DEVELOPMENT OF FINITE ELEMENT FORMULATION FOR PLATE BUCKLING STRUCTURE

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ABSTRACT

The analytical or exact mathematical formulation of stresses and displacements for plate buckling structure become impossible to develop if the plate geometry is so complicated. Numerical technique is one another approach to solve this problem and it is chosen in this study for plate structure under in-plane and out-plane load. The formulation of elastic stiffness matrix (ke) and geometric nonlinear stiffness matrix (kg) of the plate structure due to buckling is developed and based on virtual displacement principle. The geometric nonlinear stiffness matrix (kg) is found function of internal stresses. The direct iteration technique is applied in order to find nodal displacements. Under this technique, the Gauss points stresses are initialize as zeros, the kg matrix is updated, and then a new nodal displacement vector is found for the next approximation of internal stresses. Iterative process is done until convergence of displacement is satisfied. The rectangular plate with one fixed edge supported is used to test the proposed nonlinear formulation and procedure. The compressive in-plane load and moment is considered and applied for the tested plate. The plate is discretized with appropriate number of triangular finite element mesh. It is found that, the convergence of displacement is satisfied by using direct iteration technique. The load - deflection curve shows nonlinear relationship and approach to critical load. This finding shows that the direct iteration method can be accepted for the analyzing of plate buckling by considering geometric nonlinear assumption.

ABSTRAK

Formulasi analitik matematik bagi tegasan dan anjakan bagi struktur lengkokan plat adalah mustahil untuk dibangunkan jika geometri plat amat rumit. Teknik berangka adalah satu lagi pendekatan untuk menyelesaikan masalah ini dan ia dipilih dalam kajian ini untuk struktur plat dibebani secara sesatah dan luar satah. Pembentukan formulasi matrik kekukuhan elastik (ke) dan matrik kekukuhan tak linear geometrik (kg) bagi struktur plat disebabkan oleh lengkokan dibentuk berpandukan kepada prinsip anjakan maya. Bagi matrik kekukuhan tak linear geometrik (kg) ditemui fungsi kepada tegasan dalam. Teknik lelaran terus digunakan untuk mencari anjakan nod. Dalam teknik ini, tegasan pada titik Gauss dianggap sebagai sifar, matriks (kg) seterusnya dikemaskini dan seterusnya vektor anjakan nod baru didapati untuk anggaran tegasan dalam yang seterusnya. Proses lelaran dibuat sehingga penumpuan anjakan dicapai. Plat berbentuk segi empat tepat dengan satu sokong tepi terikat tegar digunakan untuk mengkaji formulasi tak linear dan prosedur yang dicadangkan. Bebanan sesatah dan momen dikenakan kepada plat yang diuji. Plat tersebut dipecahkan kepada bilangan unsur terhingga segitiga yang sesuai. Adalah ditemui bahawa, penumpuan anjakan berjaya disperolehi dengan menggunakan teknik lelaran terus. Lenkung beban-anjakan menunjukkan hubungan tak linear dan menumpu kepada beban kritikal. Penemuan ini menunjukkan bahawa kaedah lelaran terus boleh diterima bagi tujuan analisis lengkokan plat dengan mengambil kira andaian geomatri tak linear.

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LIST OF SYMBOLS AND ABBREVIATION

FEA	-	Finite Element Analysis
FEM	-	Finite Element Modelling
Lx, Ly	-	Dimension of Plate
Nx,Ny	-	In-plane compression Point Load
F	-	Out-plane load
x,y,z	-	Finite Element Coordinate
h	-	Thickness of Plate
v	-	Poisson's Ratio
E	-	Modulus of Elasticity
A	-	Cross sectional Area
I	-	Moment of inertia
u,v	-	In-plane displacement at node
w	-	Deflection –out-plane displacement at node
$\theta x, \theta y$	-	Rotation –out-plane displacement at node
ε	-	Strain
σ	-	Stress
ξ, η	-	Coordinate for Mapping using Parent Element
[N]	-	Shape function matrix
[q]	-	Bending displacement matrix
[qu]	-	In-plane displacement at x direction matrix
[qv]	-	In-plane displacement at y direction matrix

[D]	-	Strain-Stress Matrix
[Ke]	-	Stiffness Matrix first order analysis
[Kg]	-	Stiffness Matrix second order analysis
a,b,c,d	-	Gauss quadrature points
α	-	Area coordinate

CHAPTER 1

INTODUCTION

1.1 Background Of Study

Plate is two dimensional structure which its thickness *h* is very small compared to other dimension. Moreover, they can bond together either by straight or curved lines. Plate can serve either as complete structural like Slab Bridge or as structural component. one typical of structural components in engineering structures can be classified as plates is that the web of I beam (Szilard 2004).

