

DEVELOPMENT OF HYBRID SKINNING SURFACE METHOD FOR SHIP
HULL DESIGN

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To my Parents, my wife and especially,
My beloved sons, Ali and Mohamed

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ABSTRACT

Non Uniform Rational B-spline (NURBS), despite its acceptance as high-end CAD/CAM/CAGD standard for surface modeling, is still suffering from one major drawback in representing the surface of a ship hull. The present NURBS is still unable to regenerate the ship surface accurately as in the original drawing of the existing design especially with minimum number of control points. This inherent problem is often associated with the parameterization of data points in NURBS curve/surface. This research aims to develop a general NURBS skinning surface method for surfaces that comprise a set of cross sectional curves with particular application to ships hull surfaces. The developed method was based on proposed hybrid parameterization to approximately fit NURBS skinning surface to sets of cross sectional curves with minimum number of control points without sacrificing the original shape. This was achieved in three steps; the hybrid parameter values and averaging parameter knot vector were generated for each of individual cross sectional curve, then control points on NURBS curve with fitting approximation errors were then calculated, and the weights were set accordingly. The compatibility process was performed by applying knot removal within prespecified error tolerance (ε) on cross sectional curves with highest indexes, and knot refinement on cross sectional with lowest indexes. Finally the compatible cross sectional curves were arranged accordingly to get the resultant skinning surface, thus the surface can be generated. The method was tested on several types of ships hull form and other objects in representing the surface with minimum number of control points without sacrificing the original shape. Within prespecified error tolerance (ε), the results show that the proposed method has reduced number of control points for the compatible cross sections up to 90% in comparison to conventional skinning methods, up to 81.82% in comparison to Piegl method, and up to 68.63% in comparison to Hyungjun method. The method has also produced higher accuracy (75.0% – 100%) hull surfaces as compared to the surfaces generated by selected software that are widely used for ship hull design.

ABSTRAK

Splin B Rasional Tak Seragam (SBRTS), di sebalik penerimaannya sebagai piawai tertinggi di dalam CAD/CAM/CAGD untuk pemodelan permukaan, sebenarnya ia masih lagi menghadapi satu kelemahan utama di dalam mempersembahkan permukaan lengkungan badan kapal. Kelemahan utamanya ialah dalam menjana semula permukaan kapal secara tepat seperti mana bentuk asalnya terutamanya dengan menggunakan bilangan titik kawalan yang minimum. Masalah ini biasanya berkait rapat dengan pemparameteran titik kawalan di dalam lengkungan/permukaan SBRTS. Kajian ini dilaksanakan dengan tujuan untuk membangunkan satu kaedah umum penjanaan permukaan SBRTS bagi permukaan yang terdiri daripada beberapa lengkung keratan rentas dengan menumpukan penggunaannya terhadap permukaan badan kapal. Kaedah yang dicadangkan ini berdasarkan kepada kaedah pemparameteran gabungan yang menghasilkan penjanaan permukaan SBRTS yang hampir tepat terhadap beberapa lengkung keratan rentas yang mempunyai bilangan titik kawalan yang minimum tanpa mengubah bentuk asalnya. Perkara ini telah dicapai melalui tiga langkah; Penghasilan nilai parameter gabungan dan purata parameter vektor pintalan bagi setiap lengkung keratan rentas, kemudian pengiraan titik kawalan lengkung SBRTS dengan nilai ralat penyesuaian terdekat, dan seterusnya penentuan pemberat dibuat. Proses penyesuaian dilakukan dengan pembuangan ikatan bagi lengkung keratan rentas berindeks paling tinggi dalam lingkungan ralat toleransi (ϵ), yang ditentukan dan juga perbaikan pintalan bagi lengkung keratan rentas berindeks paling rendah. Akhirnya, lengkung keratan rentas yang telah disesuaikan disusun bagi menghasilkan paduan penjanaan permukaan dan seterusnya membolehkan penghasilan permukaan. Kaedah ini telah diuji terhadap beberapa jenis permukaan badan kapal dan objek lain dengan bilangan titik kawalan yang minimum tanpa mengubah bentuk asalnya. Hasil kajian menunjukkan bahawa, dalam lingkungan ralat terendah (ϵ), kaedah tersebut telah mengurangkan bilangan titik kawalan sehingga 90.0% berbanding dengan kaedah penjanaan konvensional, sehingga 81.82% dengan kaedah Piegl, dan sehingga 68.63% dengan kaedah Hyungjun. Kaedah ini juga menghasilkan permukaan badan kapal yang lebih tepat (75.0% - 100%) berbanding dengan permukaan yang dihasilkan oleh perisian yang sering digunakan bagi merekabentuk kapal.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	vi
	ABSTRAK	vii
	TABLE OF CONTENTS	viii
	LIST OF TABLES	xi
	LIST OF FIGURES	xiii
	LIST OF SYMBOLS	xv
	LIST OF ABBREVIATIONS	xvi
	LIST OF APPENDICES	xvii
1	INTRODUCTION	1
	1.1 Introduction	1
	1.2 Rationale	2
	1.3 Back Ground of the Study	3
	1.4 Problem Statement	8
	1.5 Objectives of Research	10
	1.6 Scope of Research	10
	1.7 Research Methodology	11
	1.8 Organization of Thesis	13
	1.9 Summary	14
2	MATHEMATICAL MODELS FOR SURFACE REPRESENTATION	16
	2.1 Introduction	16

