsequently, the inverse Fuzzy State Space algorithm is enhanced to address the optimization of input parameters for a MIMO system. For the MIMO system, an Extension of Optimized Defuzzified Value Theorem is derived and proven. In this study, a dynamic system is one whose inputs and outputs are related by a set of differential (or difference) equations.

S2. Software development of the Fuzzy State Space algorithm.

Matlab® programming facilities are used in the development of a semi-automated computational tool based on the inverse Fuzzy State Space algorithm. For MISO and MIMO systems, two main programs are developed where the number of input parameters and the number of intervals used for accuracy are specified by the user. Obviously, these values will affect the computation time of the programs. Computation of the percentage errors for the input and output parameters are also included in the programs. This semi-automated computational tool allows users to evaluate easily more alternatives in less time and at the same time; the users can obtain more information on the performance of each of these alternatives.

S3. Properties of the induced solution of the FSSM

Two important properties of the induced solution of the FSSM that are investigated in this study are convexity and normality. Other properties that are studied include bounded-input bounded-output stability (BIBO) of the induced solution of the FSSM.

S4. Implementation of FSSM in the furnace system of a combined cycle power plant

The theoretical concepts and the mathematical equations governing the process in the furnace system are used to develop the state space model. The dynamic behavior of the furnace system is studied through state space analysis, which is similar to forward or direct modeling. Subsequently, a FSSM for the furnace system is developed and tested using steady state operating data. Next, the inverse Fuzzy State Space algorithms for MISO and MIMO systems are implemented in the furnace system. These results are compared to the normally accepted simulation method.

S5. Systems of FSSM

The properties of the induced solution of a single FSSM are generalized to multi-connected systems of FSSM. The induced properties considered in the present study are convexity, normality, BIBO stability and optimized input value parameter. In addition, some algebraic views related to the systems of FSSM are discussed. In particular, the characteristics of a system of FSSM are developed based on the basic properties of divisors and relations.

1.5 Outline of the Thesis

The thesis is organized into seven chapters: this introductory chapter, five main chapters and a chapter of overall conclusion. Each main chapter is self contained, starting with an introduction and culminating with a summary. The present chapter gives a description of the background and rationale in developing the new modeling technique, approaches in model construction, objectives, implications and the overall scope of the study. A literature review on work related to the study is also presented.

In Chapter 2, an overview of system models and the mathematical modeling of control systems are discussed. This leads to a discussion on state space representation and state space analysis of a multivariable dynamic system. The chapter concludes with a description of uncertainties and inverse problems in system modeling, which are among the rationale behind the formulation of the system modeling framework in this study. Chapter 3 describes the theoretical concepts and principles in fuzzy sets, which are particularly useful in the development of the Fuzzy State Space Modeling for solving inverse problems in multivariable dynamic systems. This discussion includes the operations of fuzzy numbers and fuzzy arithmetic, as well as a literature review of some related studies on fuzzy arithmetic.

The main contributions of this study are presented in the next three chapters. Chapter 4 describes the development of a new modeling technique known as the Fuzzy State Space Model. The formulations of the Fuzzy State Space algorithms are presented in detail. To facilitate the implementation of this model, the algorithms are coded using Matlab® software package to form a semi-automated computational tool for MISO and MIMO systems. Five new theorems related to convexity, normality, optimized input value parameter and BIBO stability are derived and proven. To show the effectiveness of this new modeling technique, the inverse Fuzzy State Space algorithms are implemented in the furnace system of a combined cycle power plant. A detailed description of this procedure is illustrated in Chapter 5. Subsequently, a comparison between the results obtained using the inverse Fuzzy State Space algorithms and the forward simulation approach is presented.

Chapter 6 is primarily concerned with multi-connected systems of FSSM, which are the composition of subsystems to form a complex system. Five new theorems are derived and proven, which utilize the properties of FSSM for a single system discussed in Chapter 4. In addition, some foundation for the possible theory of systems of FSSM from the algebraic point of view is presented. In particular, the study is based on the ideas of divisor and relation in number theory. Finally, Chapter 7 contains a summary of research, the main contributions and findings of this study and several recommendations for further research work. The organization of the thesis is summarized in Figure 1.3. All the references in this thesis are listed in the reference section at the end of Chapter 7, which is followed by several appendices.