Furthermore, due to its light weight, it results in lighter structures. That's why they have increased its popularity application in recent years. In this study, one of the practical applications of plates is the web of I beam which is susceptible to buckle. While a plate is subjected to in plane load, initial deformation occurs in-plane directions which called in plane displacement. But, increasing the in plane load leads to changing the behaviour of plate from flat state to curved state (transverse deformation). This load must be considered as load buckle which make the plate unstable, however, the plate can withstand larger load which called post buckling (Jones 2006).

In this present paper investigation of plate buckling was analyzed for combined in-plane bending and compression load by using finite element approach-virtual work. The behaviour of a plate buckling under in plane bending is quite similar to Euler approach of bar buckling (Jones 2006).

Generally, there are not many studies carried out about the behaviour of plate under compression load to satisfy a good level of safety because any changing of the web configuration lead to the failure. The majority of previous studies were presented rely on the differential equation approach. this study investigates analysis of plate buckling based on finite element method considering descend order analysis (Szilard2004).

A few numerical method investigations to obtain the critical buckling loads of isotropic plates under in-plane loads are presented in recent years. Also, Finite element was the best choice to analysis any type of structures and it gives a accurate result.

In the present work, a formulation for nonlinear geometric analysis of plate structure is develop and presented. The formulation is derived based onvirtual displacement principle. Based on sample plate structure, the analysis is conducted by applying direct iteration method. The plate structure is modelled by finite element method, stresses at Gauss quadrature points are initialize as zeros, the direct iteration method is applied until convergent of stresses is satisfied.

A quadrature triangular element is employed to study the buckling of a plate under in-plane compression load because triangular shape is useful with irregular shape of structures. Moreover, a cubic order polynomial shape function is used to interpolate the in-plane and out-plane displacements for better accuracy of results (Robert 2001).

Also, Gauss Quadrature used in generating element matrix and it will be applied to triangular element so that it is called quadrature triangular element. This methods locates sampling points and assigns weights so as to minimize integration error when integration is a general polyminal(Hutton 2004).

On other hand, MATLAB was used as a tool to analysis. MATLAB was developed by Math Works; MATLAB allows matrix manipulations, plotting of

functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages, including C, C++, and Fortran 32-Bit Math Works Products.

Moreover, MATLAB is very convenient for writing simple finite element programs. It provides the standard constructs, such as loops and conditionals and plotting results.

MATLAB was first adopted by researchers and practitioners in control engineering, Little's specialty, but quickly spread to many other domains. It is now also used in education, in particular the teaching of linear algebra and numerical analysis, and is popular amongst scientists involved in image processing.

1.2 Problem Statement

Generally, there are many methods to analyse any structural member such as beam or plate either by using differential equation or numerical methods, but considerations the stability of plate under in plane compression load are limited and the majority of previous studies were presented rely on the finite different approach.

1.3 Aim Of Study

The aim of this study is to consider second order effect of plate.

1.4 Objectives

The principle objectives of this study are:

1. To formulate stiffness matrix for plate buckling by using finite element approach. The finite element formulation is provided based on virtual displacement principle.
2. To verify the iteration process by using direct iteration technique based on proposed finite element formulation can achieve satisfied convergence.
3. To verify the critical load can be found.
4. To develop MATLAB software program to analyse and calculate stresses and displacement on the plate.

1.5 Scope and Limitation

The scope of this study is to develop and analysis web of I beam under in plane compression load which similar to rectangular thin plate. Also, it can not only apply to I beam but any application of thin plate under in compression load.

The scope of this study is based on:

- Static analysis.
- Elastic.
- Thin plate will be analysis based on Kirchhoff theory.
- In-plane uniform compression load.
- MATLAB Program.

1.6 Significant of Study

In the previous study, partial differential equation were used to formulate stresses, however, it is a quit complicated. Nowadays, by using modern computer, finite element approach is important choices to solve any problem. With this study it will create a new programme based on finite element and MATLAB software program.Another thing is that, this considers stability of the beam's web under in plane load and This case can prevent beam's web to buckle during life structures.

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