2.2	Explicit Function Representation	16
2.3	Implicit Function Representation	17
2.4	Parametric Representation	17
2.5	Bezier Curves	18
2.6	Rational Bezier Curves	21
2.7	Bezier Surfaces	23
2.8	B-spline Curves	24
2.9	Influence of Knots on the B-spline Curve	30
2.10	B-spline Surfaces	31
2.11	NURBS Representation	34
2.11.1	Formulations and Properties of NURBS	36
2.11.1.1	NURBS Curves	36
2.11.1.2	NURBS Surfaces	41
2.11.2	Factors Influence NURBS Curves/Surfaces	45
2.11.2.1	Control Points (CP)	45
2.11.2.2	Parameterization	45
2.11.2.3	Knot Vector	47
2.11.2.4	Weight	48
2.11.3	Operation on NURBS	48
2.11.3.1	Knot Insertion/Knot Refinement	49
2.11.3.2	Knot Removal	49
2.12	Summary	50
3	PRESENT SKINNING SHIP HULL SURFACE GENERATION	
	GENERATION	51
3.1	Introduction	51
3.2	Skinning Surface	51
3.3	Ship Hull Form	53
3.4	Ship Hull surface Design – A Survey	55
3.5	Development of Skinning Method for Ship Hull surface Design	57
3.5.1	Conventional Skinning Method	58
3.5.2	Piegl Skinning Method	60
3.5.3	Hyungjun Skinning Method	64

3.6	Parameterization Methods	68
3.6.1	Uniform Parameterization Method	68
3.6.2	Chord Length Parameterization Method	68
3.6.3	Centripetal Parameterization Method	69
3.7	Ship Hull Design Software	71
3.7.1	Maxsurf Software	71
3.7.2	ProSurf Software	73
3.8	Summary	74
4	PROPOSED SKINNING SHIP HULL METHOD	75
4.1	Introduction	75
4.2	Proposed Hybrid Skinning Surface Method	75
4.3	Proposed Hybrid Parameterization Method	81
4.4	Proposed Approximation Error	87
4.5	Summary	96
5	RESULTS AND DISCUSSION	97
5.1	Introduction	97
5.2	Skinning Data Reduction	98
5.3	Discussion and Analysis of Error Tolerance, (ε)	109
5.4	Accuracy Analysis	111
5.5	Summary	115
6	CONCLUSION AND FUTURE WORK	116
6.1	Contribution of the Study	116
6.2	Strengths and Limitations of the Study	118
6.3	Future Work	119
	REFERENCES	120
	Appendices A - E	131-146

LIST OF TABLES

TABLE NO.	TITLE	PAGE
2.1	Control Points for the Numerical Example	26
2.2	B-spline Basis functions showing one maximum value within each span index	27
3.1	Comparison between Existing Skinning Methods	67
3.2	Comparison Between Conventional Parameterization Methods	70
3.3	Summary of Merits and Limitations of Maxsurf & Prosurf	74
4.1	Data Points of Cross Sectional Curve	85
4.2	Example of Data Points used in Approximation Error Calculation	88
5.1	Highest and lowest indices for Frd Regions of ship hull	98
5.2	Data Reduction Results for Frd Regions of Oil Tanker, Fishing Vessel and Buoy Tender Vessel (BTV)	99
5.3	Data Reduction of Frd Regions of Fishing Vessel and Buoy Tender Vessel (BTV) Using Hybrid Method in Comparison to Piegl Method	100
5.4	Data Reduction of Frd Regions of Fishing Vessel and Buoy Tender Vessel (BTV) Using Hybrid Method in Comparison to Hyungjun Method	100
5.5	Data Reduction Results of Small Old Yacht Vessel	103
5.6	Data Reduction of Small Old Yacht Vessel Using Hybrid Method in Comparison to Hyungjun Method	103
5.7	Highest and lowest indices for Aft Regions of ship hull	105