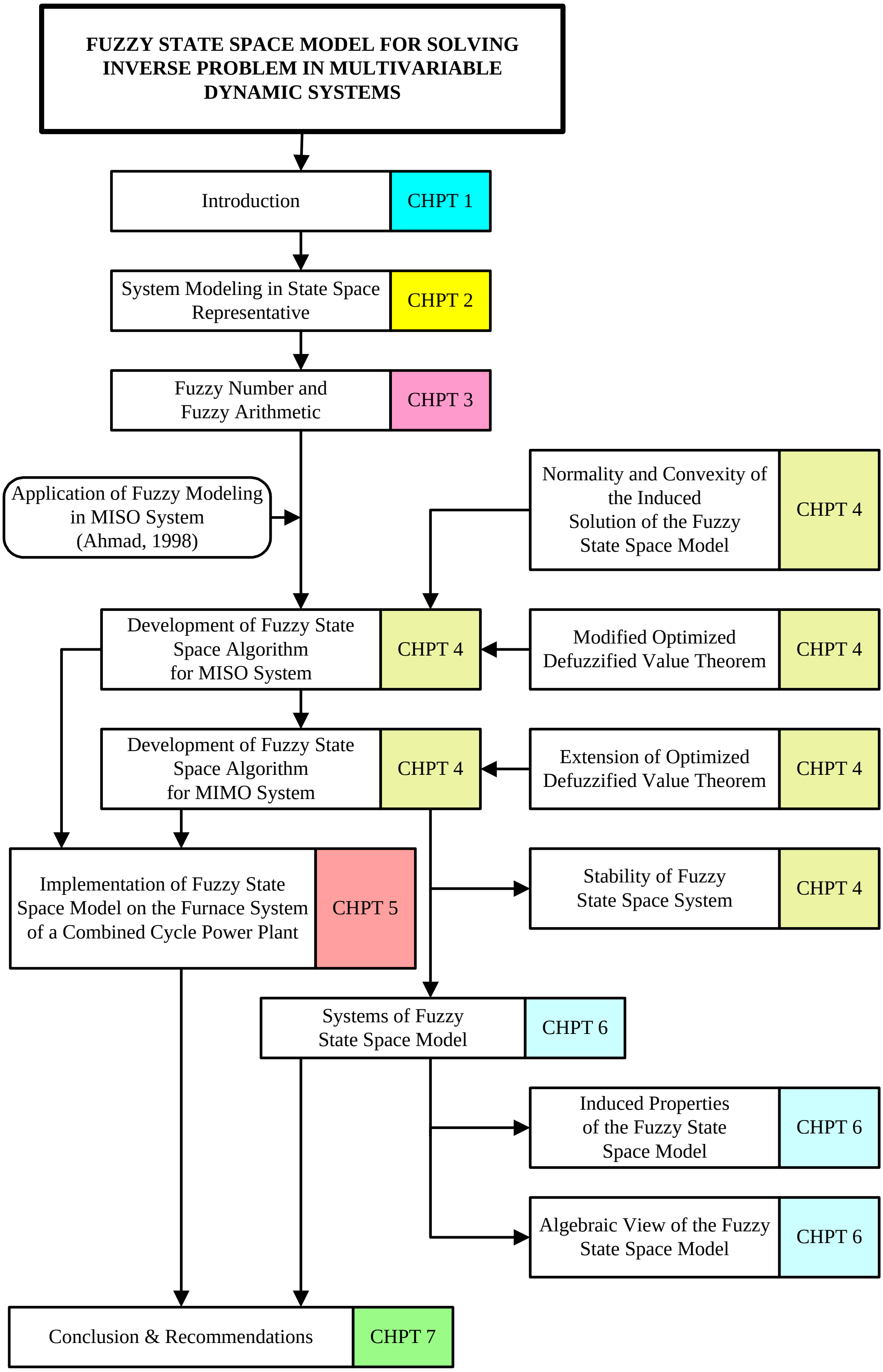

Figure 1.3 Organization of the Thesis

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