5.8	Data Reduction Results for Aft Regions of Fishing Vessel and Buoy Tender Vessel (BTV) and Fireboat	105
5.9	Data Reduction of Aft Regions of Fishing Vessel and Buoy Tender Vessel (BTV) and Fireboat Using Hybrid Method in Comparison to Piegl Method	106
5.10	Data Reduction of Aft Regions of Fishing Vessel and Buoy Tender Vessel (BTV) and Fireboat Using Hybrid Method in Comparison to Hyungjun Method	106
5.11	Range of Error Tolerance for Ship hull Data Reduction	111
5.12	Accuracy Comparison for Frd Regions of Oil Tanker, Fishing Vessel and Buoy Tender Vessel (BTV)	112
5.13	Accuracy Comparison for Aft Regions of Fishing Vessel and Buoy Tender Vessel (BTV)	113
5.14	Accuracy Comparison for Small Old Yacht Vessel and Fireboat	114

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
1.1	Original Body Plan Drawing from Fishing Vessel	6
2.1	A Cubic Bezier Curve	18
2.2	Variation Diminishing Property for Bezier Curve	20
2.3	A Rational Cubic B-spline Curve	22
2.4	Cubic B-spline Basis Functions	27
2.5	A Cubic B-spline Curve as a cubic Bezier Curve	28
2.6	End Point Interpolation and Strong Convex Hull Property for a cubic B-spline Curve	29
2.7	Local Effect of Changing the Position of Control Points	29
2.8	B-spline Curves Using the Same Control Points but with Different Degrees and Different Knot Vectors	29
2.9	Cubic B-spline Curves Defined on the Same Control Points but Different Knot Vectors	30
2.10	Cubic B-spline Curves Defined on the Same Control Points but Different Knot Vectors (Open Nonuniform)	31
2.11	A Biquadratic B-spline Surface	32
2.12	A Surface and its Parameter space	36
2.13	Cubic Rational B-spline Basis Functions	38
2.14	A cubic NURBS curve, which shows End Point Interpolation and Strong Convex Hull Property	39
2.15	Cubic NURBS Curves with Varying Weights	40
2.16	A Biquadratic NURBS Surface with its Control Net	42
3.1	A set of Cross sectional curves	52
3.2	Skinning Surface	52

3.3	A typical Lines Plan Drawing	54
3.4	3D Wireframe Ship Hull Surface	54
3.5	Framework of the Conventional Skinning Method	59
3.6	Framework of Piegl Skinning Method	62
3.7	Framework of Hyungjun Skinning Method	66
3.8	A typical Ship Hull Surface generated Using Maxsurf	72
3.9	A typical Ship Hull Surface generated Using Prosurf	73
4.1	Framework of the Proposed Hybrid Skinning Method	79
4.2	Highest B-spline Basis Functions within Span Indices	82
4.3	Highest Rational B-spline Basis Functions within Span Indices	82
4.4	Framework of the Proposed Hybrid Parameterization Method	84
4.5	Control Polygon of the Cross Sectional Curve	85
4.6	Framework of the Integrated Proposed Hybrid Skinning Method	89
4.7	Algorithm for Computing Hybrid Parameter Values	90
4.8	Algorithm for Knot Removal Operation	91
4.9	Algorithm for Knot Refinement Operation	94
5.1	Bicubic NURBS Surfaces for Frd Region of Oil Tanker	101
5.2	Bicubic NURBS Surfaces for Frd Region of Fishing Vessel	101
5.3	Bicubic NURBS Surfaces for Frd Region of Buoy Tender Vessel	101
5.4	Bicubic NURBS Surfaces for Small Old Yacht Vessel	104
5.5	Bicubic NURBS Surfaces for Aft Region of Fishing Vessel	107
5.6	Bicubic NURBS Surfaces for Aft Region of Buoy Tender Vessel	107
5.7	Bicubic NURBS Surfaces for Fireboat	107
6.1	Difference between the Proposed and Existing Skinning Method	117

LIST OF SYMBOLS

SYMBOL	DISCRIPTION
ε	Error Tolerance
d_i	Chord Length
k	Number of Cross Section
\hat{m}, \hat{n}	Highest Index
\hat{m}', \hat{n}'	Compatibility High Index
p, q	Degree
u, v, t	Parameter value
w_i	Weight
$B_{i,n}(u), B_{j,m}(v)$	Bernstein Basis Function
$C(u)$	Point on the Curve
$N_{i,p}(u), N_{j,q}(v)$	B-spline Basis Function
P^w	Weighted Control Points
$P_i, P_{i,j}, Q_{i,j}$	Control Point
$R_{i,p}(u), R_{i,j}(u, v)$	Rational Basis Function
$S(u, v)$	Point on the Surface
U, V	Knot Vector
T_{cp}	Parameter values due to Centripetal method
T_h	Parameter values due to Hybrid method
U_{cp}	Averaging Parameter Knot Vector Due to Centripetal Method
U_h	Averaging Parameter Knot Vector Due to Hybrid Method

LIST OF ABBREVIATIONS

ABBREVIATION	DISCRIPTION
2D	Two-Dimensional
3D	Three-Dimensional
4D	Four-Dimensional
Aft	After
BTV	Buoy Tank Vessel
CAD	Computer Aided Design
CAGD	Computer Aided Geometric Design
CAM	Computer Aided Manufacturing
CASHSD	Computer Aided Ship Hull Surface Design
Frd	Forward
Max	Maximum
Mid	Middle
Min	Minimum
NUBS	Non-Uniform B-Spline
NURBS	Non-Uniform Rational B-Spline
VT	Voronoi Triangulation

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A1	Algorithm for Computing Nonzero B-spline Basis Functions	131
A2	Algorithm for Computing Averaging Parameter Knot Vector	133
A3	Algorithm for Computing Control points and Approximation Error	134
B1	Sample of Input Offset Data Point	136
B2	Sample of Incompatible Cross Sectional Curves	138
B3	Sample of Compatible Cross Sectional Curves	141
C1	Data Reduction of Frd Region of Oil Tank and Small Old Yacht Vessel Using Hybrid Method in Comparison to Uniform Method	144
C2	Data Reduction of Frd Region of Oil Tank and Small Old Yacht Vessel Using Hybrid Method in Comparison to Chord Length Method	144
C3	Data Reduction of Frd Region of Oil Tank and Small Old Yacht Vessel Using Hybrid Method in Comparison to Centripetal Method	144
D1	Original Body Plan from Fireboat	145
D2	Original body plan drawing from Buoy tender vessel	145
E	List of Publications	146

CHAPTER 1

INTRODUCTION

1.1 Introduction

The application of geometrical modeling to represent an object can be traced back since the era of ancient Egypt. Since then, much advancement has been made, but serious effort on geometrical modeling using computer (computer graphics) only began to emerge in early sixties. One of the factors that contributed significantly to the advancement of Computer Aided Design (CAD), Computer Aided Manufacturing (CAM) and Computer Aided Geometric Design (CAGD) is the development of mathematical models and their algorithms that were able to generate and represent the curves and surfaces of the objects effectively. From the application of simple polynomials in the early sixties to more complicated modeling methods including Bezier and B-Splines developed in mid eighties, much advancement had occurred. To date, Non Uniform Rational B-Splines (NURBS), is the international accepted standard mathematical modeling method for geometrical design and geometrical data exchange. However, despite such advancement, in certain application, there is still need for more accurate and efficient mathematical methods. This research is therefore dedicated to seek further enhancement to the present methods in an effort to develop a more effective tool for computer aided geometric design.

One of the applications that utilized in CAD, CAM and CAGD is ship hull design. The ship hull gained the importance of the fact that the surface of the ship

hull shape has strong influenced on the ship performance at sea. Therefore, it requires accurate and smooth shape of the ship hull to be designed, which in turns leads to the need of suitable mathematical method that can generate the desired surface.

Mathematically, surfaces can be represented either in closed forms (functional) or others as physical models and physical experiments. Functional representation is divided into three classes: explicit form, implicit form and parametric form. Among these, the parametric representation is most useful and widely applied, since it is axis independent and applicable to complex surfaces.

Another important aspect in surface modeling is the representation of surfaces in data set of ordered points, since these data can be raised from direct measurements of physical models, experiments or mathematical analysis of desired properties. Details discussion about the methods of surface representation can be found in (Faux and Pratt, 1979; Mortenson, 1985; Hoffman, 1989; Beach, 1991; Andrew, 1999; Inge, 1999).

The ultimate aim of this research is to develop Non-Uniform Rational B-spline (NURBS) skinning surface approach based on proposed hybrid parameterization method. This approach aims to fit a skinning surface of the ship hull surface that comprises a set of cross sectional curves share a common degree, possibly represented by different number of control points and defined over different knot vectors.

1.2 Rationale

Presently, various approaches have been applied on a set of control points to generate smooth, fair and accurate surface for the ship hull form. Although these methods are acceptable to be applied but there is one major problem that is a large number of control points is still required to represent the ship hull surface efficiently. Otherwise, more powerful computer is required to manipulate the surface in order to

achieve the desired accuracy. This is mainly due to the parameterization methods that may not in all cases suitable to be used to represent complex surfaces like ship hull form. Therefore there is need to improve the existing skinning surface methods in order to generate accurate, smooth and fair ship hull surface using an adequate minimum number of control points.

1.3 Background of the Study

Generally, the goal of surface reconstruction is to obtain a mathematical description that accurately and concisely represents a shape of physical surface. The description of the object surfaces plays a critical role in designing and manufacturing of automobile bodies, ship hull, aircraft, fuselages, wings, shoes, bottles and sculptures.

A surface may require to be reconstructed from many curves (cross sectional curves). These curves may be rational or non-rational, defined on different number of control points on different knot vectors. One problem that still being addressed is, how to construct a surface (skinning surface) that passes through all these curves that be able to represent the original shape accurately and efficiently with minimum number of control points. Many researchers have investigated the problem that related to the effectiveness of surface skinning method as summarized below.

Earlier Ball (1974, 1975, 1977) developed a computerized systems incorporated with lofting procedure known as CONSURF, which is based on rational cubic curves. Filip and Ball (1989) suggested a procedural method for lofting based on arbitrary parametric cross sectional curves. Also one way to solve this problem is to interpolate each row with B-spline curve, and pass a smooth surface through these curves using surface skinning method, which was approved by (Tiller, 1983; Woodward, 1987, 1988; Hohmeyer and Barsky, 1991). Though this method is acceptable solution, but the number of control points tends to be unreasonably large, especially when the numbers of rows (curves) are large.

Hoschek (1987), Hoschek and Lasser (1993), Patrikalakis (1989), and Wolter and Tuohy (1992), investigated the approximation of rational curves with non-rational B-splines. The algorithms produced inconsistent parameterization and a large number of control points when merging all knots. This is due to independent approximation of the cross sectional curves. Piegl and Tiller (1996) developed an algorithm to overcome the above problem.

Jeong *et al.*, (1999) proposed a B-spline method for smooth surface approximation to a set of 2D cross sections based on distance map to handle the branching problem through a set of intermediate cross sections. Although the generated surface is smooth and accurate, the surface is suboptimal and the control points are still large.

Cong and Parvin (2001) proposed a new approach for surface recovery from cross sections (planar contours) based on the equal importance criterion using the method of partial differential equation and smoothing the generated surface using Gaussian filter. Although the results showed the efficiency of the approach to construct surfaces ranging from simple to complex geometry, more accurate numerical methods can be incorporated to handle more accurate results and it is limitation to Voronoi Triangulation (VT).

Siddiqui and Sclaroff (2002) proposed an algorithm for reconstructing 3D rational B-spline surfaces from multiple views. Taking the advantages of rational B-spline, which are the property of invariant under affine map and local control property, the algorithm used to extract the 3D rational B-spline surface from 2D rational B-spline via bundle of procedures (e.g. factorization Methods, Sturm and Triggs, 1996). Although the approaches capable to extract the basic shape of the surfaces modeled as 3D rational B-spline, the visual quality of the reconstruction depends on how well features of the views were extracted, oscillation (smoothness) appears in places of surface when there were not enough or poorly tracked features.

Dmitrii and Ichiro, (2002) investigated reconstruction of surfaces from cross sections (planar contours) depending on minimal area as optimal criterion. The surface generated using arbitrary homotopic interpolation and straight-line homotopy

surface generation techniques with minimal area criterion leads to conclusion that in general case, surface reconstruction from cross sections are defective, bad-looking surfaces and hence unacceptable.

Piegl and Tiller (2002) developed a technique of surface skinning method, considering the problem on how to reduce the large number of control points remaining having a precise interpolation. The idea depends on how to give the knots some elasticity, so that each row (curve) can be interpolated with a few new knots added as possible. The result of applying the method shows that there is a growth in the number of the control points relevant to the original data curves, which is seem to be acceptable in their opinion due to the availability of computing standards.

According to Piegl and Tiller (2002), due to inconsistency in parameterization and use of the rational form of NURBS, the problem of skinning surface can be summarized as:

- The parameterizations of the cross sectional curves must be synchronous.
- The section curves must be placed in 3D coordinate with care.
- The cross sectional curves must continuous in 4D homogeneous.
- The weights must be chosen with extreme caution.

Nasri *et al.*, (2003) developed a skinning approach based on surface subdivision. However the method avoids the process of knots merging and gives efficient data reduction, it is produce three times more vertices than necessary if all of the cross section relatively having the same number of control points.

Hyungjun *et al.*, (2004) developed a new approach to generate skinning surface of cross sectional curves based on lofted B-spline surface interpolation using the universal parameterizations. Although the approach provide a compact representation beside preserving the surface shape, the approach fixed the number of

control points for each curve as the highest number of control points of any curve among the cross sectional curves, which may increase number of control points in the resultant skinning surface.

Skinning approach can be applied to generate variety of object with complicated surfaces. One of the surfaces, which will be dealt with in this research, is the ship hull surface and accordingly, the background below is followed.

Even the method of lofting/skinning, has been used many decades before the advent of computers, it is still widely used in shipbuilding, automotives, and aircrafts industries. One particular applications of skinning surface methods that will be dealt in this research work is the ship's hull surface reconstruction from several cross sectional curves. Ship hull form is a complex form due to its 3D non-linear curvature. In Practice, the hull form is represented in a set of lines or curves known as "*lines plan*" which consists of "*body plan, half breadth plan and Sheer Plan*". An original body plan of Fishing Vessel is shown in Figure 1.1.

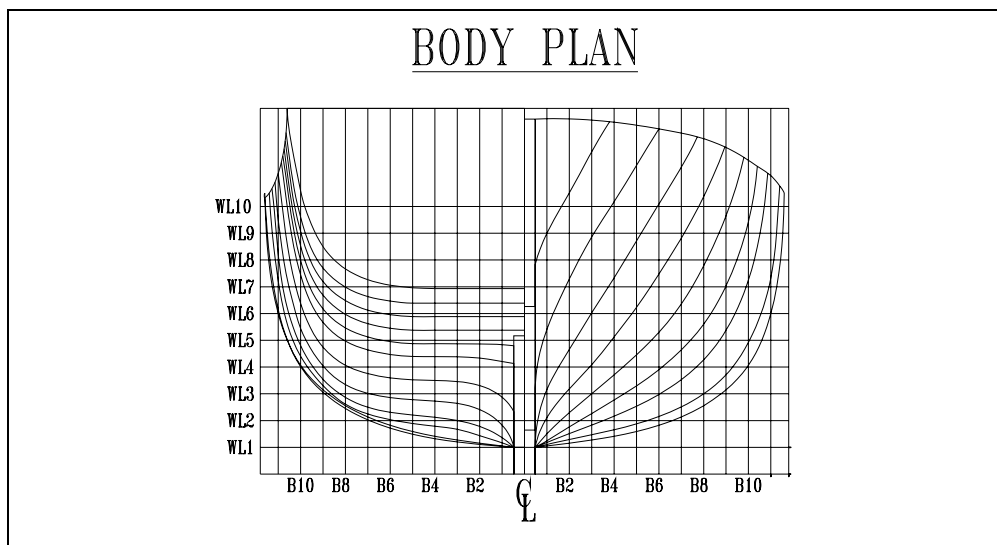


Figure 1.1 Original body plan drawing from fishing vessel

During the past decade several methods of fairing and generating lines for curves and surface for use in generating surface patches have been developed. All of these methods suffer to one extent or another from a number of difficulties as unwanted polynomial oscillation, limited continuity at the joins, limited local control,

inconvenient available for use in shaping the curve or surface by non-mathematical users, excessive computational requirements, excessive computer storage required to hold the curve, the necessity to break curves at sharp corner or knuckles and the necessity to represent curves in a piecewise manner (Rogers, 1977).

The effectiveness of any proposed mathematical approach, which serves the ship hull design task, is normally judged based on the following criteria (Yahya Samian, 1991):

- Capability of fitting the existing design and creating a novel design.
- Single equation: capable of representing each of the key profiles and the main ship curves in single equation without necessarily divided into several parts.
- Continuity, smoothness, and fairness. Both continuity and smoothness properties are normally determined by calculating the slope of the curve and consider smooth if the slope does not changed suddenly, whereas fairness property may be judged in various including:
 - a. Using visualization of each line or group of lines on the drawing board.
 - b. Kuo (1971) defines the fairness for the water-planes and the sectional curves as:
 - The first, second, and third difference should be smooth.
 - There should be no more than the required number of the point inflexion.
 - c. Izumida and Matida (1979) determined the surface fairness using Gaussian curvature.
 - d. Munchmeyer *et al.*, (1979) deal with fairness in two ways:

- Lines fairness criterion is based on upon the curvature of curve.
 - The surface fairness is measured using Euler's net.
-
- Interactive: to have a mathematical model that enables the designer to create, access and modify the design efficiently.
 - Global and local control property: global control is important criterion, it gives an intuitive feeling over the overall shape generated in relation with the input data, while local control allows the designer to modify the shape at the desired location without affecting the entire surface.
 - Non-mathematician: to have mathematical model that requires minimum level of mathematical understanding.

Even with the advancement of computer technology, there is a need to develop mathematical techniques for designing ship hull that needs low storage medium and performed in less execution time.

1.4 Problem Statement

The problem of surface reconstruction can be decomposed into smaller problems, which are dependent to each other. Questions like the following have to be solved:

1. How the present mathematical models should be improved in order to model the surface effectively and accurately?
2. How many points should be adequately used to represent the surface accurately?
3. How the point sets should be reduced efficiently without significant losses in the quality?

4. How does the model accurately fit into the control point?

In general, the goal of surface reconstruction is to obtain a mathematical description that accurately and effectively represents a shape of physical surface.

Having a ship hull comprises a set of cross sectional curves, each of which has different number of control points, different knot vectors. Extracting a hull surface from these curves yields a problem of making all these curves compatible i.e. defining on common knot vector, which leads to explosion in the number of control points due to the unavoidable process of merging the knot vectors to have common knot vector. Further more based on discussion with CAD designers (Formation Design System Pty Ltd – Maxsurf, 2004). For easy and fast design, the mathematical method is preferred to have the following criteria:

1. The less control points you use, the fairer the surface.
2. The higher the order (i.e. stiffness) of the surface, the fairer the surface.
3. The more evenly spaced and fair the control point net is, the fairer the surface.

Based on the above arguments, the research hypothesis can be stated as:

The NURB skinning surface can be constructed from these cross sectional curves within prespecified error tolerance (ϵ) using as minimum control points as possible while not sacrificing the surface shape to give an optimum approximation of surface that described by the cross sectional curves.

1.5 Objectives of Research

The objectives of this research can be enumerated as:

1. To develop hybrid parameterization method that inherits characteristics of different parameterization methods.
2. To develop 3D NURBS surface models with capability of generating surfaces from a set of cross sectional curves using reasonably low number of control points and knots sequence (common knot vector).
3. To propose the application of approximation error that measures the closeness of the approximant to the original shape.
4. To study the sensitivity of the prespecified error of the tolerance (ε) on data reduction without sacrificing the desired object shape via different values of (ε).
5. To compare the results of the proposed work with some of the existing methods.

1.6 Scope of research

The scope of this research covers the following:

1. The application of the proposed method is mainly focus on the ship hull in order to evaluate its effectiveness in generating the ship hull.
2. The proposed method will allow the manipulations of NURBS through curves/surfaces, knot refinement, knot removal in order to produce the skinned.
3. The proposed method is limited to open cross sectional curves.

1.7 Research Methodology

Having determined the research objectives and scope, the methodology that was adopted to carry out the research can be described as follows:

- Existing skinning methods and parameterization methods were studied and applied on various objects including ship hull.
- Preliminary results were carried out show that there is need to improve the existing methods for better results.
- Hybrid parameterization method was developed and based on this method, a hybrid skinning surface method was then developed.
- The developed hybrid skinning surface method was applied on variety of ship hull representing various types of ship hulls including simple and complex hull.
- The control points required to represent the ship hull was measured from lines plan (body plan, half breadth plan and sheer plan).
- The results achieved is verified and analyzed in different forms including, data reduction, surface generation, surface accuracy and the sensitivity of the error tolerance (ε). The comparison was also made against the conventional and the recent skinning methods.

The methodology of the research work is being defined through various steps. Initially, some of the existing methods including skinning surface methods and data parameterization methods have been implemented on various type objects including ship hull, the preliminary results show that the methods required large number of control points to generate accurate surfaces, which is time consuming. Therefore there is need to improved the existing methods for better results.

Also the preliminary results show that there some factors that influence the shape of the NURBS surface generated to represent any object. The main factors are data parameterization and knot vectors. Further more, as it stated by (Jung and Kim, 2001), the parameterization process needs to be improved since sometimes the existing parameterization methods are inadequate to represent accurate surface of the complex objects such as ship hull and automobile. This encouraged us to develop a new parameterization method that leads to improve the process of generating accurate surface that closed to the original object.

The proposed method involved various steps. The hybrid parameterization method is initially developed. Based on the hybrid parameterization method, a hybrid skinning surface method is also developed. The hybrid skinning surface method make used of the averaging parameter knot vector that generated using the hybrid parameterization method. For compatibility process the hybrid skinning surface method utilized both the knot removal operation within prespecified error tolerance (ε) to maintain the object shape, and the knot refinement operation is also utilized.

To test the significance of the developed hybrid skinning surface method in reducing the number of control points without sacrificing the original object shape, ship hulls that represent a variety of ship including simple and complex ship have been selected. The data collected for the selected ship hull was measured in terms of (x, y, z) from the lines plan (body plan, half breadth plan and sheer plan). The ship hull is divided into three regions, Frd (forward) region, Aft (After) region and middle region. However the method is mainly applied on the Frd and Aft regions of the ship hull due to their complexity in comparison to the middle region.

The results of the ship hull surfaces generated by developed hybrid skinning surface method have been verified and analyzed in various forms these include

1. How the hybrid skinning surface method is successfully and efficiently reduced the number of control points without sacrificing the original ship hull in comparison to the existing skinning methods.

2. Visualizing and rendering the surface of the ship hull generated to examine surface fairness and smoothness.
3. Check the accuracy of the ship hull surface generated using minimum number of cross sectional curves and compare to the accuracy of the ship hull generated using existing ship hull software design.
4. Analyzed the error tolerance (ϵ) and determine its range to control the data reduction of different region of the ship hull.

1.8 Organization of Thesis

This thesis is organized in the following manner.

Chapter 1 explains briefly the aim of the thesis and the rationale for the research work, back ground of the study, and statement of problem. Objectives and scope of the study are stated. The research methodology is also briefly described.

Chapter 2 presents the mathematical models for surface reconstruction that show how geometrically surface is represented using recent geometrical modeling techniques. The chapter begins with the general description of the mathematical models used to represent surfaces. Parametric representation includes Bezier, B-spline and NURBS curves/surfaces and their useful properties, is then discussed in detail. Detail discussion on data parameterization and NURBS useful operations such as influence of knots on curve shape, knot refinement and knot removal is also included.

Chapter 3 highlights the process skinning surface and discusses the present skinning ship hull surface design methods. The discussion describes how ship hull is organized in term of line plans, which can be described using skinning surface. The discussion covers studying and investigating present methods adopted for ship hull surface design and stating the need for development of Skinning Method for Ship

hull Design. Selected skinning surface methods developed by (Piegl and Tiller, 2000, 2002; Hyungjun *et al.*, 2004) together with most cited parameterization methods are also presented. The method used in the selected ship hull design software (Maxsurf and Prosurf) are also presented in the discussion.

Chapter 4 presents the proposed hybrid skinning method to solve the problem of skinning surface especially, ship hull surface. This chapter starts with defining the proposed skinning surface method, then describing the proposed hybrid parameterization method and approximation error that used to setup the weights of the control points. The integrated hybrid skinning surface method with some useful algorithms is also presented.

Chapter 5 presents and discusses the results of applying the proposed hybrid skinning surface method and compare with existing skinning surface methods. The results are presented in graphical form that includes the visualization (Wireframe surface) and rendering (solid surface) of the ship hull surfaces represented by the proposed method. The effectiveness of the error tolerance (ε) on reducing number of control points is demonstrated and analyzed via selected ships hull. The chapter also demonstrates the accuracy of the proposed method to represent the selected surfaces in comparison to the surface generated by selected ship hull software design.

Finally, the thesis summarizes the achievement in the research work, highlights the strength and weaknesses of the study. Possible future research directions and recommendations that can enhance the output of this research are also presented.

1.9 Summary

In this chapter presents, the need and rationale of the research work had been discussed. The need arose mainly due to the inadequacy of the present NURBS skinning method to represent the hull surface accurately and efficiently. Based on this need several issues that need to be addressed were then identified and hence the

objectives of the research work were presented. Finally, the research methodology that was adopted through out the research work was briefly discussed

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